Conditional Probability



DSC 40A



If we choose a random avocado from the set of 100, what is the probability that the avocado is soft given that it is green?

If we choose a random avocado from the set of 100, what is the probability that the avocado is soft given that it is green?

We can't solve this problem directly because we need to find more information about the conditional probabilities. For example, we could solve P(Green) easily because there are 75 green avocados out of 100 total which makes it

 $\mathsf{P}(\mathsf{Green}) = \frac{75}{100}$

If we choose a random avocado from the set of 100, what is the probability that the avocado is soft given that it is green?

We can't solve this problem directly because we need to find more information about the conditional probabilities. For example, we could solve P(Green) easily because there are 75 green avocados out of 100 total which makes it

$$\mathsf{P}\big(\mathsf{Green}\big) = \frac{75}{100}$$

We need to use a different method to solve the conditional probability problem like using a Venn diagram.

If we choose a random avocado from the set of 100, what is the probability that the avocado is soft given that it is green?

First we can fill in a Venn diagram of the number of avocados that are green, soft, both, and neither.

Let events G be for green avocados and S be for soft avocados.



If we choose a random avocado from the set of 100, what is the probability that the avocado is soft given that it is green?

First we can fill in a Venn diagram of the number of avocados that are green, soft, both, and neither.

Let events G be for green avocados and S be for soft avocados.



10 avocados are neither green nor soft.

If we choose a random avocado from the set of 100, what is the probability that the avocado is soft given that it is green?

First we can fill in a Venn diagram of the number of avocados that are green, soft, both, and neither.

Let events G be for green avocados and S be for soft avocados.



10 avocados are neither green nor soft.

$$|\mathsf{G}^{\mathsf{C}} \cap \mathsf{S}^{\mathsf{C}}| = 10$$

If we choose a random avocado from the set of 100, what is the probability that the avocado is soft given that it is green?

First we can fill in a Venn diagram of the number of avocados that are green, soft, both, and neither.

Let events G be for green avocados and S be for soft avocados.



10 avocados are neither green nor soft.

Since there are 100 avocados total, we need to sort out the other 90 which are either green or soft. First we need to find the number that are green and soft.

If we choose a random avocado from the set of 100, what is the probability that the avocado is soft given that it is green?

First we can fill in a Venn diagram of the number of avocados that are green, soft, both, and neither.

Let events G be for green avocados and S be for soft avocados.



We need to find how many are green and soft. We can use the addition rule:

$$|\mathsf{G} \cup \mathsf{S}| = |\mathsf{G}| + |\mathsf{S}| - |\mathsf{G} \cap \mathsf{S}|$$

If we choose a random avocado from the set of 100, what is the probability that the avocado is soft given that it is green?

First we can fill in a Venn diagram of the number of avocados that are green, soft, both, and neither.

Let events G be for green avocados and S be for soft avocados.



We need to find how many are green and soft. We can use the addition rule:

$$|G \cup S| = |G| + |S| - |G \cap S|$$

90 = 75 + 40 - |G \cap S|
|G \cap S| = 25

If we choose a random avocado from the set of 100, what is the probability that the avocado is soft given that it is green?

First we can fill in a Venn diagram of the number of avocados that are green, soft, both, and neither.

Let events G be for green avocados and S be for soft avocados.



We need to find how many are green and soft. We can use the addition rule:

$$|G \cup S| = |G| + |S| - |G \cap S|$$

90 = 75 + 40 - |G \cap S|
|G \cap S| = 25

25 avocados are both green and soft.

If we choose a random avocado from the set of 100, what is the probability that the avocado is soft given that it is green?

First we can fill in a Venn diagram of the number of avocados that are green, soft, both, and neither.

Let events G be for green avocados and S be for soft avocados.



We need to find how many are green and soft. We can use the addition rule:

$$G \cup S| = |G| + |S| - |G \cap S|$$

$$90 = 75 + 40 - |G \cap S|$$

$$|G \cap S| = 25$$

25 avocados are both green and soft.

If we choose a random avocado from the set of 100, what is the probability that the avocado is soft given that it is green?

First we can fill in a Venn diagram of the number of avocados that are green, soft, both, and neither.

Let events G be for green avocados and S be for soft avocados.



To find the number that are green and not soft, we can subtract the number of green and soft from total green.

If we choose a random avocado from the set of 100, what is the probability that the avocado is soft given that it is green?

First we can fill in a Venn diagram of the number of avocados that are green, soft, both, and neither.

Let events G be for green avocados and S be for soft avocados.



To find the number that are green and not soft, we can subtract the number of green and soft from total green.

75 green - 25 green and soft = 50 only green

50 are only green.

If we choose a random avocado from the set of 100, what is the probability that the avocado is soft given that it is green?

First we can fill in a Venn diagram of the number of avocados that are green, soft, both, and neither.

Let events G be for green avocados and S be for soft avocados.



If we choose a random avocado from the set of 100, what is the probability that the avocado is soft given that it is green?

First we can fill in a Venn diagram of the number of avocados that are green, soft, both, and neither.

Let events G be for green avocados and S be for soft avocados.



To find the number that are soft and not green, we can subtract the number of green and soft from total soft.

If we choose a random avocado from the set of 100, what is the probability that the avocado is soft given that it is green?

First we can fill in a Venn diagram of the number of avocados that are green, soft, both, and neither.

Let events G be for green avocados and S be for soft avocados.



To find the number that are soft and not green, we can subtract the number of green and soft from total soft.

40 soft - 25 green and soft = 15 only soft

15 are only soft.

If we choose a random avocado from the set of 100, what is the probability that the avocado is soft given that it is green?

First we can fill in a Venn diagram of the number of avocados that are green, soft, both, and neither.

Let events G be for green avocados and S be for soft avocados.



If we choose a random avocado from the set of 100, what is the probability that the avocado is soft given that it is green?

Now we can find this probability. Since we are given that the avocado is green, we can just look at that part of the Venn diagram.



If we choose a random avocado from the set of 100, what is the probability that the avocado is soft given that it is green?

Now we can find this probability. Since we are given that the avocado is green, we can just look at that part of the Venn diagram.



In this section we can see that 25 avocados are soft. In total there are 75 avocados.

If we choose a random avocado from the set of 100, what is the probability that the avocado is soft given that it is green?

Now we can find this probability. Since we are given that the avocado is green, we can just look at that part of the Venn diagram.



In this section we can see that 25 avocados are soft. In total there are 75 avocados.

```
Therefore,
```

```
\mathsf{P}(\mathsf{S}|\mathsf{G}) = \frac{25}{75}
```

If we choose a random avocado from the set of 100, what is the probability that the avocado is soft given that it is green?

Now we can find this probability. Since we are given that the avocado is green, we can just look at that part of the Venn diagram.



In this section we can see that 25 avocados are soft. In total there are 75 avocados.

Therefore,

$$\mathsf{P}(\mathsf{S}|\mathsf{G}) = \frac{25}{75}$$

We have found the probability that the avocado is soft given it is green.

We can also use a tree in this situation.



We can also use a tree in this situation.

75 avocados are green so 25 are not. The probabilities of green and not green are:



We can also use a tree in this situation.

75 avocados are green so 25 are not. The probabilities of green and not green are:

 $P(G) = \frac{75}{100} P(G^{C}) = \frac{25}{100}$



We can also use a tree in this situation.

75 avocados are green so 25 are not. The probabilities of green and not green are:

$$P(G) = \frac{75}{100} P(G^{C}) = \frac{25}{100}$$

We draw the tree like this:





We can also use a tree in this situation.

Of the 75 green avocados we see that 25 are soft and 50 are not in the Venn diagram.





We can also use a tree in this situation.

Of the 75 green avocados we see that 25 are soft and 50 are not in the Venn diagram.

 $P(S|G) = \frac{25}{75} P(S^{C}|G) = \frac{50}{75}$





We can also use a tree in this situation.

Of the 75 green avocados we see that 25 are soft and 50 are not in the Venn diagram.



 $\mathsf{P}(\mathsf{S}|\mathsf{G}) = \frac{25}{75} \quad \mathsf{P}(\mathsf{S}^{\mathsf{C}}|\mathsf{G}) = \frac{50}{75}$



We can also use a tree in this situation.

Of the 25 not green avocados we see that 15 are soft and 10 are not in the Venn diagram.





We can also use a tree in this situation.

 $\mathsf{P}(\mathsf{S}|\mathsf{G}^{\mathsf{C}}) = \frac{15}{25} \quad \mathsf{P}(\mathsf{S}^{\mathsf{C}}|\mathsf{G}^{\mathsf{C}}) = \frac{10}{25}$

Of the 25 not green avocados we see that 15 are soft and 10 are not in the Venn diagram.





We can also use a tree in this situation.

 $\mathsf{P}(\mathsf{S}|\mathsf{G}^{\mathsf{C}}) = \frac{15}{25} \quad \mathsf{P}(\mathsf{S}^{\mathsf{C}}|\mathsf{G}^{\mathsf{C}}) = \frac{10}{25}$

Of the 25 not green avocados we see that 15 are soft and 10 are not in the Venn diagram.





We can also use a tree in this situation.



If we choose a random avocado from the set of 100, what is the probability that the avocado is soft given that it is not green?



If we choose a random avocado from the set of 100, what is the probability that the avocado is soft given that it is not green?

First we go down the branch of not green.



If we choose a random avocado from the set of 100, what is the probability that the avocado is soft given that it is not green?

First we go down the branch of not green.

Next we go down the branch of soft.



If we choose a random avocado from the set of 100, what is the probability that the avocado is soft given that it is not green?

First we go down the branch of not green.

Next we go down the branch of soft.



If we choose a random avocado from the set of 100, what is the probability that the avocado is soft given that it is not green? First we go down the branch of not green.

Next we go down the branch of soft.



If we choose a random avocado from the set of 100, what is the probability that the avocado is soft given that it is not green? First we go down the branch of not green.

Next we go down the branch of soft.



Now we take the probability of this branch.

 $\mathsf{P}(\mathsf{S}|\mathsf{G}^{\mathsf{C}}) = \frac{15}{25}$

This is how we use a tree to find a conditional probability.

If we choose a random avocado from the set of 100, what is the probability that the avocado is green and not soft?

If we choose a random avocado from the set of 100, what is the probability that the avocado is green and not soft?



If we choose a random avocado from the set of 100, what is the probability that the avocado is green and not soft?

First we go down the branch of green.



If we choose a random avocado from the set of 100, what is the probability that the avocado is green and not soft?

First we go down the branch of green.

Next we go down the branch of not soft.



If we choose a random avocado from the set of 100, what is the probability that the avocado is green and not soft?

First we go down the branch of green.

Next we go down the branch of not soft.



We can use the multiplication rule.

 $\mathsf{P}(\mathsf{G} \cap \mathsf{S}^{\mathsf{C}}) = \mathsf{P}(\mathsf{G}) \cdot \mathsf{P}(\mathsf{S}^{\mathsf{C}}|\mathsf{G})$

If we choose a random avocado from the set of 100, what is the probability that the avocado is green and not soft?

First we go down the branch of green.

Next we go down the branch of not soft.



We can use the multiplication rule.

 $P(G \cap S^{C}) = P(G) \cdot P(S^{C}|G)$

We find the probability of green with this branch.

 $\mathsf{P}(\mathsf{G}) = \frac{75}{100}$

If we choose a random avocado from the set of 100, what is the probability that the avocado is green and not soft?

First we go down the branch of green.

Next we go down the branch of not soft.



We can use the multiplication rule.

 $\mathsf{P}(\mathsf{G} \cap \mathsf{S}^{\mathsf{C}}) = \mathsf{P}(\mathsf{G}) \cdot \mathsf{P}(\mathsf{S}^{\mathsf{C}}|\mathsf{G})$

We find the probability of green with this branch.

 $\mathsf{P}(\mathsf{G}) = \frac{75}{100}$

We find the probability of not soft given green with this branch.

 $\mathsf{P}(\mathsf{S}^{\mathsf{C}}|\mathsf{G}) = \frac{50}{75}$

If we choose a random avocado from the set of 100, what is the probability that the avocado is green and not soft?

First we go down the branch of green.

Next we go down the branch of not soft.



We can use the multiplication rule.

$$P(G \cap S^{C}) = P(G) \cdot P(S^{C}|G)$$
$$P(G) = \frac{75}{100} \quad P(S^{C}|G) = \frac{50}{75}$$

If we choose a random avocado from the set of 100, what is the probability that the avocado is green and not soft?

First we go down the branch of green.

Next we go down the branch of not soft.



We can use the multiplication rule.

$$P(G \cap S^{C}) = P(G) \cdot P(S^{C}|G)$$
$$P(G) = \frac{75}{100} \quad P(S^{C}|G) = \frac{50}{75}$$
$$P(G \cap S^{C}) = \frac{75}{100} \cdot \frac{50}{75}$$
$$P(G \cap S^{C}) = \frac{50}{100}$$

If we choose a random avocado from the set of 100, what is the probability that the avocado is green and not soft?

When you take a single branch's probability, it is the probability that event happens given that the entire path before it happened.



If we choose a random avocado from the set of 100, what is the probability that the avocado is green and not soft?

When you take a single branch's probability, it is the probability that event happens given that the entire path before it happened.





If we choose a random avocado from the set of 100, what is the probability that the avocado is green and not soft?



When you take the product of probabilities of an entire path, it is the probability of the intersection of all events on the path, or the probability that every event on the path happens given no assumptions about events on that path.

If we choose a random avocado from the set of 100, what is the probability that the avocado is green and not soft?



When you take the product of probabilities of an entire path, it is the probability of the intersection of all events on the path, or the probability that every event on the path happens given no assumptions about events on that path.

The probability at this path is the probability that the avocado is not soft and green.

If we choose a random avocado from the set of 100, what is the probability that the avocado is not green given it is soft?

If we choose a random avocado from the set of 100, what is the probability that the avocado is not green given it is soft?

Since we know the probability that an avocado is soft given not green, the probability of not green, and the probability of soft, we can use Bayes' Theorem.

$$P(G^{C}|S) = \frac{P(S|G^{C})P(G^{C})}{P(S)}$$

$$P = \frac{75}{100} \qquad Green \qquad P = \frac{25}{75} \qquad Soft$$

$$P = \frac{75}{100} \qquad Soft$$

$$P = \frac{15}{25} \qquad Soft$$

$$P = \frac{15}{25} \qquad Soft$$

$$P = \frac{10}{25} \qquad Not$$

$$P = \frac{10}{25} \qquad Not$$

$$P = \frac{10}{25} \qquad Not$$

$$Soft$$

If we choose a random avocado from the set of 100, what is the probability that the avocado is not green given it is soft?

Since we know the probability that an avocado is soft given not green, the probability of not green, and the probability of soft, we can use Bayes' Theorem.

40

100

$$P(G^{C}|S) = \frac{P(S|G^{C})P(G^{C})}{P(S)} \qquad P(S|G^{C}) = \frac{15}{25} \qquad P(G^{C}) = \frac{25}{100} \qquad P(S) = \frac{15}{100} \qquad P(S) = \frac{15}$$



C> / C>

If we choose a random avocado from the set of 100, what is the probability that the avocado is not green given it is soft?

Since we know the probability that an avocado is soft given not green, the probability of not green, and the probability of soft, we can use Bayes' Theorem.

$$P(G^{C}|S) = \frac{P(S|G^{C})P(G^{C})}{P(S)}$$

$$P(S|G^{C}) = \frac{15}{25} \quad P(G^{C}) = \frac{25}{100} \quad P(S) = \frac{40}{100}$$

$$P(S|G^{C}) = \frac{15}{25} \cdot \frac{25}{100} = \frac{15}{40}$$

C) (C)

 $\mathsf{P} = \frac{15}{25}$

 $P = \frac{10}{25}$

Soft

Not Soft

Not

Green

This is how we can use Bayes' Theorem to find conditional probabilities.



Other Resources



Here are some other resources for conditional probabilities:

- DSC 40A Probability Roadmap
- <u>Visual Example</u>
- <u>Previous Lecture Video</u>