Discussion of propeller-assisted straight-downwind land sailing faster than the wind

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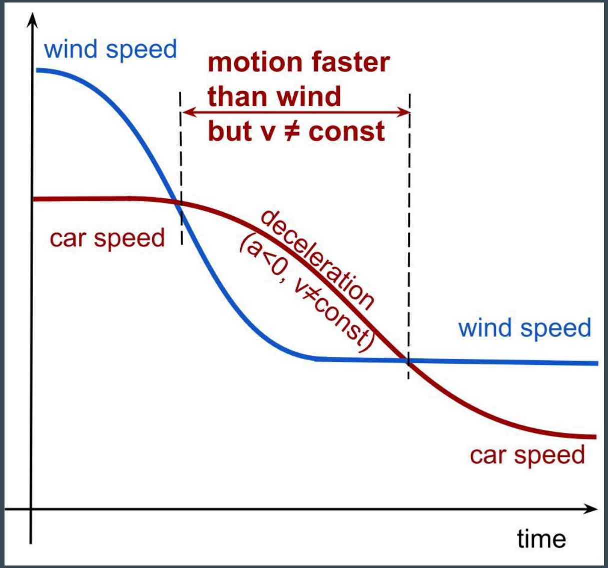
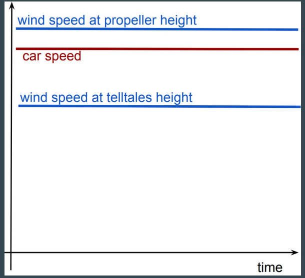
Why the Veritasium video was inconclusive

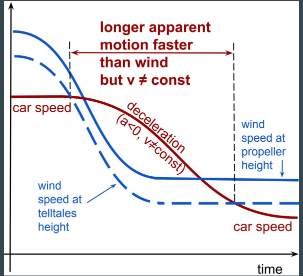
The Veritasium video:
https://youtu.be/jyQwgBAaBag

- In a **variable** wind, the car accelerates during a gust, then moves by inertia and decelerates when the wind decreases. For a limited time, the car can move faster than the wind, while its acceleration is negative (**a < 0, v ≠ const**).

- The wind at the level of the propeller can be faster than the wind at the level of the telltales by about ~10-15%.

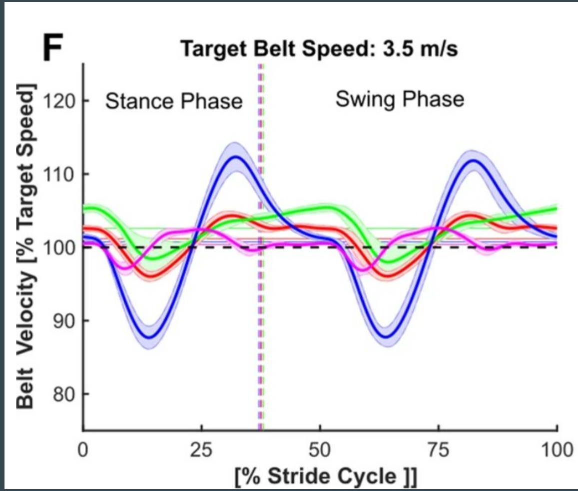






Problems with the treadmill experiments

- Standing waves and running waves on the belt (next slide).
- The level accuracy is limited. The level is attached to the frame, but the belt can be inclined with respect to the frame. The belt may not be flat: the belts sag between the supports.
- Treadmill speed fluctuates, affecting the car's speed.
- The human adjusting the car with a spork can (subconsciously) bias the motion in favor of the desired outcome.
- Videos of the models advancing on the treadmill often show the forward speed which is small in comparison with the belt speed. The slow drift suggests that the model might be driven by some small effects, such as a small incline, vibration, waves of excitation...



Willwacher et al, https://www.nature.com/articles/s41598-021-81951-9

Effects of vibration (for small or zero thrust)

Vibrations, averaged over time scales >> period, contribute to the effective potential energy.

Example: inverted pendulum (link)

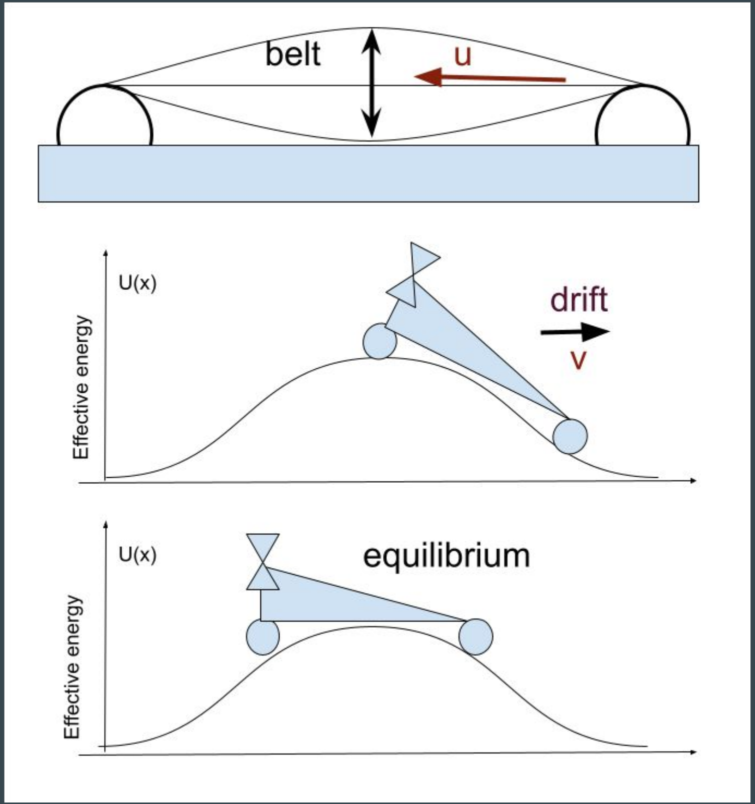
In the case of a standing wave,

$$y = A\sin(\pi x/L)\cos(\omega t)$$

$$U(x) \propto \langle \dot{y}^2 \rangle = \frac{A^2}{2}\omega^2 \sin^2(\pi x/L)$$

The forward **drift** (0:00-0:53) and **equilibrium** (from 0:53) could be due to vibrations: inconclusive

https://youtu.be/7xL8gRJ5F6k



If it's a "fan", not "wind turbine" (powered by wheels alone)

In the ground frame of reference, energy conservation implies that the power delivered to the propeller equals the power expanded by the propeller:

$$\eta F_{\text{friction}} v = P_{\text{air}} + P_{\text{car}} = F_{\text{thrust}} \frac{u_1 + u_2}{2} + F_{\text{thrust}} \cdot v$$

Solve for the thrust:

$$F_{\text{thrust}} = \eta F_{\text{friction}} \frac{1}{1 + \frac{u_1 + u_2}{2v}} < \eta F_{\text{friction}} < F_{\text{friction}}$$

Newton's 2nd Law:

$$a = \frac{F_{\text{net}}}{M} = (F_{\text{thrust}} - F_{\text{friction}} - F_{\text{air drag}})/M < 0$$

$$a < 0$$

negative

However, there is additional power from a “wind turbine”

When the car is moving faster than the wind, the passing air pushes the propeller in the same direction as the wheels push it. This has been a subject of discussion, and Blackbird has a ratchet to prevent the propeller from actively spinning the wheels, but the ratchet does not keep the propeller’s wind power from spinning the propeller itself, adding the torque in the same direction as the wheels.

When the car moves faster than both the air and the ground, the two media drive the propeller together. The total power delivered to the propeller is:

This alters the energy budget and allows sailing faster than the wind.

$$P_{\text{prop}} = \eta F_{\text{friction}} v + P_{\text{turbine}}$$

$$P_{\text{turbine}} \approx \lambda \times \left(v - \frac{u_1 + u_2}{2}\right)^{\beta}$$

The added power from a “wind turbine” increases the thrust

$$F_{\text{thrust}} = \eta F_{\text{friction}} \frac{1}{1+\frac{u}{v}} + \boxed{\frac{P_{\text{turbine}}}{v+u}} = \eta F_{\text{friction}} \frac{1}{1+\frac{u}{v}} + \boxed{\lambda \frac{v-u}{v+u}}$$

$$(\text{for} \quad \beta = 1, \mathrm{u} = \frac{\mathrm{u}_1 + \mathrm{u}_2}{2})$$

- It is possible for the thrust to exceed the friction. Therefore, it is possible for the vehicle to move faster than the wind.
- The success depends on λ and η, which are determined by the propeller and the drivetrain design.
- The positive contribution starts at speeds v>u, but u is the average of the air speeds in front and behind the propeller, while the wind speed is the greater of the two. So, if the tailwind can push the car close to the wind speed, the car can be in the v>u regime necessary for the acceleration..

Comments on some other solutions

- M. Drela (unpublished) starts with an ill-defined equation that uses the propeller efficiency, leading to an undefined limit for v->u. While the propeller efficiency vanishes for zero wind speed, it is because of how "useful work" is defined in various sources on prop efficiency. Usually, it is the power that advances the airplane to a destination that is called "useful", and that is why (useful_power)=(thrust)*(flying_speed). This vanishes for zero speed not because the propeller is somehow "inefficient", but because the plane is not flying anywhere (speed=0). In fact, the thrust generated by the propeller is close to maximal at zero speed (see Fig. 2 of http://www.epi-eng.com/propeller_technology/selecting_a_propeller.htm) The equation quoted by Drela eventually for the thrust is not derived from the equations using the efficiencies.

- It is possible and useful to consider the problem in the moving frame of reference, in which case one should not forget to include the work of friction done on the ground (transferring the momentum to the Earth). Some solutions found online do neglect this contribution, leading to unphysical results.

The wager

Technically, my wager with Derek Muller stated as part of the claim that *“the propeller works like a fan rather than a wind turbine”*, which is incorrect. The propeller acts as both a fan and a turbine, and the power is contributed by both moving media in a somewhat symmetrical manner. Also, the video falls short of providing evidence that the motion faster than the wind occurs with a non-negative acceleration and that it is not caused by the wind variability.

However, I would not attempt to haggle over a technicality. I have stated that I deserve to win only if I am right in all of my claims, which was not the case.
(Professors don’t deserve a partial credit.)

Therefore, I concede the wager.

I thank all the people who looked at this problem and contributed to the discussion.