

Numerical differentiation of a polynomial

$$f(x) = 4x^4 + 8x^3 - x + 9$$

whose first derivative is

$$f'(x) = 16x^3 + 24x^2 - 1$$

Calculations

Step 1						Step 0.5						Step 0.1						
Exact						Exact						Exact						
x	y(xi)	y(xi+1)	dy/dx	dydx	Error %	x	y(xi)	y(xi+1)	dy/dx	dydx	Error %	x	y(xi)	y(xi+1)	dy/dx	dydx	Error %	
0	9	20	11	-1	1200	0	9	9.75	1.5	-1	250	0	9	8.9084	-0.916	-1	8.4	
1	20	135	115	39	194.8717	0.5	9.75	20	20.5	7	192.85714	0.1	8.9084	8.8704	-0.38	-0.744	48.924731	
2	135	546	411	223	84.30493	1	20	54.75	69.5	39	78.205128	0.2	8.8704	8.9484	0.78	0.088	786.36363	
3	546	1541	995	647	53.78670	1.5	54.75	135	160.5	107	50	0.3	8.9484	9.2144	2.66	1.592	67.085427	
4	1541	3504	1963	1407	39.51670	2	135	287.75	305.5	223	36.995515	0.4	9.2144	9.75	5.356	3.864	38.612836	
5	3504	6915	3411	2599	31.24278	2.5	287.75	546	516.5	399	29.448621	0.5	9.75	10.6464	8.964	7	28.057142	
6	6915	12350	5435	4319	25.83931	3	546	948.75	805.5	647	24.497681	0.6	10.6464	12.0044	13.58	11.096	22.386445	
7	12350	20481	8131	6663	22.03211	3.5	948.75	1541	1184.5	979	20.990806	0.7	12.0044	13.9344	19.3	16.248	18.783850	
8	20481	32076	11595	9727	19.20427	4	1541	2373.75	1665.5	1407	18.372423	0.8	13.9344	16.5564	26.22	22.552	16.264632	
9	32076	47999	15923	13607	17.02065	4.5	2373.75	3504	2260.5	1943	16.340710	0.9	16.5564	20	34.436	30.104	14.390114	
10	47999	69210	21211	18399	15.28343	5	3504	4994.75	2981.5	2599	14.717198	1	20	24.4044	44.044	39	12.933333	
11	69210	96765	27555	24199	13.86834	5.5	4994.75	6915	3840.5	3387	13.389430	1.1	24.4044	29.9184	55.14	49.336	11.764228	
12	96765	131816	35051	31103	12.69330	6	6915	9339.75	4849.5	4319	12.282935	1.2	29.9184	36.7004	67.82	61.208	10.802509	
13						6.5	9339.75	12350	6020.5	5407	11.346402	1.3	36.7004	44.9184	82.18	74.712	9.9957168	
						7	12350	16032.75	7365.5	6663	10.543298	1.4	44.9184	54.75	98.316	89.944	9.3080138	
						7.5	16032.75	20481	8896.5	8099	9.8468946	1.5	54.75	66.3824	116.324	107	8.7140186	
						8	20481	25793.75	10625.5	9727	9.2371748	1.6	66.3824	80.0124	136.3	125.976	8.1952117	
						8.5	25793.75	32076	12564.5	11559	8.6988493	1.7	80.0124	95.8464	158.34	146.968	7.7377388	
						9	32076	39438.75	14725.5	13607	8.2200338	1.8	95.8464	114.1004	182.54	170.072	7.3310127	
						9.5	39438.75	47999	17120.5	15883	7.7913492	1.9	114.100	135	208.996	195.384	6.9667935	
						10	47999	57879.75	19761.5	18399	7.4052937	2	135	158.7804	237.804	223	6.6385650	
						10.5	57879.75	69210	22660.5	21167	7.0557943	2.1	158.780	185.6864	269.06	253.016	6.3411009	
						11	69210	82124.75	25829.5	24199	6.7378817	2.2	185.686	215.9724	302.86	285.528	6.0701577	
						11.5	82124.75	96765	29280.5	27507	6.4474497	2.3	215.972	249.9024	339.3	320.632	5.8222510	
						12	96765	113277.75	33025.5	31103	6.1810757	2.4	249.902	287.75	378.476	358.424	5.5944914	
						12.5						2.5	287.75	329.7984	420.484	399	5.3844611	
												2.6	329.798	376.3404	465.42	442.456	5.1901206	
												2.7	376.340	427.6784	513.38	488.888	5.0097363	
												2.8	427.678	484.1244	564.46	538.392	4.8418252	
												2.9	484.124	546	618.756	591.064	4.6851102	
												3	546	613.6364	676.364	647	4.5384853	
												3.1	613.636	687.3744	737.38	706.296	4.4009876	
												3.2	687.374	767.5644	801.9	769.048	4.2717749	
												3.3	767.564	854.5664	870.02	835.352	4.1501067	
												3.4	854.566	948.75	941.836	905.304	4.0353295	
												3.5	948.75	1050.4944	1017.44	979	3.9268641	
												3.6	1050.49	1160.1884	1096.94	1056.536	3.8241952	
												3.7	1160.18	1278.2304	1180.42	1138.008	3.7268630	
												3.8	1278.23	1405.0284	1267.98	1223.512	3.6344555	
												3.9	1405.02	1541	1359.71	1313.144	3.5466026	
												4	1541	1686.5724	1455.72	1407	3.4629708	
												4.1	1686.57	1842.1824	1556.1	1505.176	3.3832588	
												4.2	1842.18	2008.2764	1660.94	1607.768	3.3071935	
												4.3	2008.27	2185.3104	1770.34	1714.872	3.2345271	
												4.4	2185.31	2373.75	1884.39	1826.584	3.1650337	
												4.5						