Measures of Dispersion

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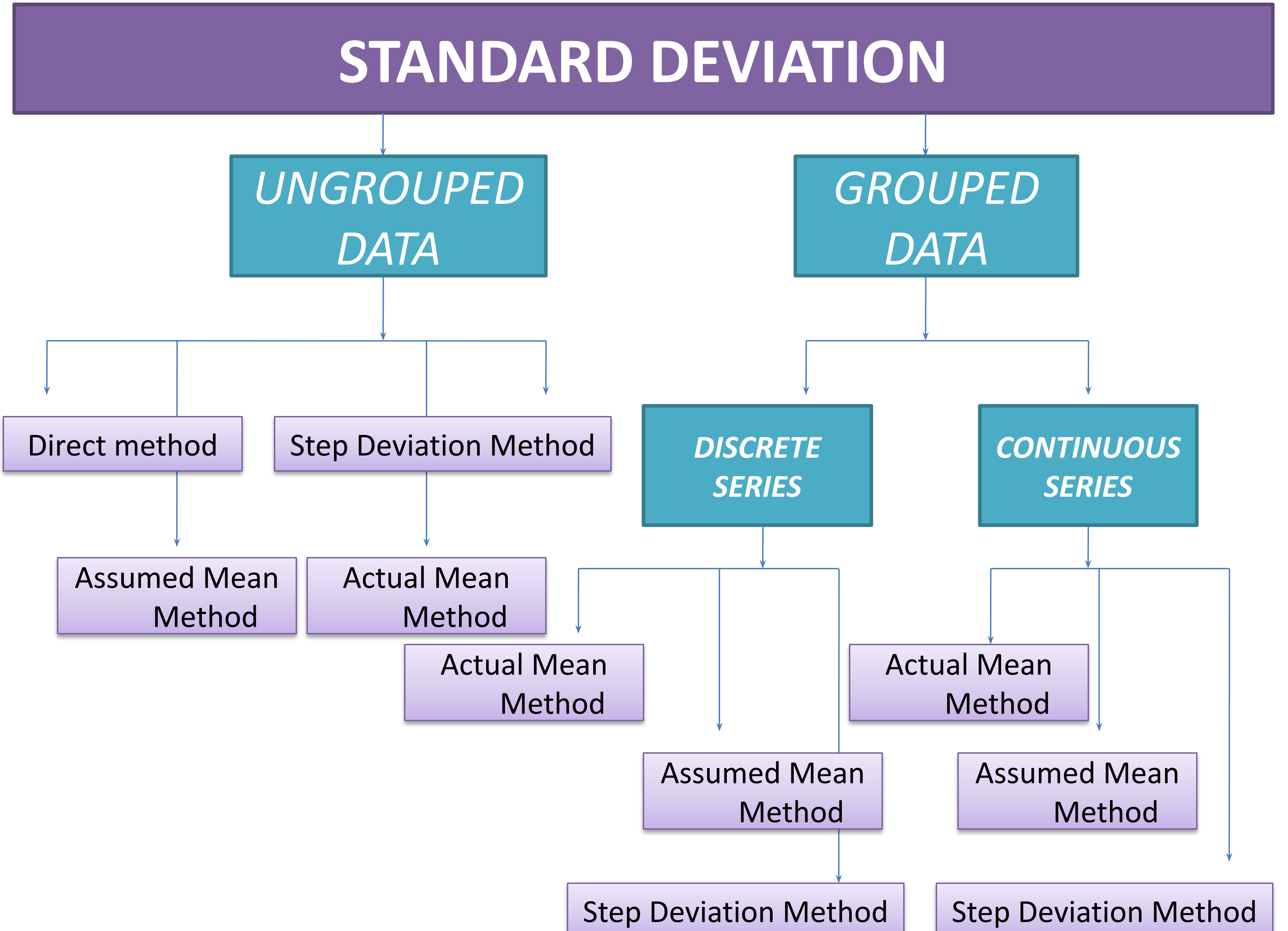
Standard Deviation

- Standard Deviation is the positive square root of the mean of squared deviations from mean. So if there are five values x1, x2, x3, x4 and x5, first their mean is calculated. Then deviations of the values from mean are calculated. These deviations are then squared. The mean of these squared deviations is the variance. Positive square root of the *variance is the* standard deviation. **(Note that Standard Deviation is calculated on the basis of the mean only).**


STANDARD DEVIATION
UNGROUPED DATA
GROUPED DATA
Direct method
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DISCRETE SERIES
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Assumed Mean Method
Actual Mean Method
Actual Mean Method
Actual Mean Method
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Assumed Mean Method
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Step Deviation Method

Calculation of Standard Deviation for ungrouped data

- Four alternative methods are available for the calculation of standard deviation of individual values. All these methods result in the same value of standard deviation. These are:
  - Actual Mean Method
  - Assumed Mean Method
  - Direct Method
  - Step-Deviation Method

Actual Mean Method

- Suppose you have to calculate the standard deviation of the following values:
- 5, 10, 25, 30, 50

X		
5		
10		
25		
30		
50		

Actual Mean Method

- Suppose you have to calculate the standard deviation of the following values:
- 5, 10, 25, 30, 50

X		
5		
10		
25		
30		
50		

120/5 = 24

Actual Mean Method

- Suppose you have to calculate the standard deviation of the following values:
- 5, 10, 25, 30, 50

X	**d (X-X)**	
5	-19	
10	-14	
25	+1	
30	+6	
50	+26	
	0	

Actual Mean Method

- Suppose you have to calculate the standard deviation of the following values:
- 5, 10, 25, 30, 50

X	**d (X-X)**	**d^2**
5	-19	361
10	-14	196
25	+1	1
30	+6	36
50	+26	676
	0	

Actual Mean Method

- Suppose you have to calculate the standard deviation of the following values:
- 5, 10, 25, 30, 50

X	**d (X-X)**	**d^2**
5	-19	361
10	-14	196
25	+1	1
30	+6	36
50	+26	676
	0	**1270**

Actual Mean Method

- Suppose you have to calculate the standard deviation of the following values:
- 5, 10, 25, 30, 50

X	d (X-X)	d^2
5	-19	361
10	-14	196
25	+1	1
30	+6	36
50	+26	676
	0	**1270**

$$\sigma = \sqrt{\frac{\Sigma d^2}{n}}$$

Assumed Mean Method

- For the same values, deviations may be calculated from any arbitrary value A x such that d = X – A x . Taking A x = 25, the computation of the standard deviation is shown below:

X		
5		
10		
25		
30		
50		

Assumed Mean Method

- For the same values, deviations may be calculated from any arbitrary value A x such that d = X – A x . Taking A x = 25, the computation of the standard deviation is shown below:

X	d(X-X)	
5	-20	
10	-15	
25	0	
30	+5	
50	+25	

Assumed Mean Method

- For the same values, deviations may be calculated from any arbitrary value A x such that d = X – A x . Taking A x = 25, the computation of the standard deviation is shown below:

X	**d(X-X)**	
5	-20	
10	-15	
25	0	
30	+5	
50	+25	
	-5	

Assumed Mean Method

- For the same values, deviations may be calculated from any arbitrary value A x such that d = X – A x . Taking A x = 25, the computation of the standard deviation is shown below:

X	**d(X-X)**	**d^2**
5	-20	400
10	-15	225
25	0	0
30	+5	25
50	+25	625
	-5	**1275**

Assumed Mean Method

- For the same values, deviations may be calculated from any arbitrary value A x such that d = X – A x . Taking A x = 25, the computation of the standard deviation is shown below:

X	**d(X-X)**	**d^2**
5	-20	400
10	-15	225
25	0	0
30	+5	25
50	+25	625
	-5	**1275**

Formula for Standard Deviation

$$\sigma = \sqrt{\frac{\Sigma d^2}{n} - \left(\frac{\Sigma d}{n}\right)^2}$$

Direct Method

- Standard Deviation can also be calculated from the values directly, i.e., without taking deviations, as shown below:

X	
5	
10	
25	
30	
50	

Direct Method

- Standard Deviation can also be calculated from the values directly, i.e., without taking deviations, as shown below:

X	
5	
10	
25	
30	
50	
120	

Direct Method

- Standard Deviation can also be calculated from the values directly, i.e., without taking deviations, as shown below:

X	**X^2**
5	25
10	100
25	625
30	900
50	2500
120	

Direct Method

- Standard Deviation can also be calculated from the values directly, i.e., without taking deviations, as shown below:

X	X^2
5	25
10	100
25	625
30	900
50	2500
120	**4150**

Direct Method

- Standard Deviation can also be calculated from the values directly, i.e., without taking deviations, as shown below:

X	**X^2**
5	25
10	100
25	625
30	900
50	2500
120	**4150**

Following formula is used.

$$\sigma = \sqrt{\frac{\Sigma x^2}{n} - (\bar{x})^2}$$

Step-deviation Method

- If the values are divisible by a common factor, they can be so divided and standard deviation can be calculated from the resultant values as follows:

X			
5			
10			
25			
30			
50			

Step-deviation Method

- If the values are divisible by a common factor, they can be so divided and standard deviation can be calculated from the resultant values as follows:

X	X′		
5	1		
10	2		
25	5		
30	6		
50	10		

Step-deviation Method

- If the values are divisible by a common factor, they can be so divided and standard deviation can be calculated from the resultant values as follows:

X	**X′**		
5	1		
10	2		
25	5		
30	6		
50	10		
	24		

Step-deviation Method

- If the values are divisible by a common factor, they can be so divided and standard deviation can be calculated from the resultant values as follows:

X	**X′**	**d (X′ – Mean X′)**	
5	1	-3.8	
10	2	-2.8	
25	5	0.2	
30	6	+1.2	
50	10	+5.2	
	24		

Step-deviation Method

- If the values are divisible by a common factor, they can be so divided and standard deviation can be calculated from the resultant values as follows:

X	**X'**	**d (X' – Mean X')**	**d^2**
5	1	-3.8	14.44
10	2	-2.8	7.84
25	5	0.2	0.04
30	6	+1.2	1.44
50	10	+5.2	27.04
	24		

Step-deviation Method

- If the values are divisible by a common factor, they can be so divided and standard deviation can be calculated from the resultant values as follows:

X	**X′**	**d (X′ – Mean X′)**	**d^2**
5	1	-3.8	14.44
10	2	-2.8	7.84
25	5	0.2	0.04
30	6	+1.2	1.44
50	10	+5.2	27.04
	24		**50.80**

Step-deviation Method

- If the values are divisible by a common factor, they can be so divided and standard deviation can be calculated from the resultant values as follows:

X	X'	d (X' – Mean X')	d^2
5	1	-3.8	14.44
10	2	-2.8	7.84
25	5	0.2	0.04
30	6	+1.2	1.44
50	10	+5.2	27.04
	24		**50.80**

$$\sigma = \sqrt{\frac{\Sigma d^2}{n}} \times c$$

Standard Deviation in Discrete Series (Actual Mean Method):

x	**f**				
3.5	3				
4.5	7				
5.5	22				
6.5	60				
7.5	85				
8.5	32				
9.5	8				
	N=217				

Standard Deviation in Discrete Series (Actual Mean Method):

x	f	fx			
3.5	3	10.5			
4.5	7	31.5			
5.5	22	121			
6.5	60	390			
7.5	85	637.5			
8.5	32	272			
9.5	8	76			
	N=217				

Standard Deviation in Discrete Series (Actual Mean Method):

x	**f**	**fx**			
3.5	3	10.5			
4.5	7	31.5			
5.5	22	121			
6.5	60	390			
7.5	85	637.5			
8.5	32	272			
9.5	8	76			
	N=217	**1538.5**			

Standard Deviation in Discrete Series (Actual Mean Method):

x	f	fx	d		
3.5	3	10.5	-3.58		
4.5	7	31.5	-2.58		
5.5	22	121	-1.58		
6.5	60	390	-0.58		
7.5	85	637.5	+0.41		
8.5	32	272	+1.41		
9.5	8	76	+2.41		
	N=217	**1538.5**			

Standard Deviation in Discrete Series (Actual Mean Method):

x	**f**	**fx**	**d**	**d^2**	
3.5	3	10.5	-3.58	12.81	
4.5	7	31.5	-2.58	6.65	
5.5	22	121	-1.58	2.49	
6.5	60	390	-0.58	0.34	
7.5	85	637.5	+0.41	0.16	
8.5	32	272	+1.41	1.99	
9.5	8	76	+2.41	5.80	
	N=217	**1538.5**			

Standard Deviation in Discrete Series (Actual Mean Method):

x	**f**	**fx**	**d**	**d^2**	**fd^2**
3.5	3	10.5	-3.58	12.81	38.43
4.5	7	31.5	-2.58	6.65	46.55
5.5	22	121	-1.58	2.49	54.78
6.5	60	390	-0.58	0.34	20.40
7.5	85	637.5	+0.41	0.16	13.60
8.5	32	272	+1.41	1.99	63.68
9.5	8	76	+2.41	5.80	46.46
	N=217	**1538.5**			

Standard Deviation in Discrete Series (Actual Mean Method):

x	f	fx	d	d^2	fd^2
3.5	3	10.5	-3.58	12.81	38.43
4.5	7	31.5	-2.58	6.65	46.55
5.5	22	121	-1.58	2.49	54.78
6.5	60	390	-0.58	0.34	20.40
7.5	85	637.5	+0.41	0.16	13.60
8.5	32	272	+1.41	1.99	63.68
9.5	8	76	+2.41	5.80	46.46
	N=217	**1538.5**			**$\sum fd^2 = 283.9$**

Standard Deviation in Discrete Series (Actual Mean Method):

x	f	fx	d	d^2	fd^2
3.5	3	10.5	-3.58	12.81	38.43
4.5	7	31.5	-2.58	6.65	46.55
5.5	22	121	-1.58	2.49	54.78
6.5	60	390	-0.58	0.34	20.40
7.5	85	637.5	+0.41	0.16	13.60
8.5	32	272	+1.41	1.99	63.68
9.5	8	76	+2.41	5.80	46.46
	N=217	**1538.5**			**$\sum fd^2$ = 283.9**

$$\sigma = \sqrt{\frac{\Sigma fd^2}{n}}$$

Standard Deviation in Discrete Series (Assumed Mean Method):

x	**f**				
3.5	3				
4.5	7				
5.5	22				
6.5	60				
7.5	85				
8.5	32				
9.5	8				
	N=217				

Standard Deviation in Discrete Series (Assumed Mean Method):

x	f	d (X-Mean)			
3.5	3	-3			
4.5	7	-2			
5.5	22	-1			
6.5	60	0			
7.5	85	+1			
8.5	32	+2			
9.5	8	+3			
	N=217				

Standard Deviation in Discrete Series (Assumed Mean Method):

x	**f**	**d (X-Mean)**	**d^2**		
3.5	3	-3	9		
4.5	7	-2	4		
5.5	22	-1	1		
6.5	60	0	0		
7.5	85	+1	1		
8.5	32	+2	4		
9.5	8	+3	9		
	N=217				

Standard Deviation in Discrete Series (Assumed Mean Method):

x	**f**	**d (X-Mean)**	**d^2**	**fd**	
3.5	3	-3	9	-9	
4.5	7	-2	4	-14	
5.5	22	-1	1	-22	
6.5	60	0	0	0	
7.5	85	+1	1	+85	
8.5	32	+2	4	+64	
9.5	8	+3	9	+24	
	N=217			**$\sum$fd = +128**	

Standard Deviation in Discrete Series (Assumed Mean Method):

x	**f**	**d (X-Mean)**	**d^2**	**fd**	**fd^2**
3.5	3	-3	9	-9	27
4.5	7	-2	4	-14	28
5.5	22	-1	1	-22	22
6.5	60	0	0	0	0
7.5	85	+1	1	+85	85
8.5	32	+2	4	+64	128
9.5	8	+3	9	+24	72
	N=217			**$\sum fd = +128$**	

Standard Deviation in Discrete Series (Assumed Mean Method):

x	f	d (X-Mean)	d^2	fd	fd^2
3.5	3	-3	9	-9	27
4.5	7	-2	4	-14	28
5.5	22	-1	1	-22	22
6.5	60	0	0	0	0
7.5	85	+1	1	+85	85
8.5	32	+2	4	+64	128
9.5	8	+3	9	+24	72
	N=217			**$\sum fd = +128$**	**$\sum fd^2 = 362$**

Standard Deviation in Discrete Series (Assumed Mean Method):

x	f	d (X-Mean)	d^2	fd	fd^2
3.5	3	-3	9	-9	27
4.5	7	-2	4	-14	28
5.5	22	-1	1	-22	22
6.5	60	0	0	0	0
7.5	85	+1	1	+85	85
8.5	32	+2	4	+64	128
9.5	8	+3	9	+24	72
	N=217			**$\sum fd = +128$**	**$\sum fd^2 = 362$**

$$\sigma = \sqrt{\frac{\Sigma fd^2}{n} - \left(\frac{\Sigma fd}{n}\right)^2}$$

Standard Deviation in Discrete Series (Step Deviation Method):

x	**f**					
45	3					
50	5					
55	8					
60	7					
65	9					
70	7					
75	4					
80	7					
	N=50					

Standard Deviation in Discrete Series (Step Deviation Method):

x	**f**	**d {X-Mean (60)}**				
45	3	-15				
50	5	-10				
55	8	-5				
60	7	0				
65	9	5				
70	7	10				
75	4	15				
80	7	20				
	N=50					

Standard Deviation in Discrete Series (Step Deviation Method):

x	f	d {X-Mean (60)}	d’ (d/c), c=5			
45	3	-15	-3			
50	5	-10	-2			
55	8	-5	-1			
60	7	0	0			
65	9	5	+1			
70	7	10	+2			
75	4	15	+3			
80	7	20	+4			
	N=50					

Standard Deviation in Discrete Series (Step Deviation Method):

x	f	d {X-Mean (60)}	d' (d/c), c=5	d'^2		
45	3	-15	-3	9		
50	5	-10	-2	4		
55	8	-5	-1	1		
60	7	0	0	0		
65	9	5	+1	1		
70	7	10	+2	4		
75	4	15	+3	9		
80	7	20	+4	16		
	N=50					

Standard Deviation in Discrete Series (Step Deviation Method):

x	**f**	**d {X-Mean (60)}**	**d' (d/c), c=5**	**d'^{2}**	**fd'**	
45	3	-15	-3	9	-9	
50	5	-10	-2	4	-10	
55	8	-5	-1	1	-8	
60	7	0	0	0	0	
65	9	5	+1	1	+9	
70	7	10	+2	4	+14	
75	4	15	+3	9	+12	
80	7	20	+4	16	+28	
	N=50					

Standard Deviation in Discrete Series (Step Deviation Method):

x	**f**	**d {X-Mean (60)}**	**d' (d/c), c=5**	**d'^2**	**fd'**	
45	3	-15	-3	9	-9	
50	5	-10	-2	4	-10	
55	8	-5	-1	1	-8	
60	7	0	0	0	0	
65	9	5	+1	1	+9	
70	7	10	+2	4	+14	
75	4	15	+3	9	+12	
80	7	20	+4	16	+28	
	N=50				**$\sum fd' = 36$**	

Standard Deviation in Discrete Series (Step Deviation Method):

x	f	d {X-Mean (60)}	d’ (d/c), c=5	d'^2	fd′	fd'^2
45	3	-15	-3	9	-9	27
50	5	-10	-2	4	-10	20
55	8	-5	-1	1	-8	8
60	7	0	0	0	0	0
65	9	5	+1	1	+9	9
70	7	10	+2	4	+14	28
75	4	15	+3	9	+12	36
80	7	20	+4	16	+28	112
	N=50				**∑fd’ = 36**	

Standard Deviation in Discrete Series (Step Deviation Method):

x	f	d {X-Mean (60)}	d' (d/c), c=5	d'^2	fd'	fd'^2
45	3	-15	-3	9	-9	27
50	5	-10	-2	4	-10	20
55	8	-5	-1	1	-8	8
60	7	0	0	0	0	0
65	9	5	+1	1	+9	9
70	7	10	+2	4	+14	28
75	4	15	+3	9	+12	36
80	7	20	+4	16	+28	112
	N=50				$\sum fd' = 36$	$\sum fd'^2 = 240$

Standard Deviation in Discrete Series (Step Deviation Method):

x	f	d {X-Mean (60)}	d' (d/c), c=5	d'^2	fd'	fd'^2
45	3	-15	-3	9	-9	27
50	5	-10	-2	4	-10	20
55	8	-5	-1	1	-8	8
60	7	0	0	0	0	0
65	9	5	+1	1	+9	9
70	7	10	+2	4	+14	28
75	4	15	+3	9	+12	36
80	7	20	+4	16	+28	112
	N=50				**$\sum fd' = 36$**	**$\sum fd'^2 = 240$**

$$\sigma = \sqrt{\frac{\Sigma fd'^2}{\Sigma f} - \left(\frac{\Sigma fd'}{\Sigma f}\right)^2} \times c$$

Standard Deviation in Continuous frequency distribution:

- Like ungrouped data, S.D. can be calculated for grouped data by any of the following methods:
  - Actual Mean Method
  - Assumed Mean Method
  - Step-Deviation Method

Actual Mean Method

Following steps are required:

- Calculate the mean of the distribution.

  mean = $\Sigma fm / \Sigma f$ = 1620/40 = 40.5
- Calculate deviations of mid-values from the mean so that

  d = m – mean.
- Multiply the deviations with their corresponding frequencies to get ‘fd’ values.
- Calculate ‘fd^2’ values by multiplying ‘fd’ values with ‘d’ values. Sum up these to get Σfd^2.
- Apply the formula.

CI	f	m	fm	d	d^2	fd^2
10 – 20	5	15	75	-25.5	650.25	3251.25
20 – 30	8	25	200	-15.5	240.25	1922.00
30 – 50	16	40	640	-0.5	0.25	4.00
50 – 70	8	60	480	+19.5	380.25	3042.00
70 – 80	3	75	225	+34.5	1190.25	3570.75
	40		**1620**			**11790**

$$\sigma = \sqrt{\frac{\Sigma fd^2}{n}} = \sqrt{\frac{11790}{40}} = 17.168$$

Assumed Mean Method

- For the values in earlier example, standard deviation can be calculated by taking deviations from an assumed mean (say 40) as follows:
- The following steps are required:
- Calculate mid-points of classes
- Calculate deviations of mid-points from an assumed mean such that d = m – Assumed mean. Assumed Mean = 40.
- Multiply values of ‘d’ with corresponding frequencies to get ‘fd’ values. (note that the total of this column is not zero since deviations have been taken from assumed mean).
- Multiply ‘fd’ values with ‘d’ values to get fd^2 values. Find fd^2.
- Standard Deviation can be calculated by formula.

CI	f	m	d	fd	fd^2
10 – 20	5	15	-25	-125	3125
20 – 30	8	25	-15	-120	1800
30 – 50	16	40	0	0	0
50 – 70	8	60	+20	160	3200
70 – 80	3	75	+35	105	3675
	40			**20**	**11800**

$$\sigma = \sqrt{\frac{\Sigma fd^2}{n} - \left(\frac{\Sigma fd}{n}\right)^2}$$

$$\text{or } \sigma = \sqrt{\frac{11800}{40} - \left(\frac{20}{40}\right)^2}$$

$$\text{or } \sigma = \sqrt{294.75} = 17.168$$

Step-deviation Method

- In case the values of deviations are divisible by a common factor, the calculations can be simplified by the step-deviation method.
- *Steps required:*
- Calculate class mid-points (Col. 3) and deviations from an arbitrarily chosen value, just like in the assumed mean method. In this example, deviations have been taken from the value 40. (Col. 4)
- Divide the deviations by a common factor denoted as ‘c’. c = 5 in the above example. The values so obtained are ‘d'’ values (Col. 5).
- Multiply ‘d'’ values with corresponding ‘f'’ values (Col. 2) to obtain ‘fd'’ values (Col. 6).
- Multiply ‘fd'’ values with ‘d'’ values to get ‘fd'2’ values (Col. 7)
- Sum up values in Col. 6 and Col. 7 to get S fd' and S fd'2 values.
- Apply the following formula.

C	f	m	d	d'	d'^2	fd'	fd'^2
10 – 20	5	15	-25	-2.5	6.25	-12.5	31.25
20 – 30	8	25	-15	-1.5	2.25	-12	18
30 – 50	16	40	0	0	0	0	0
50 – 70	8	60	+20	2	4	16	32
70 – 80	3	75	+35	3.5	12.25	10.5	36.75
	40					**2**	**118**

$$\sigma = \sqrt{\frac{\Sigma fd'^2}{\Sigma f} - \left(\frac{\Sigma fd'}{\Sigma f}\right)^2} \times c$$