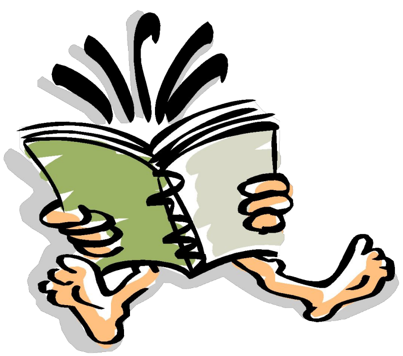
Merge Sort

Week 4 Lesson 2



Sorting

- Insertion sort
  - Design approach: incremental
  - Sorts in place: Yes
  - Best case: $\Theta(n)$
  - Worst case: $\Theta(n^2)$

- Bubble Sort
  - Design approach: incremental
  - Sorts in place: Yes
  - Running time: $\Theta(n^2)$

Sorting

- Selection sort
  - Design approach: incremental
  - Sorts in place: Yes
  - Running time: $\Theta(n^2)$
- Merge Sort
  - Design approach: divide and conquer
  - Sorts in place: No
  - Running time: *Let's see!!*

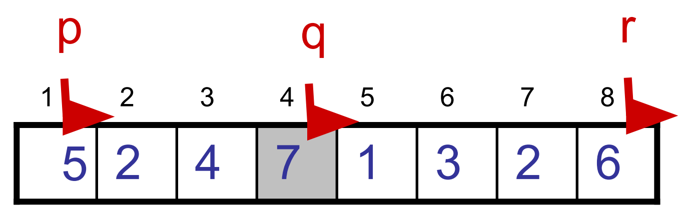
Divide-and-Conquer

- **Divide** the problem into a number of sub-problems
  - Similar sub-problems of smaller size
- **Conquer** the sub-problems
  - Solve the sub-problems recursively
  - Sub-problem size small enough $\Rightarrow$ solve the problems in straightforward manner
- **Combine** the solutions of the sub-problems
  - Obtain the solution for the original problem

Merge Sort Approach

- To sort an array $A[p \,..\, r]$:
- **Divide**
  - Divide the n-element sequence to be sorted into two subsequences of $n/2$ elements each
- **Conquer**
  - Sort the subsequences recursively using merge sort
  - When the size of the sequences is 1 there is nothing more to do
- **Combine**
  - Merge the two sorted subsequences

Merge Sort



Alg.: MERGE-SORT(*A*, p, r)

if p < r ▷ Check for base case

then *q* ← ⌊(p + r)/2⌋ ▷ Divide

MERGE-SORT(*A*, p, *q*) ▷ Conquer

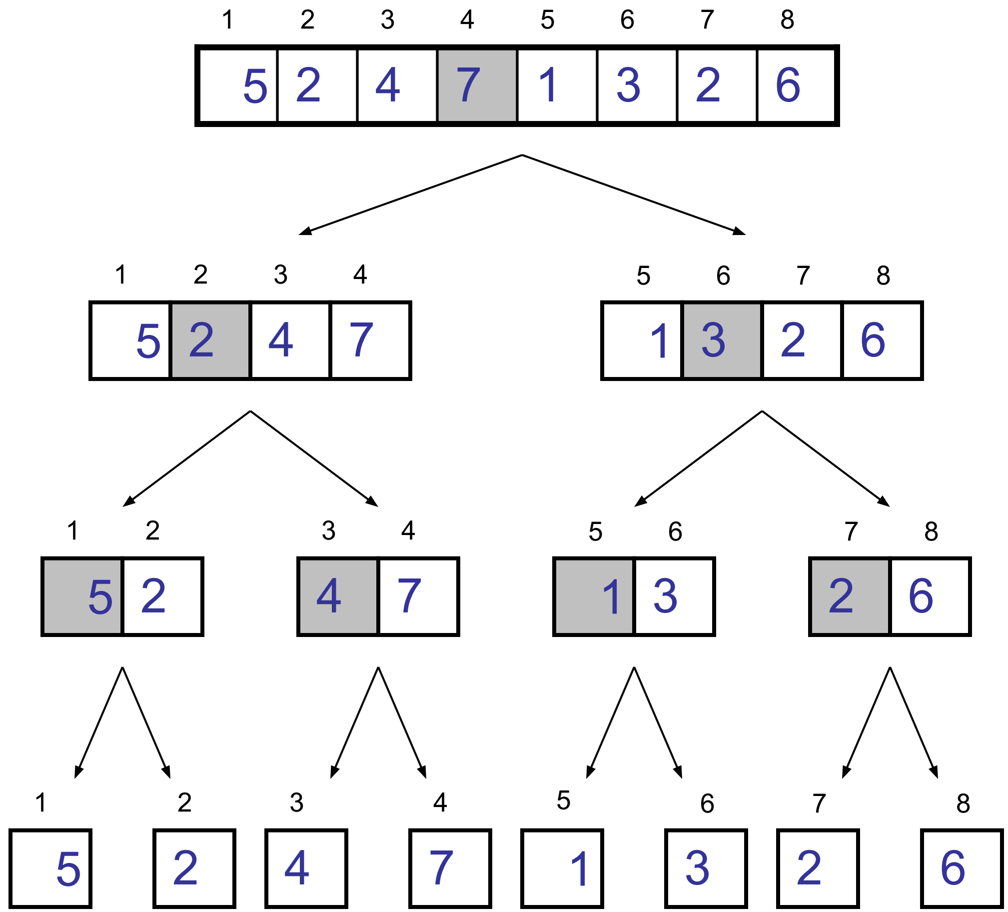
MERGE-SORT(*A*, *q* + 1, r) ▷ Conquer

MERGE(*A*, p, *q*, r) ▷ Combine

- Initial call: MERGE-SORT(*A*, 1, *n*)

Example – n Power of 2

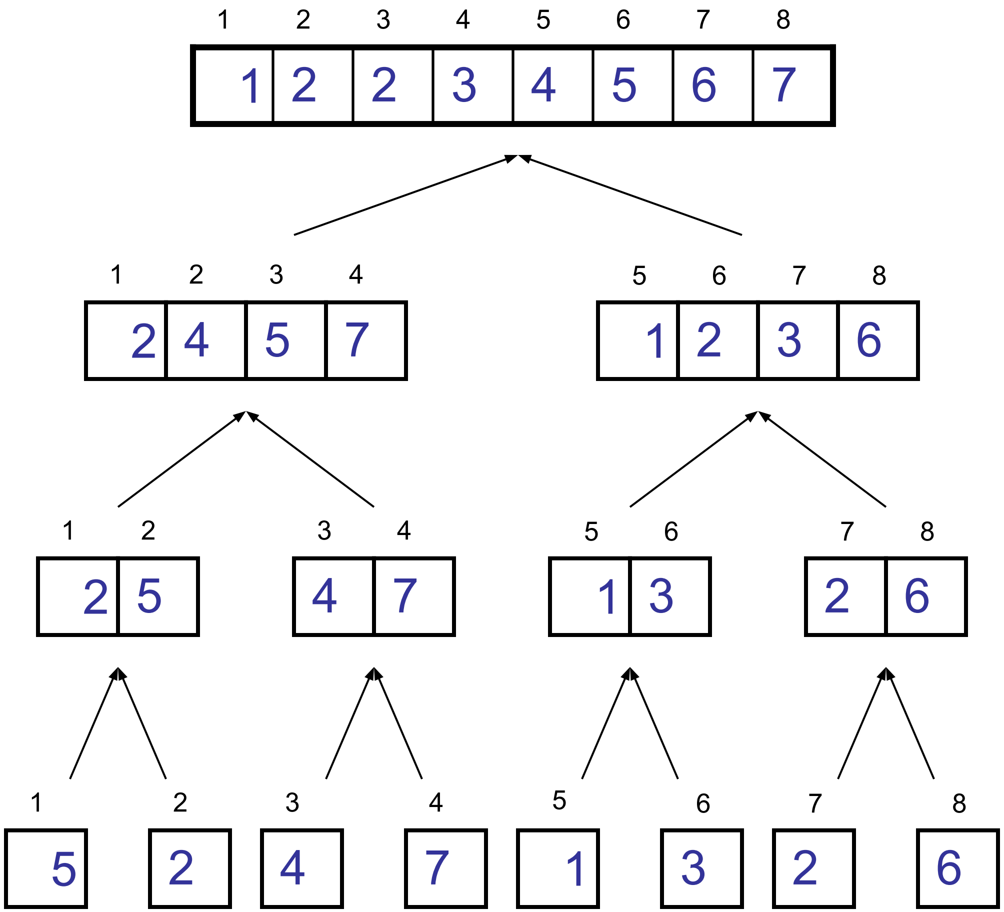
Divide



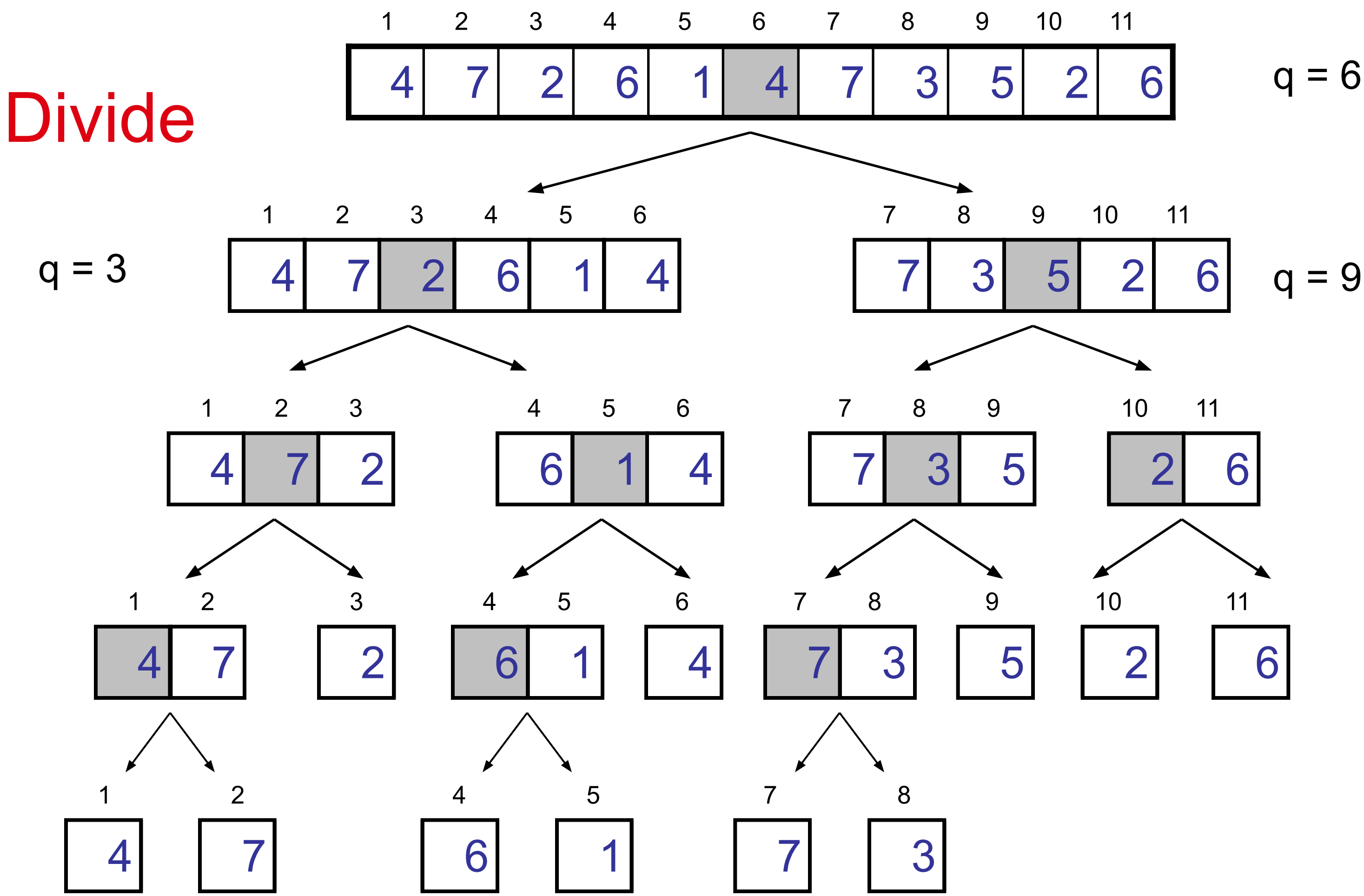
q = 4

Example – n Power of 2

Conquer and Merge

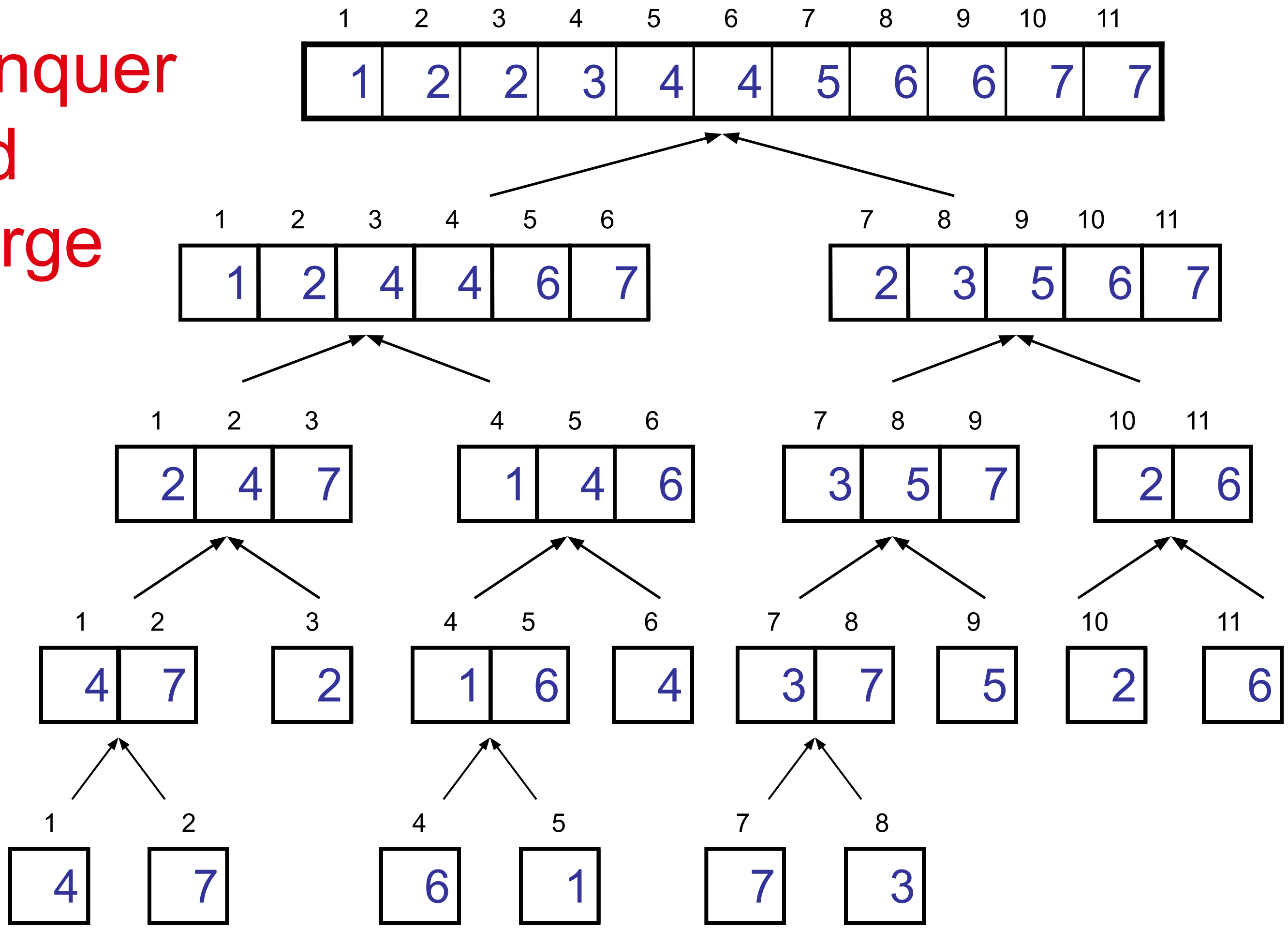


Example – n Not a Power of 2

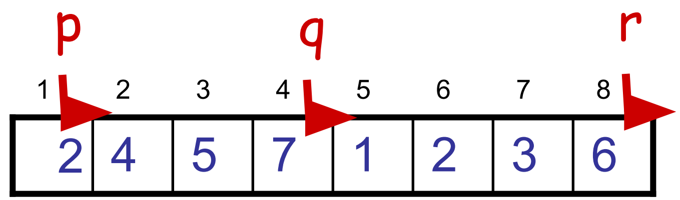


Example – *n* Not a Power of 2

Conquer and Merge



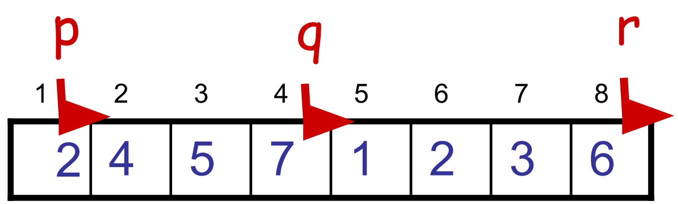
Merging



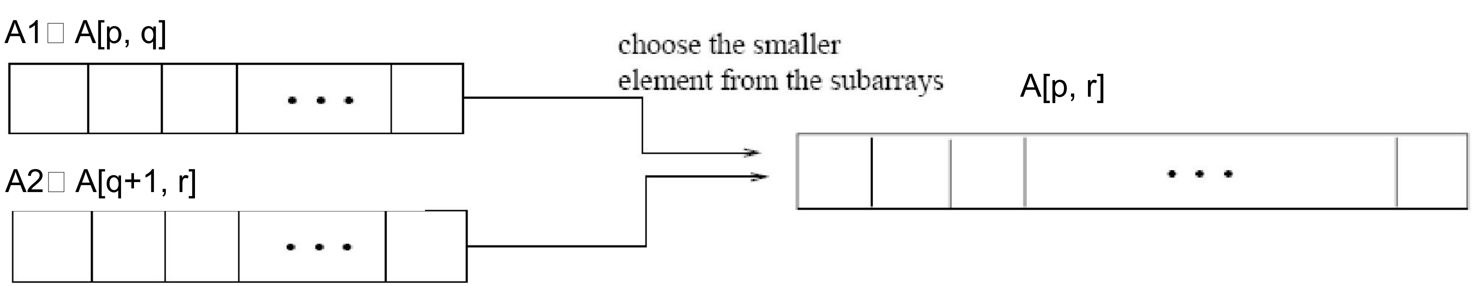
- **Input:** Array *A* and indices p, q, r such that p ≤ q < r
  - Subarrays *A*[p . . q] and *A*[q + 1 . . r] are sorted
- **Output:** One single sorted subarray *A*[p . . r]

Merging

- Idea for merging:



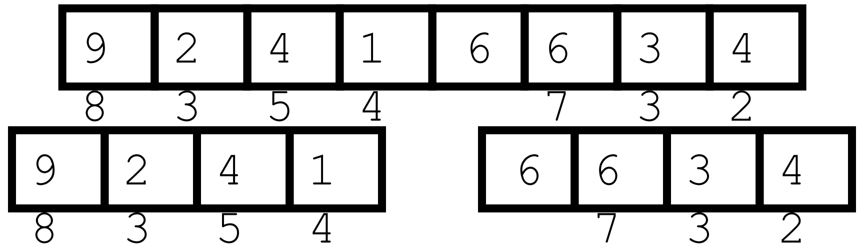
  – Two piles of sorted cards
    - Choose the smaller of the two top cards
    - Remove it and place it in the output pile
  – Repeat the process until one pile is empty
  – Take the remaining input pile and place it face-down onto the output pile



Merge Sort (recursive simulation)

98	23	45	14	6	67	33	42

Merge Sort (recursive simulation)



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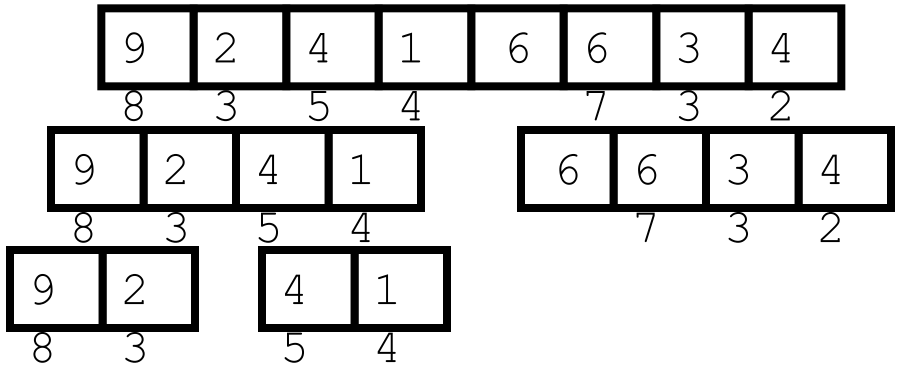

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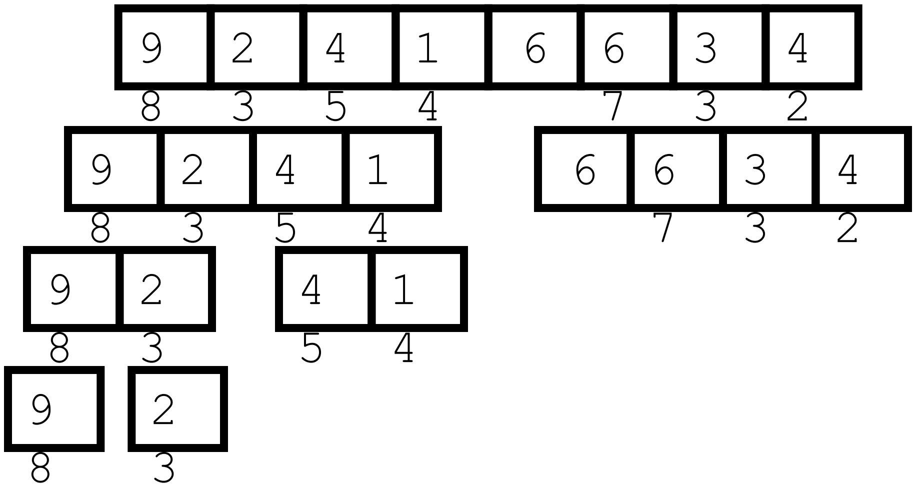

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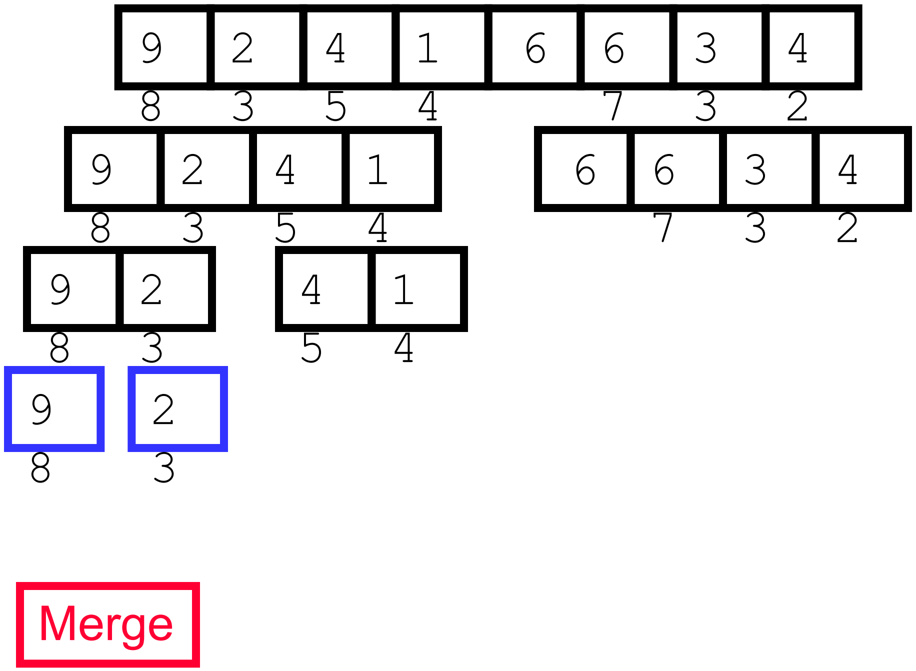

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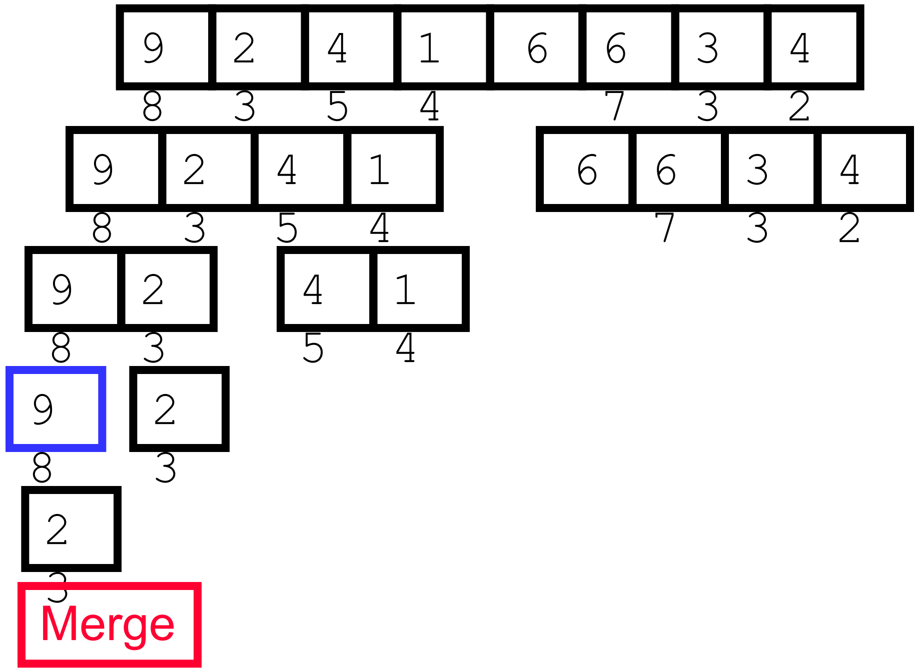

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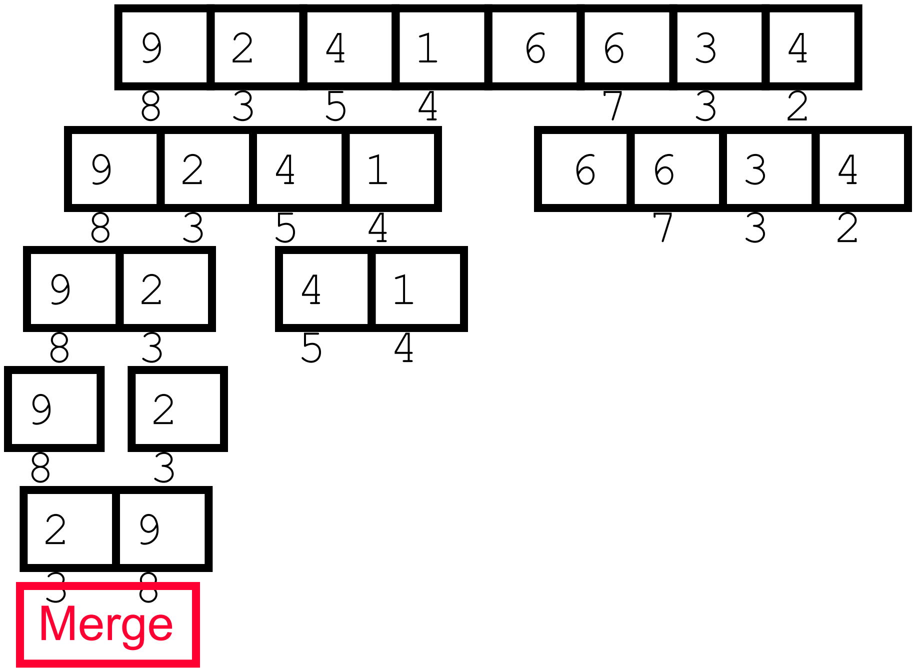

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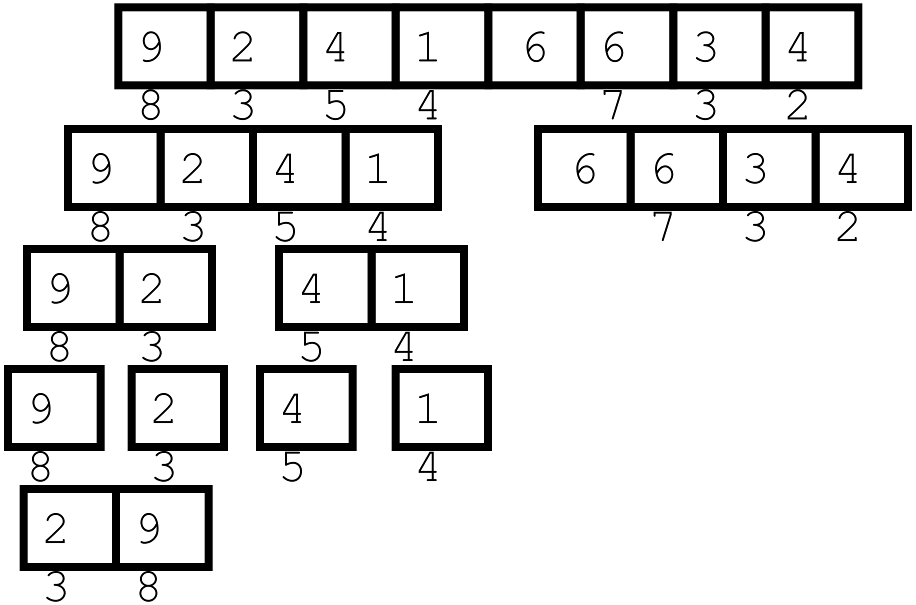

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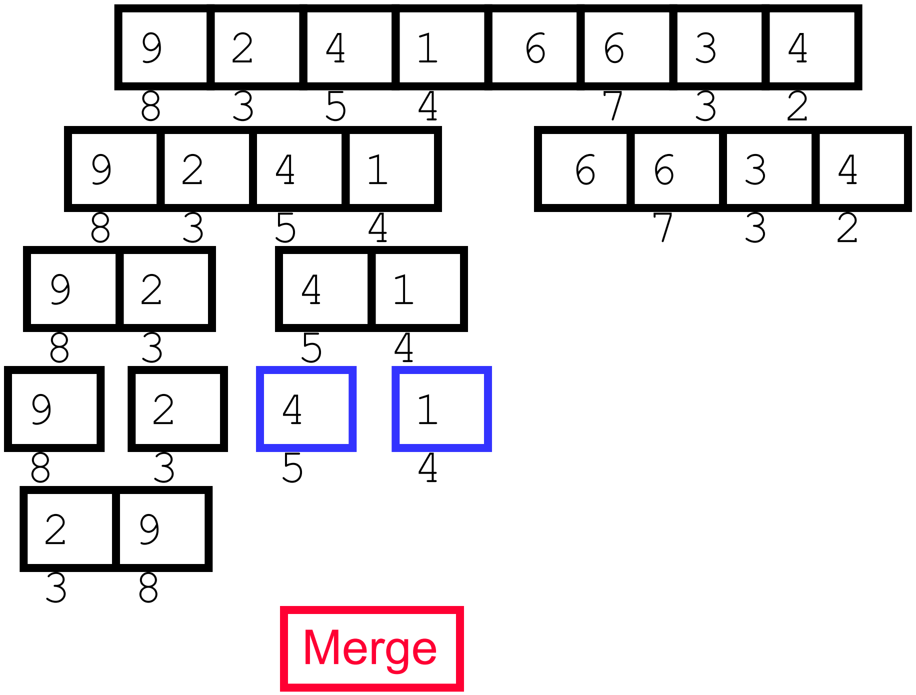

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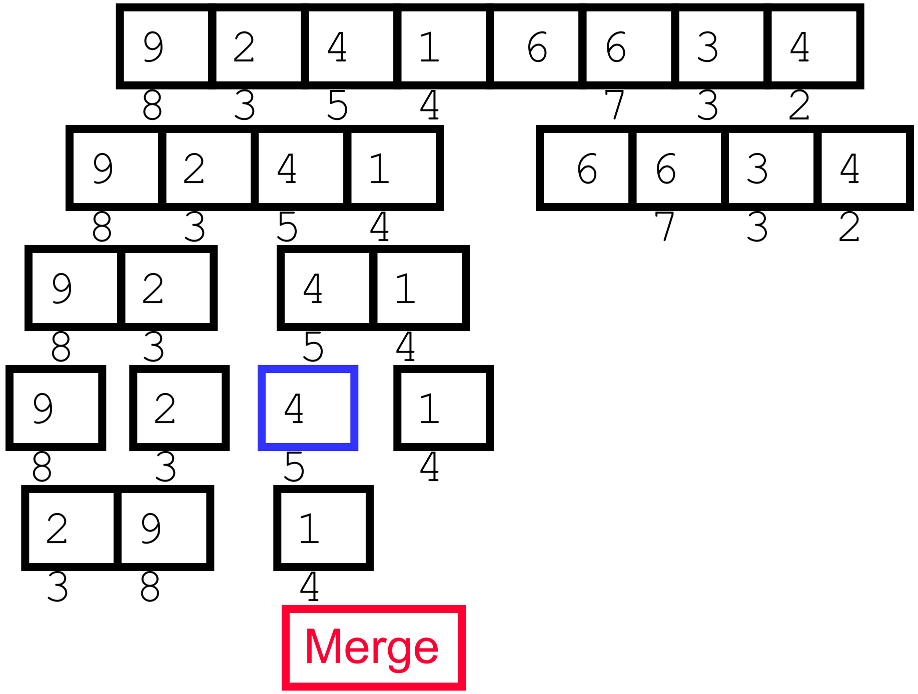

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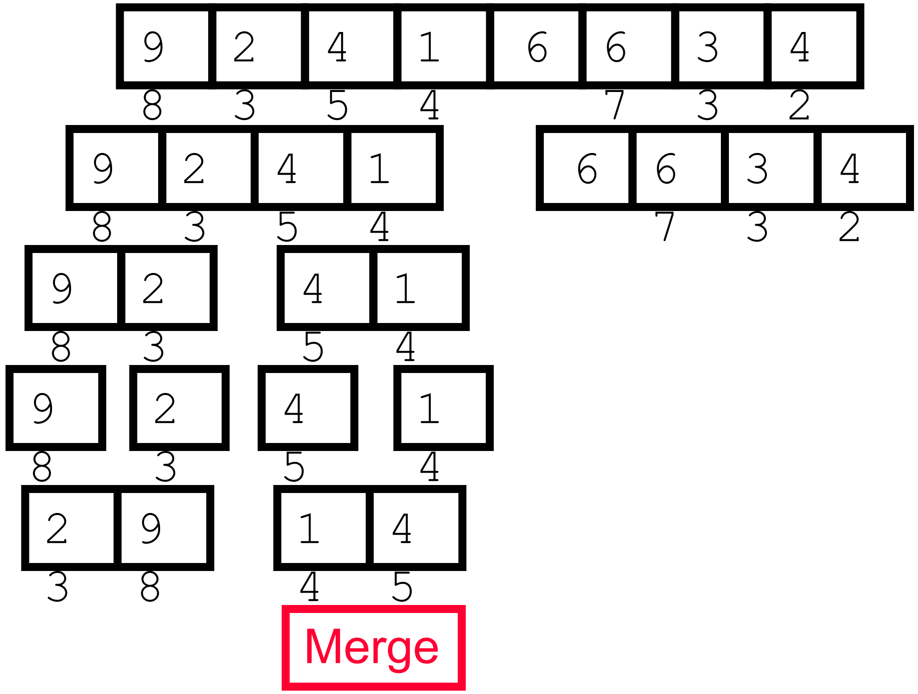

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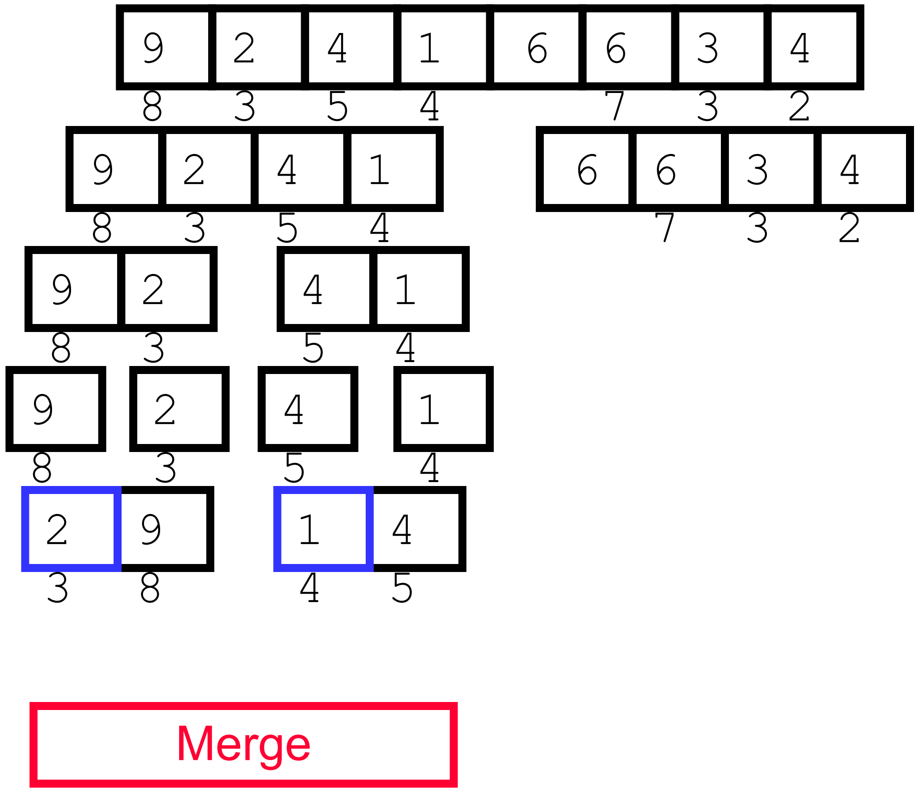

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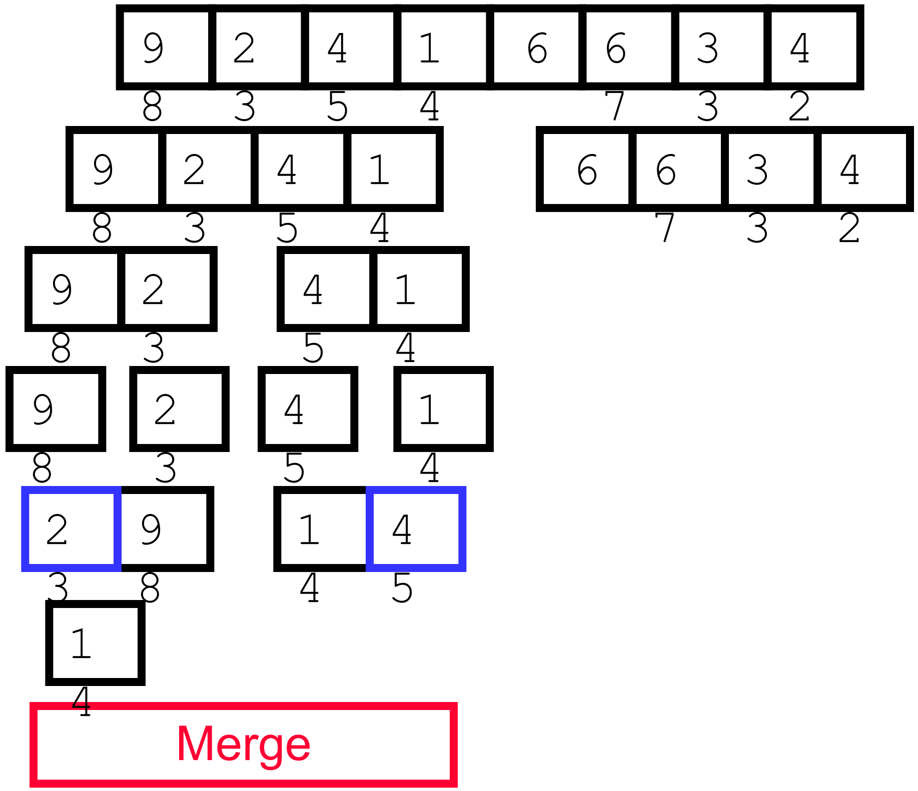

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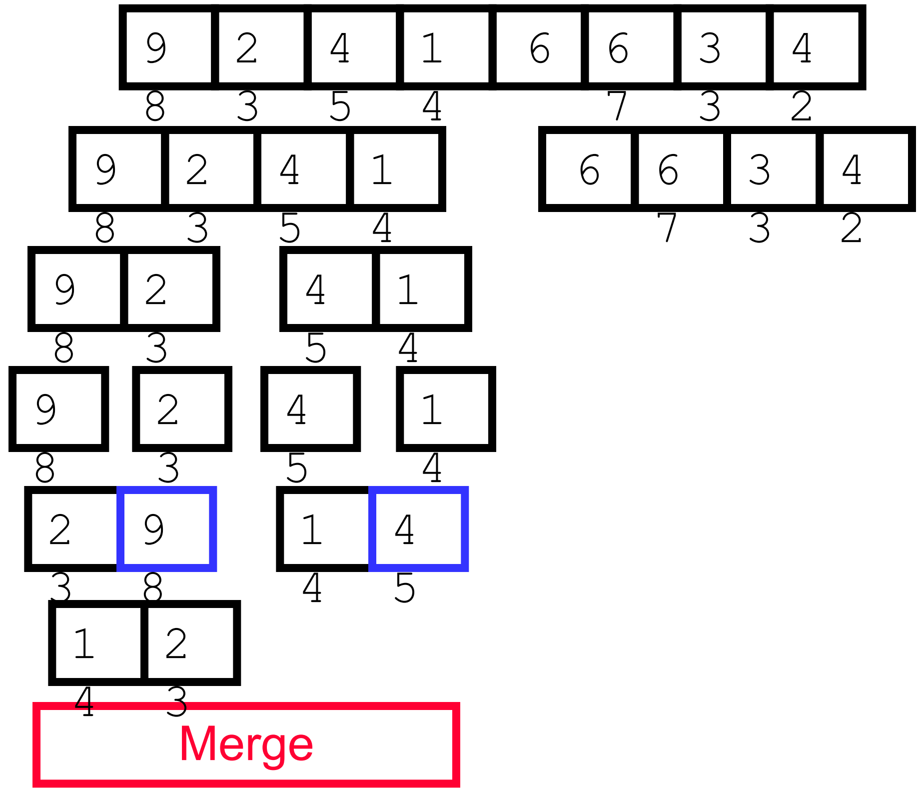

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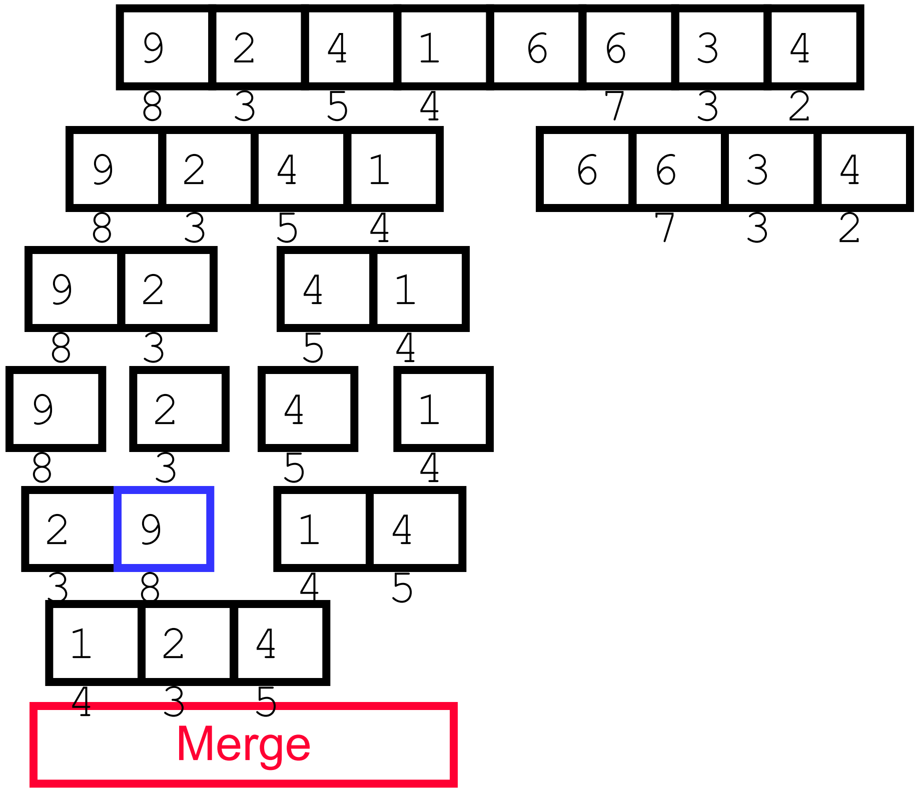

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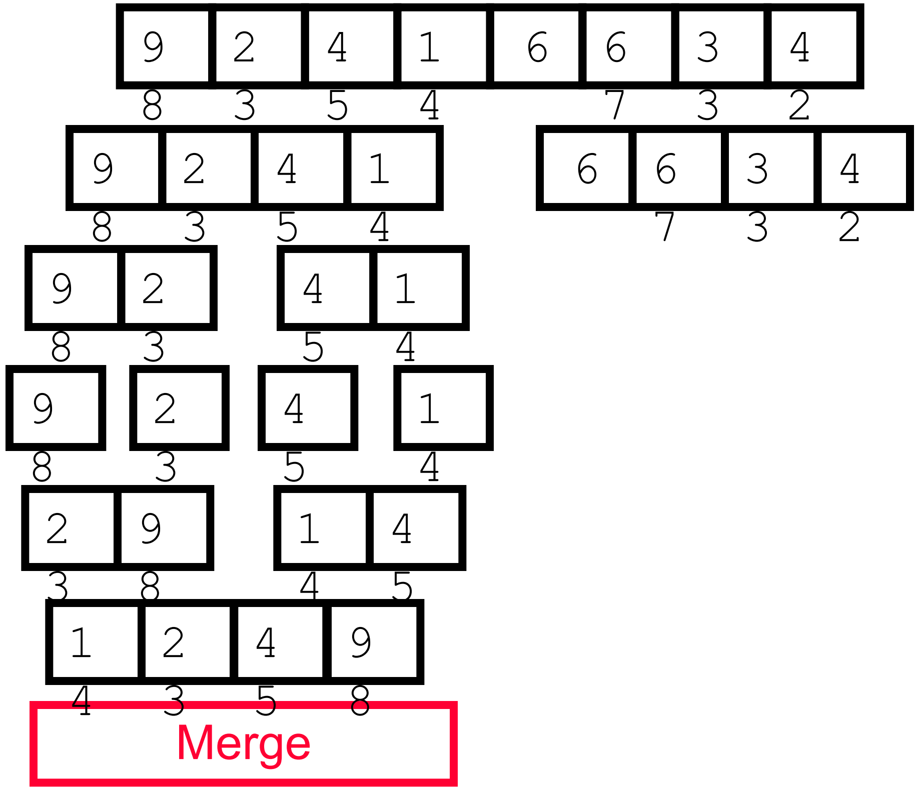

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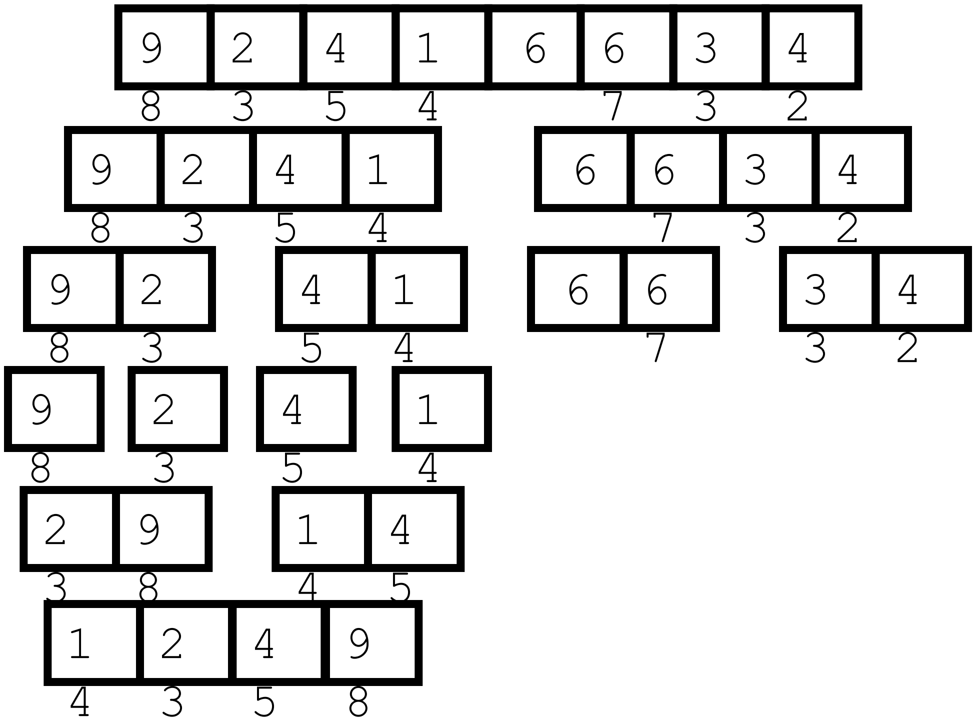

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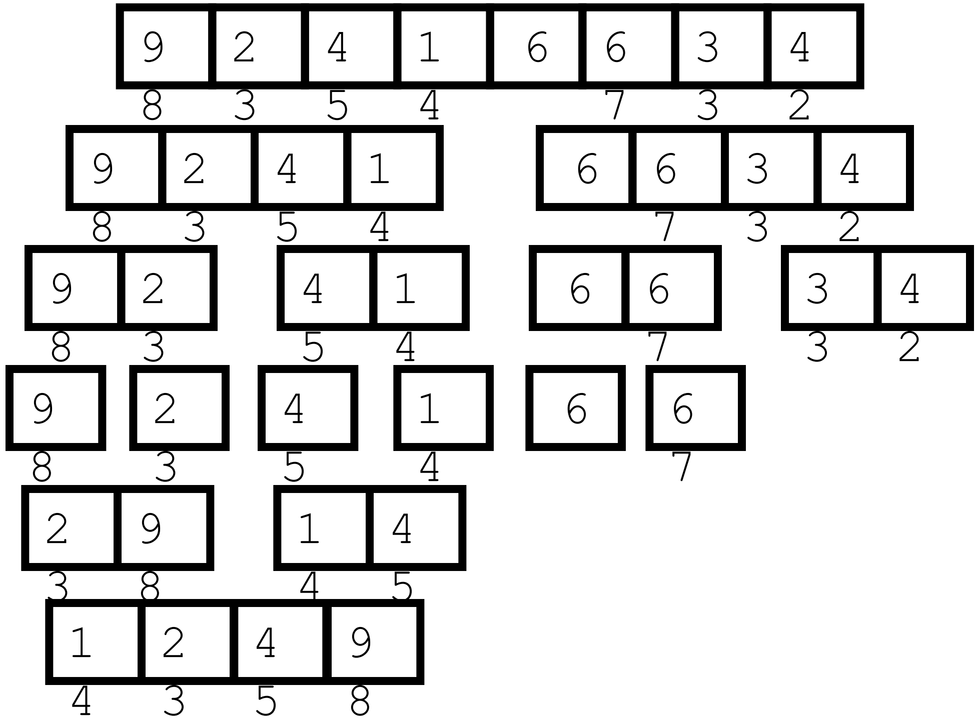

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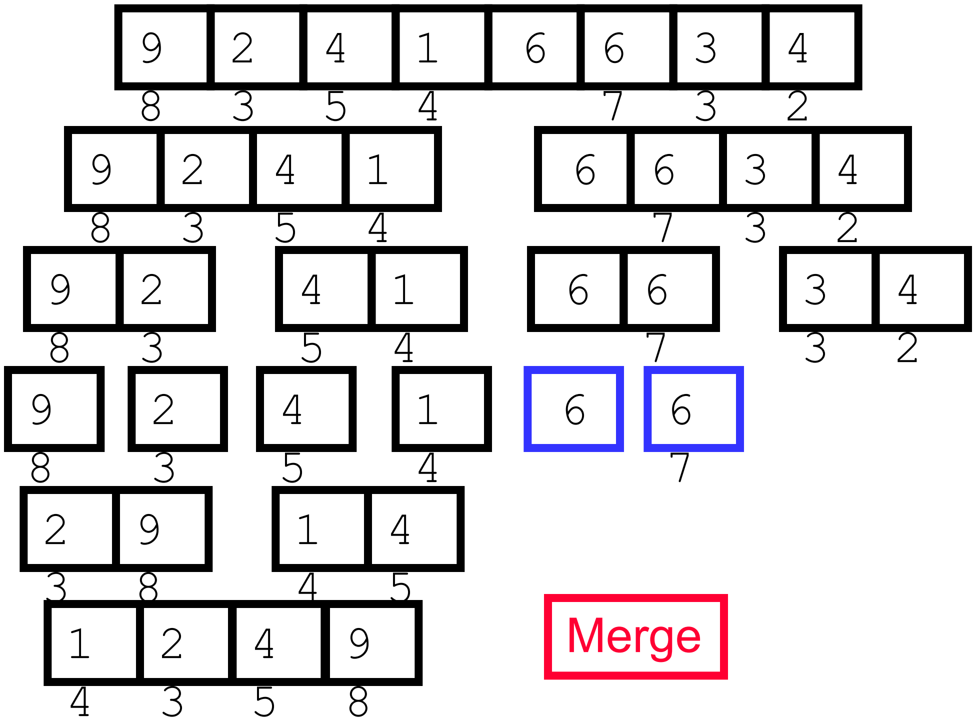

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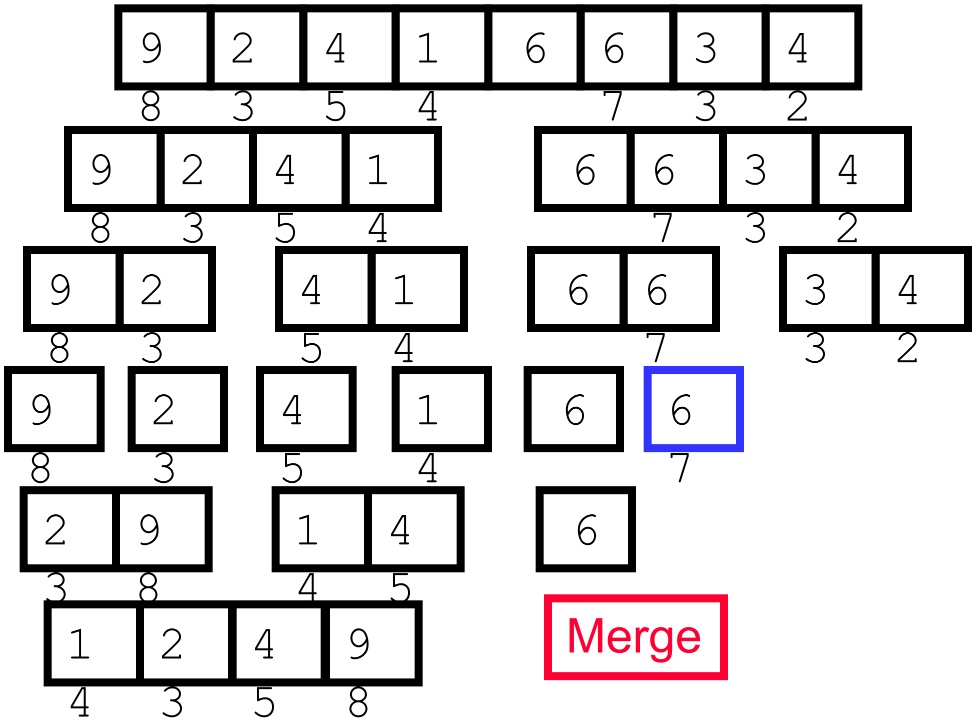

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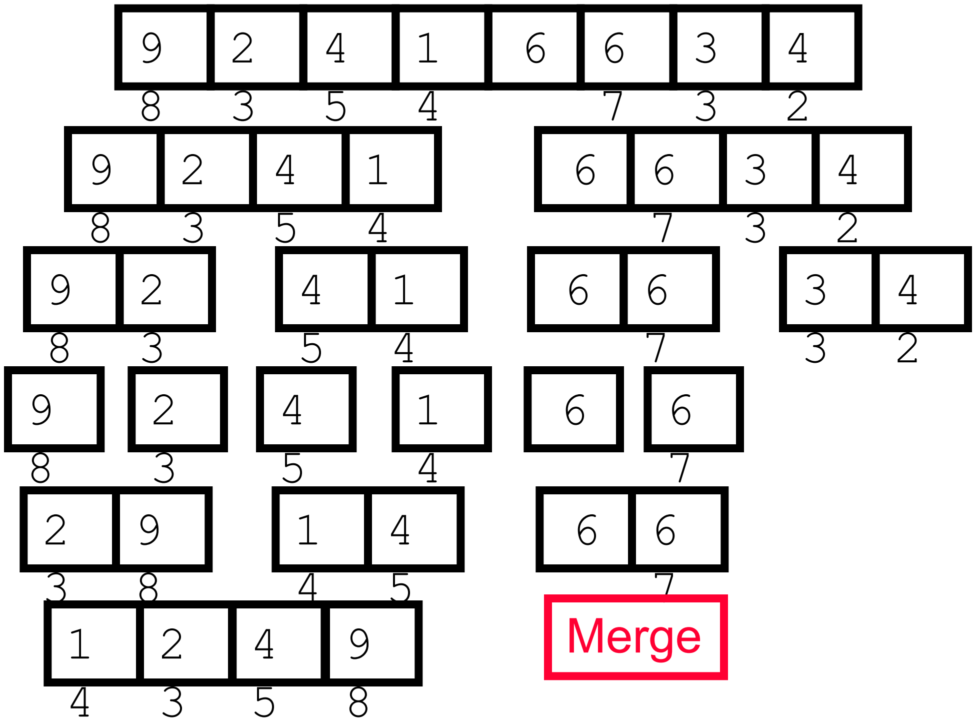

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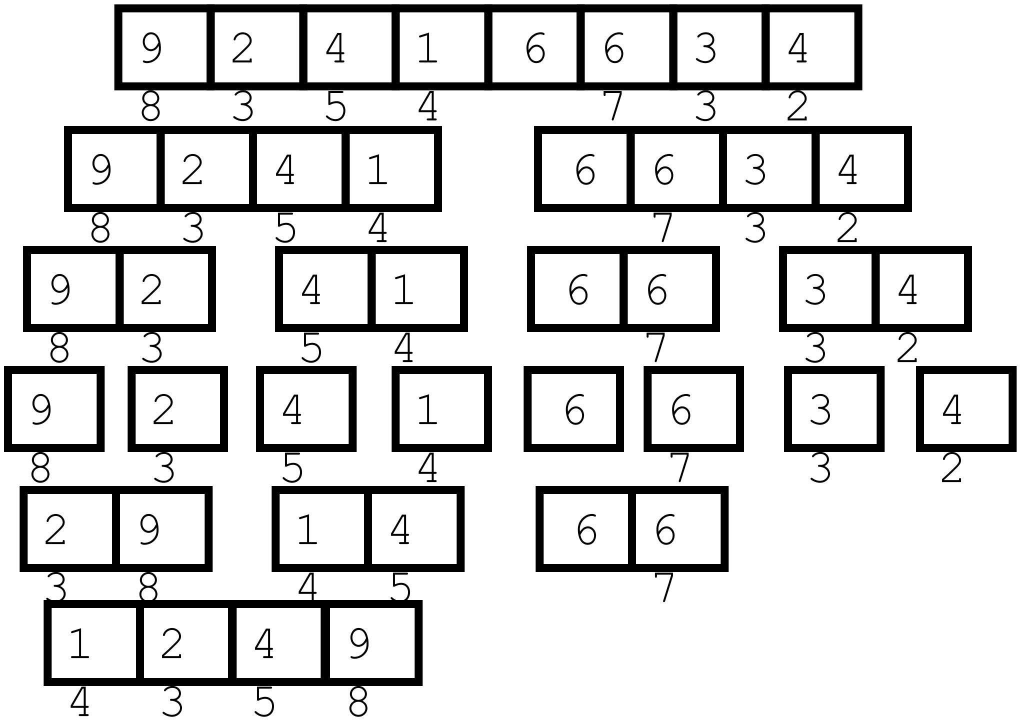

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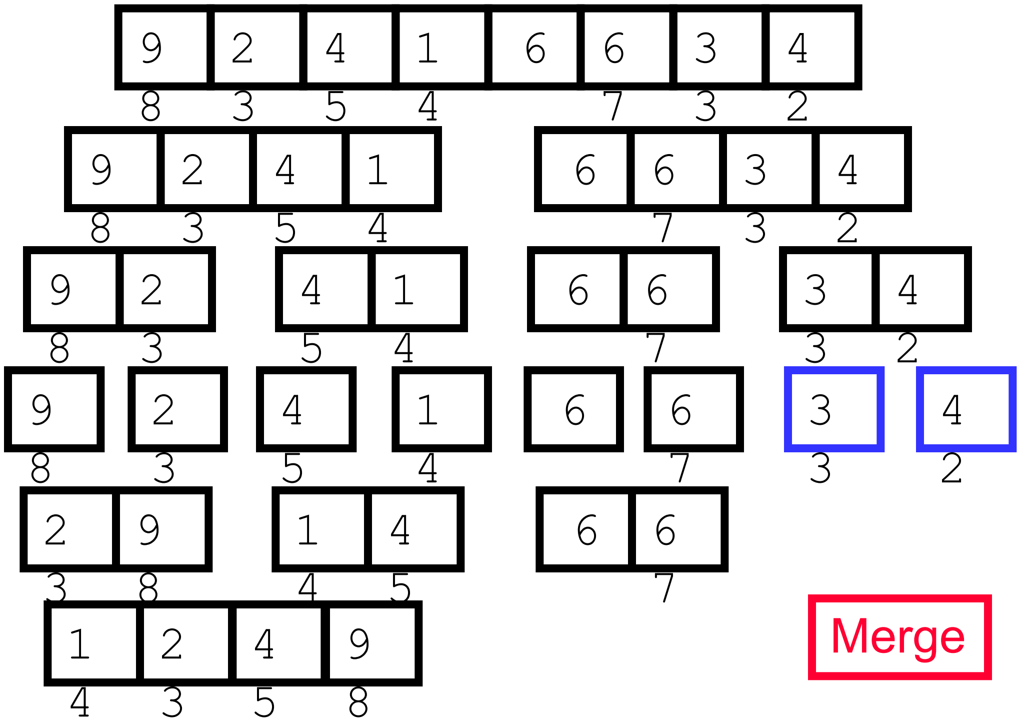

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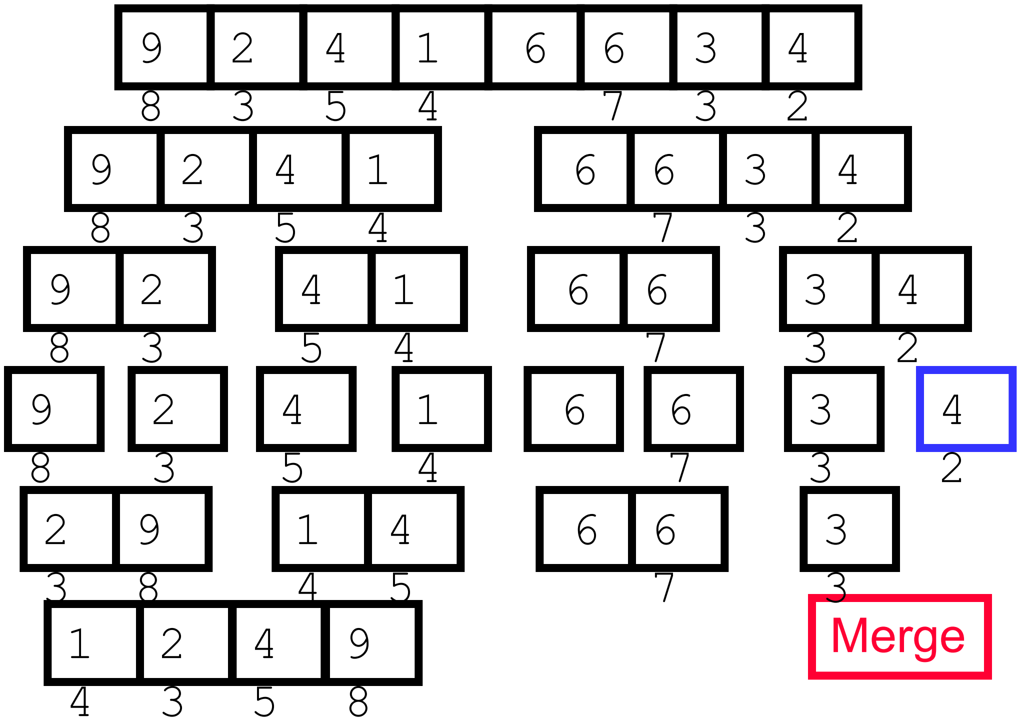

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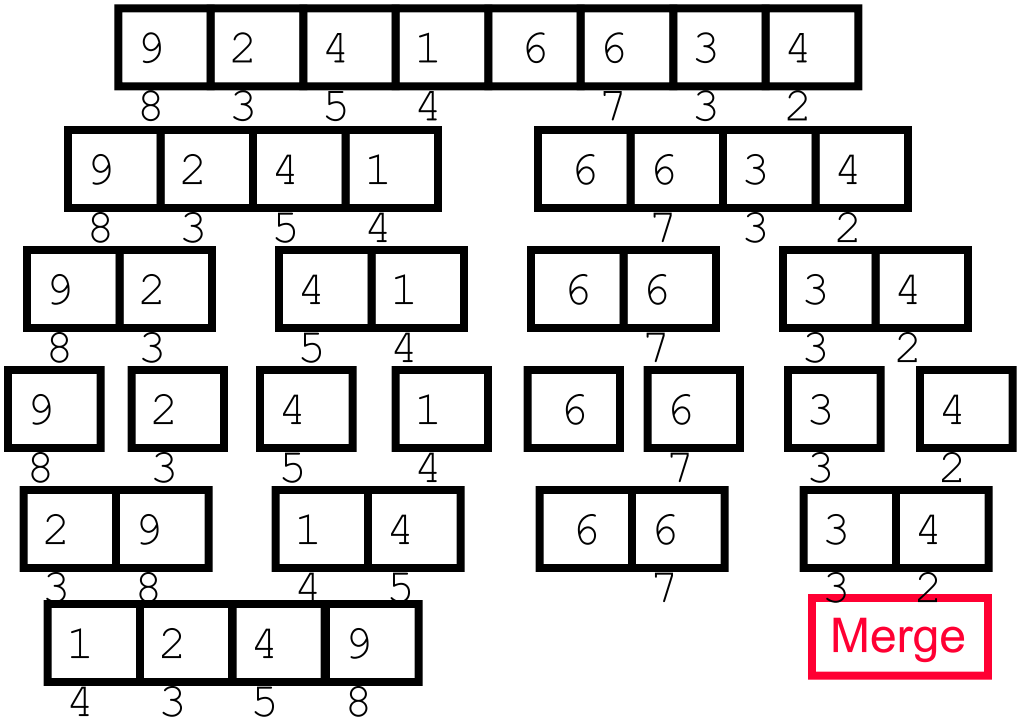

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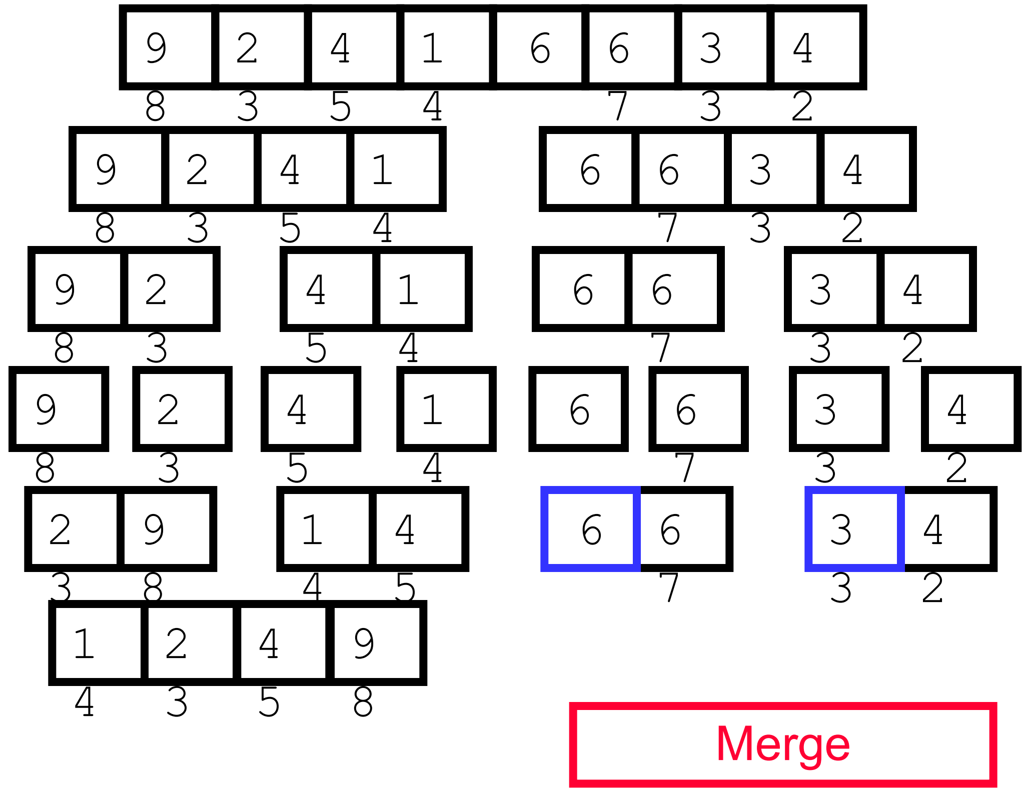

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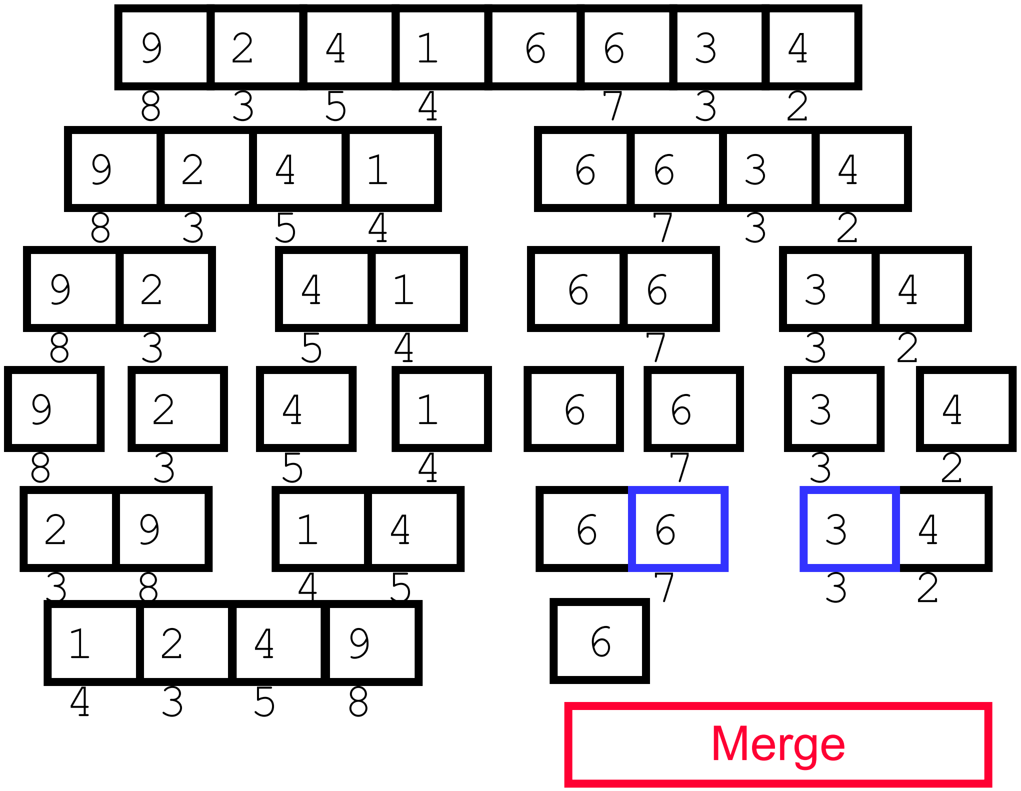

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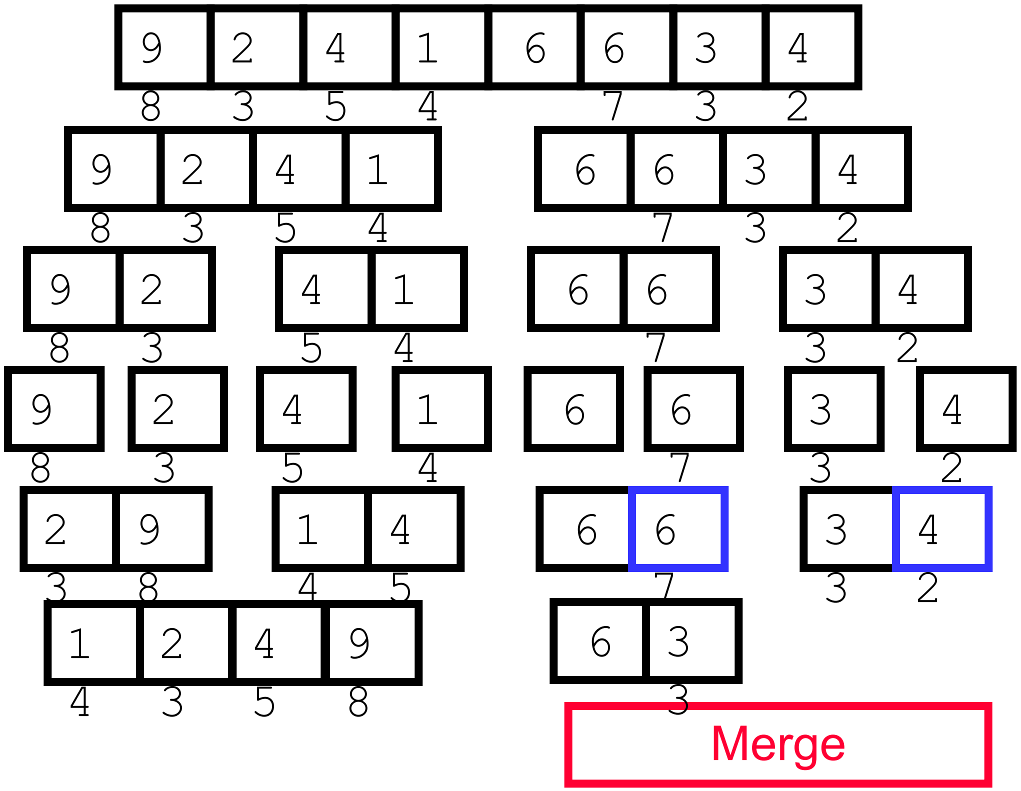

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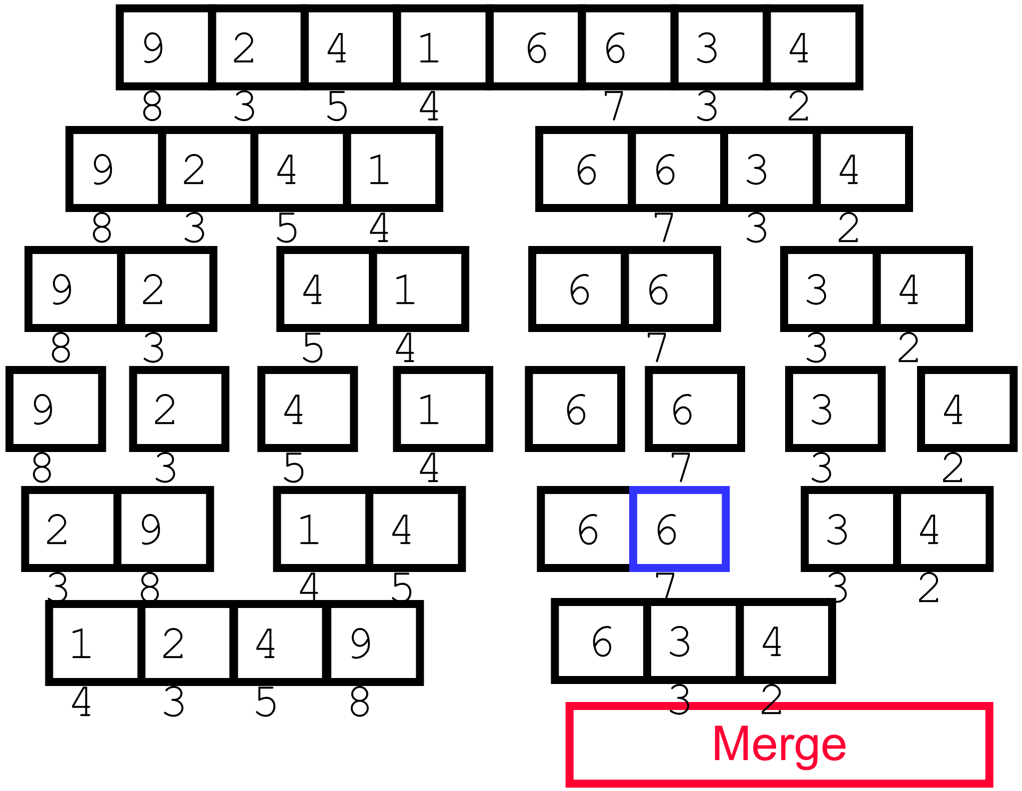

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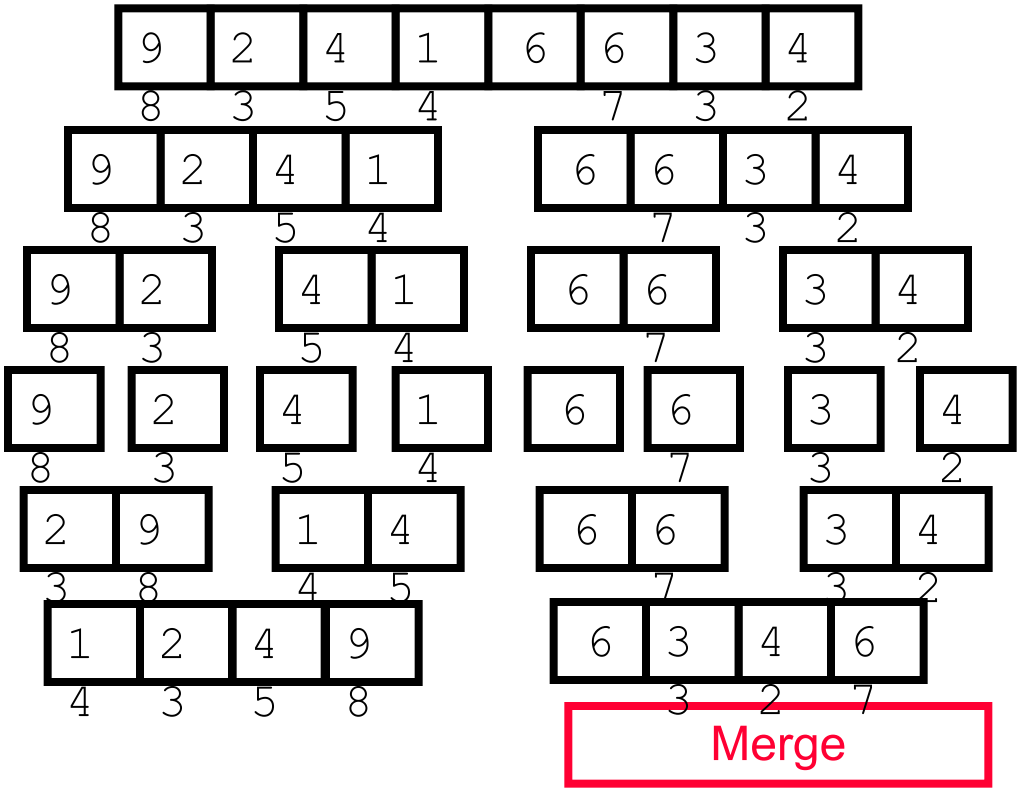

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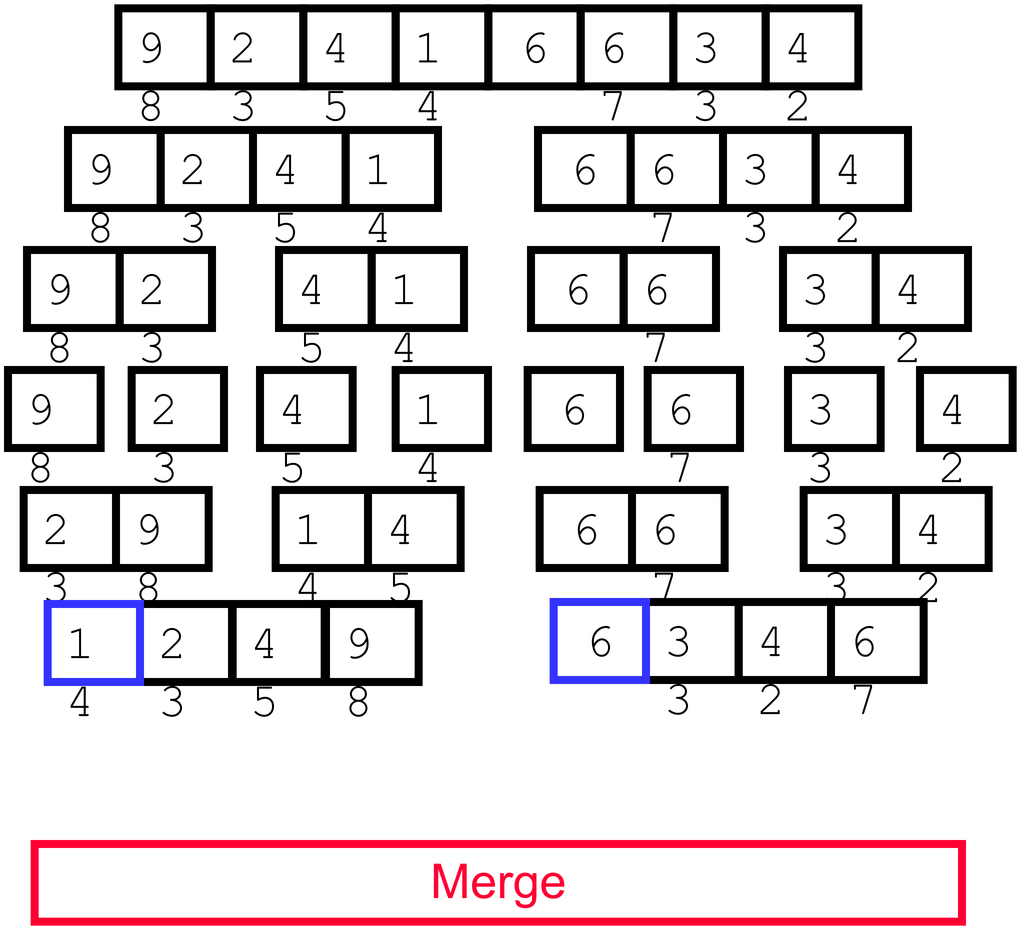

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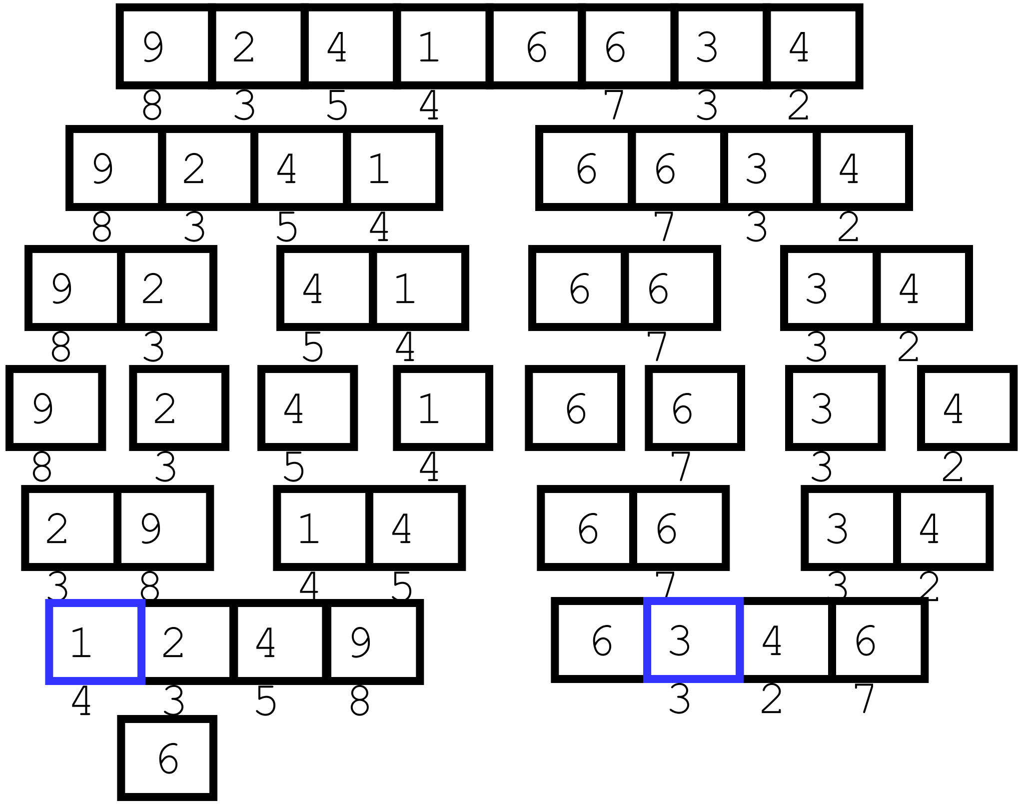
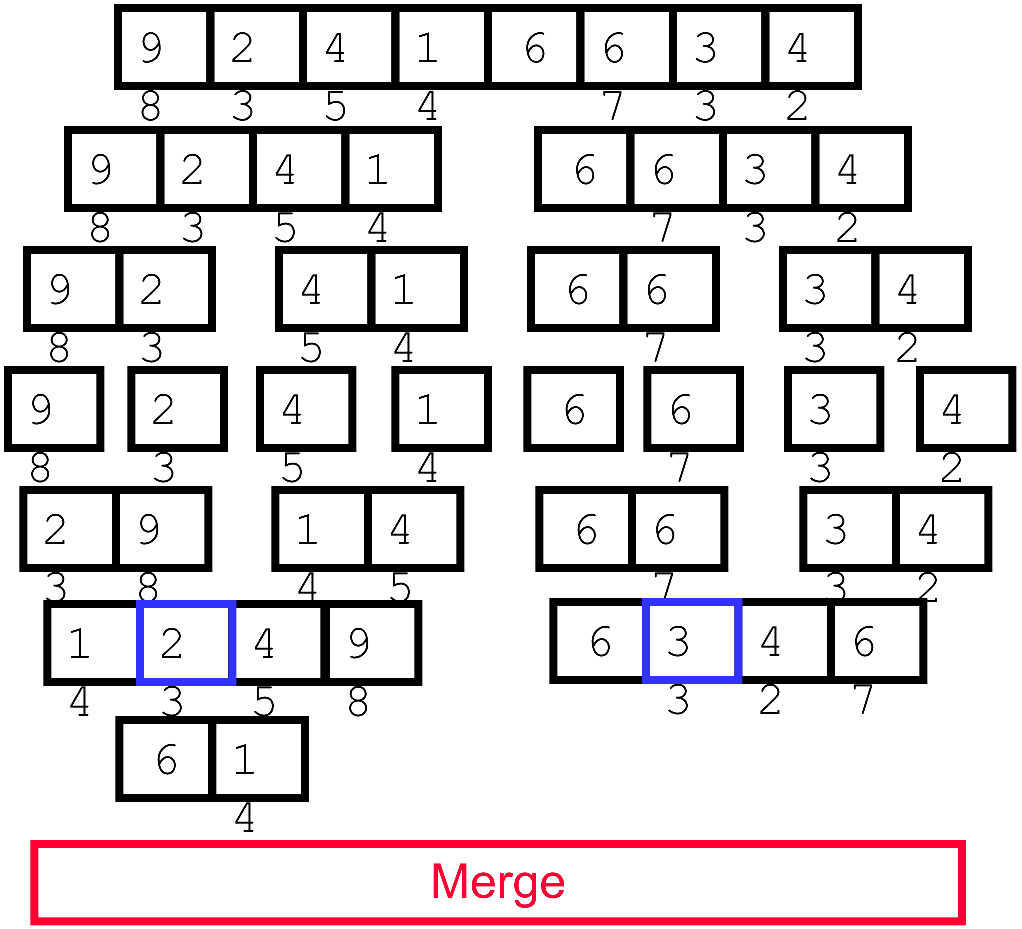


Merge

Merge Sort (recursive simulation)



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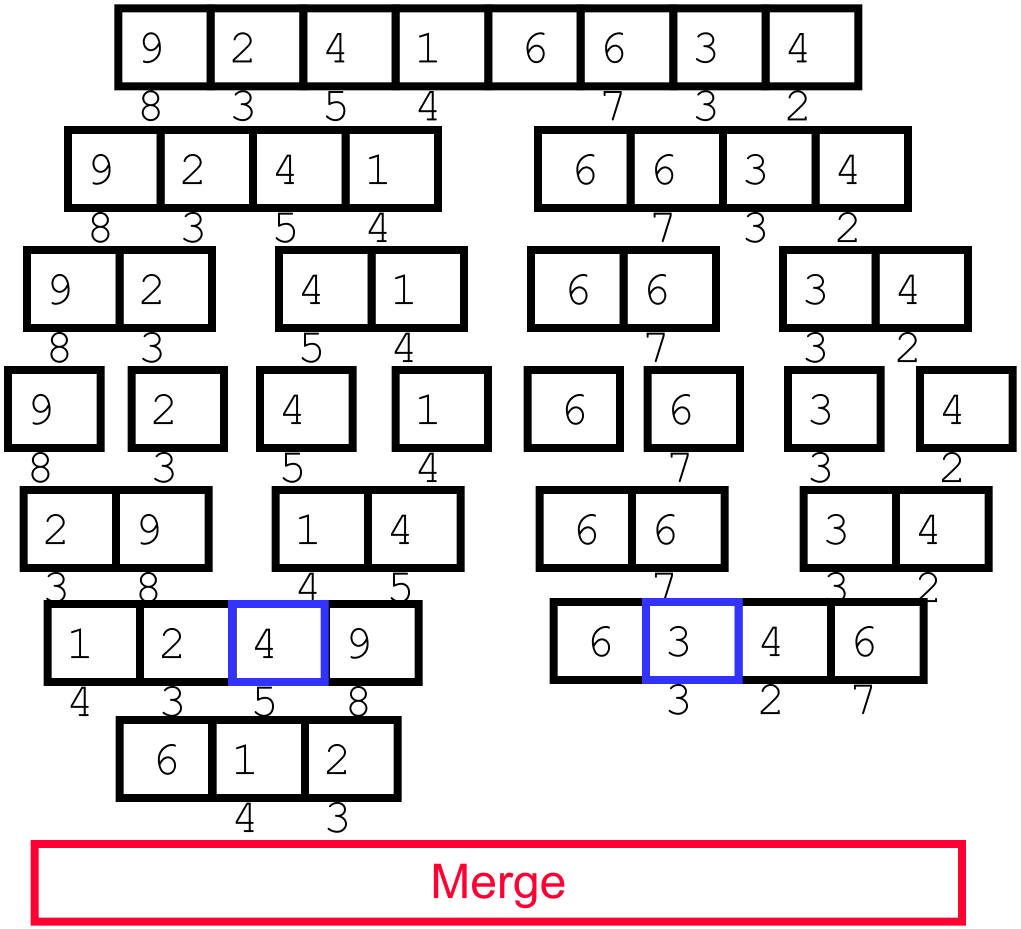

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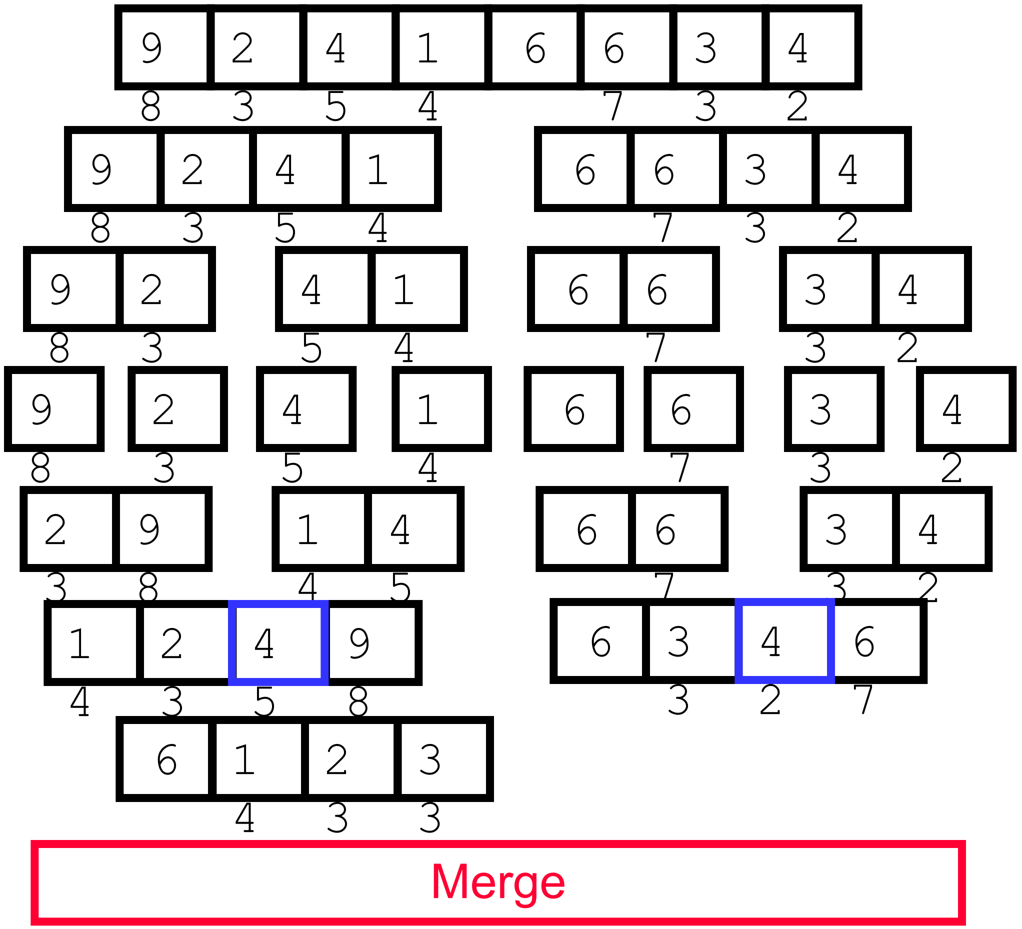

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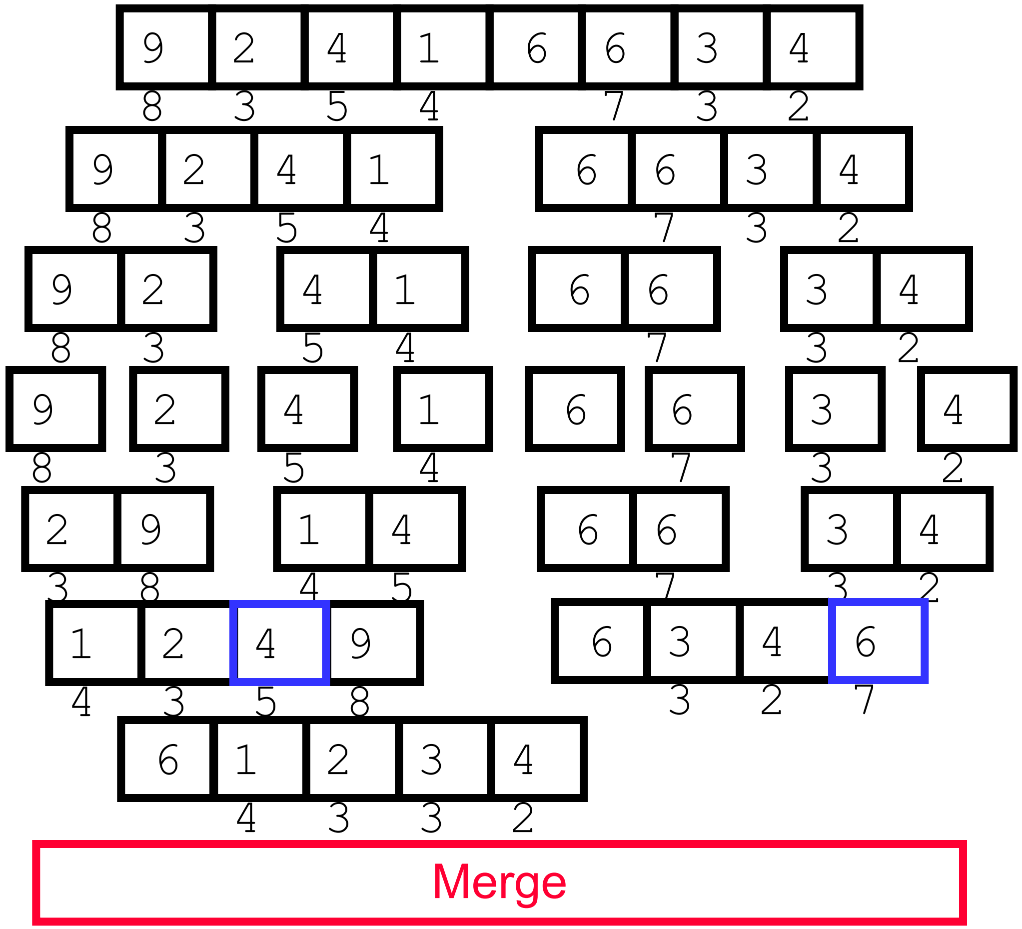

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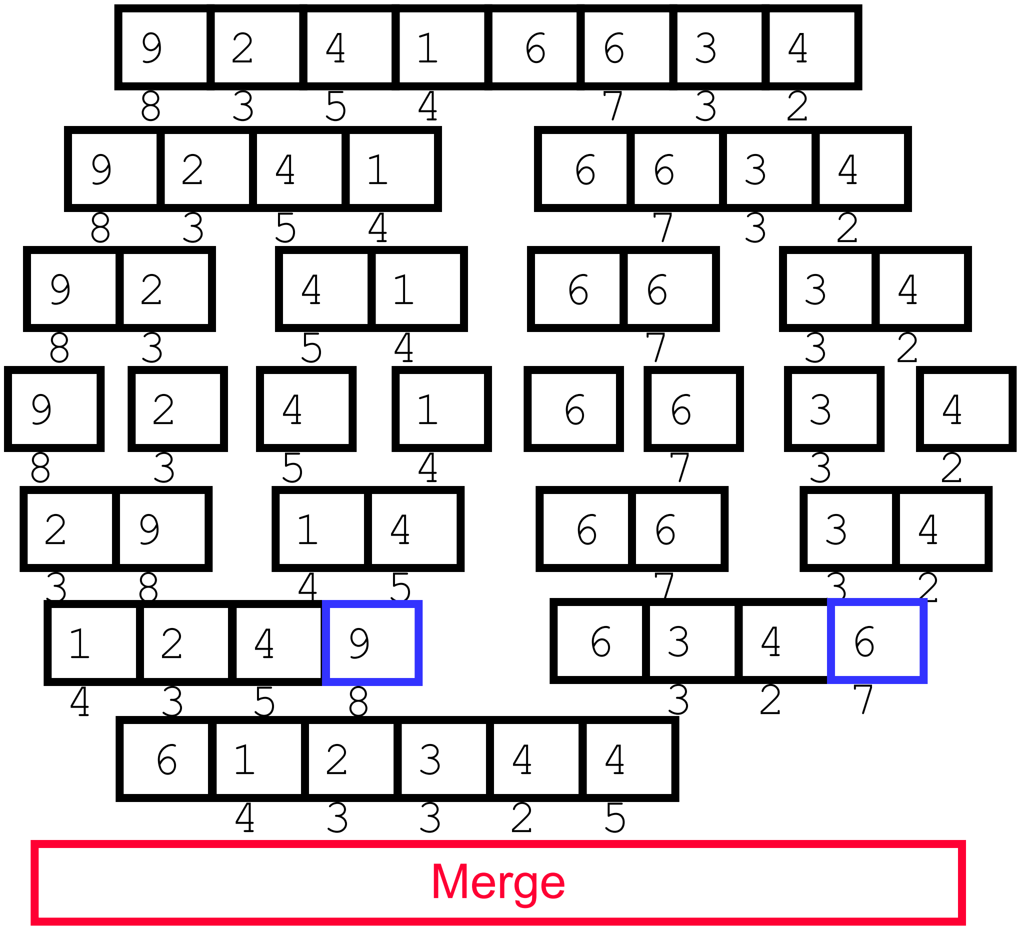

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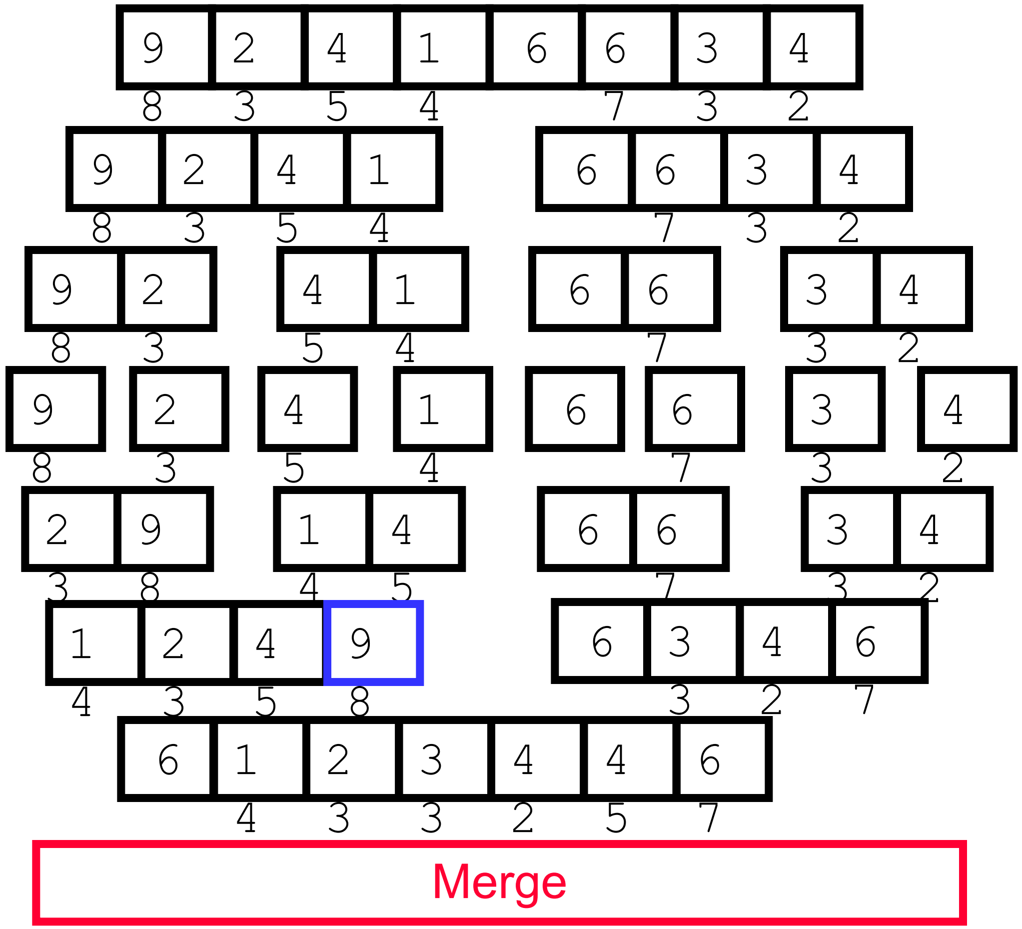

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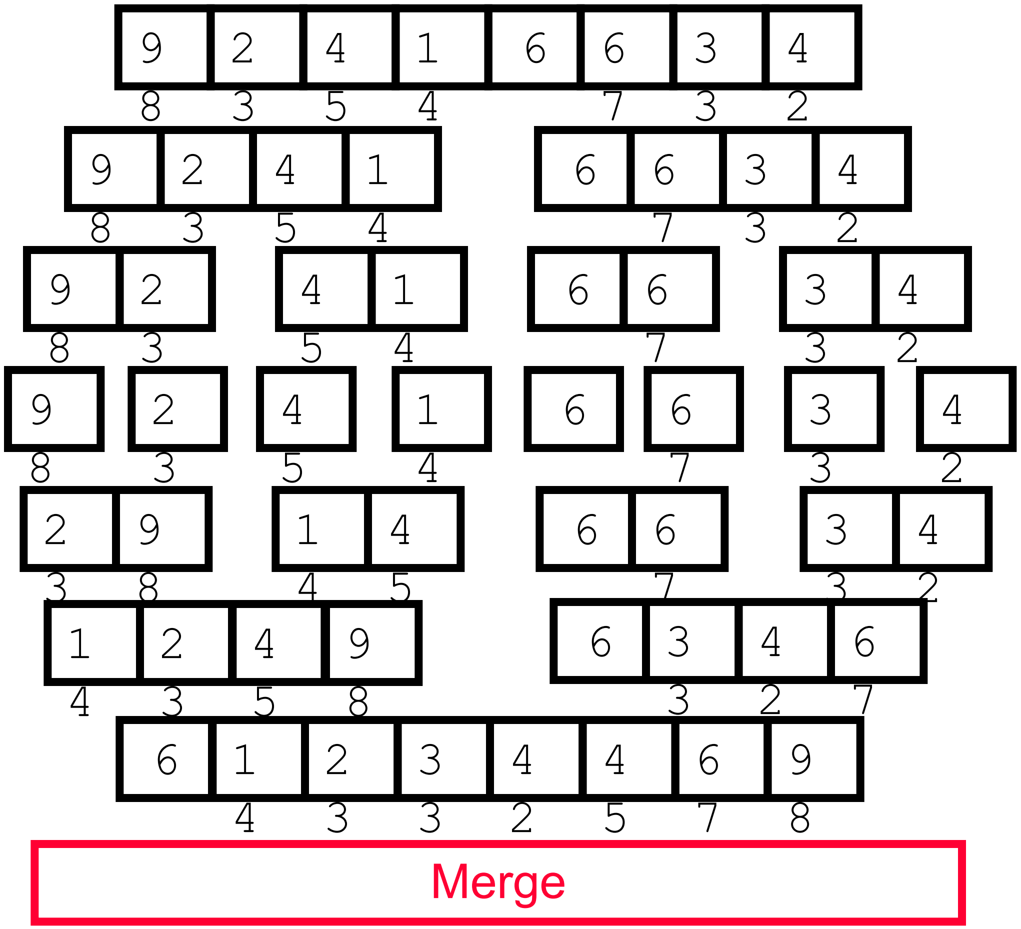

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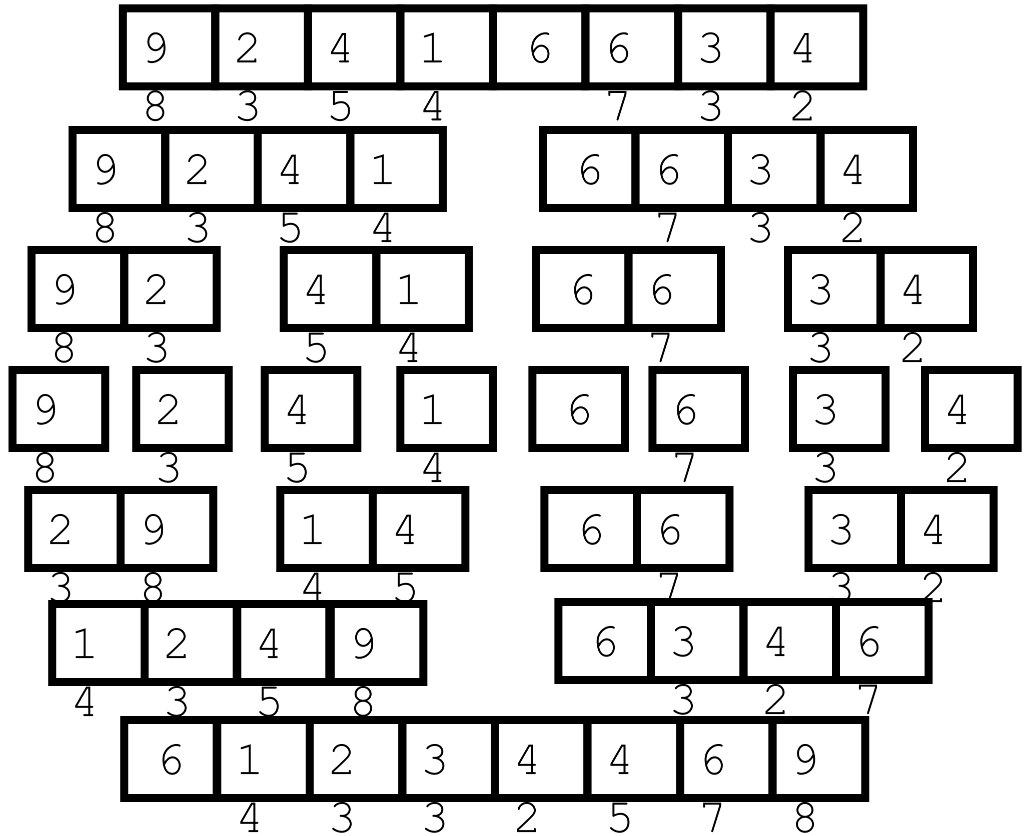
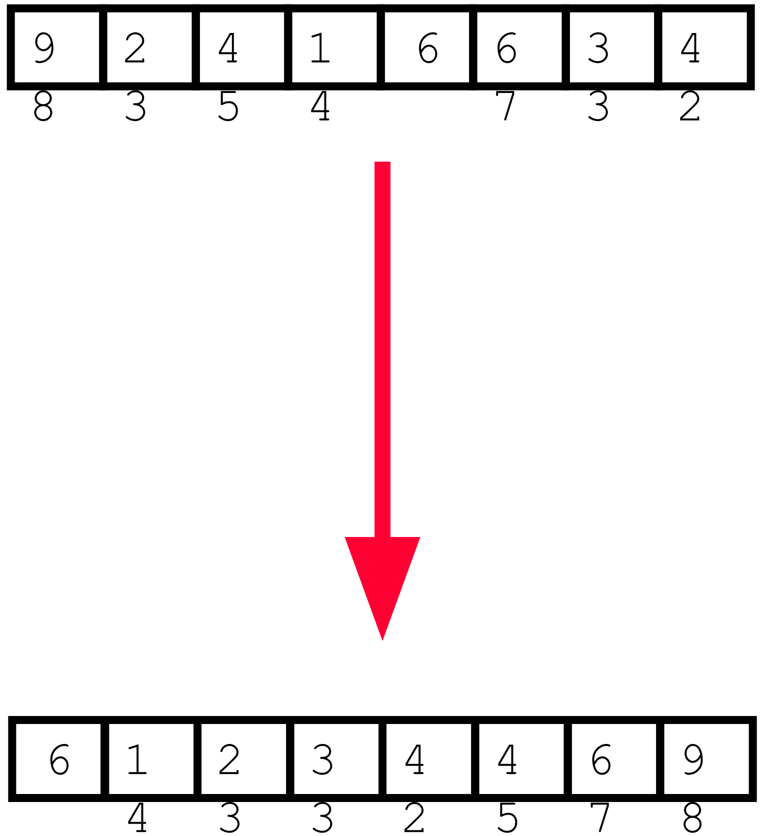
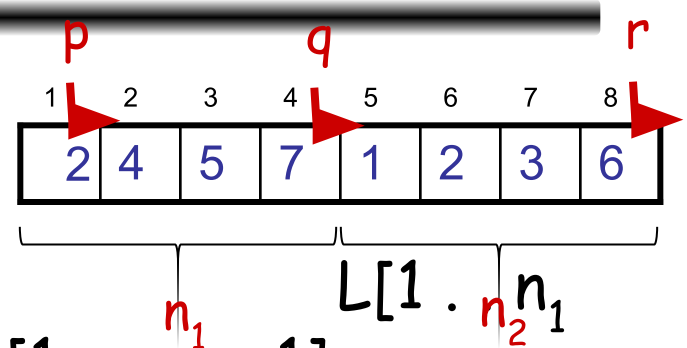


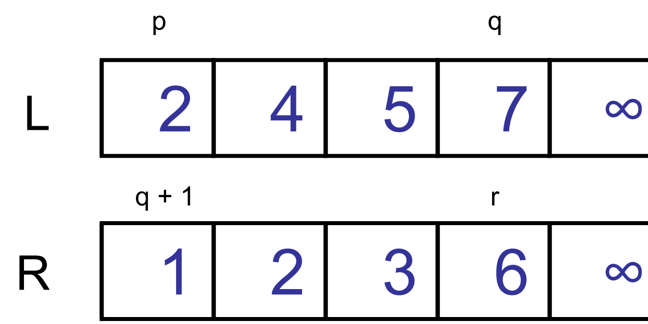

9 2 4 1 6 6 3 4
8 3 5 4 7 3 2
6 1 2 3 4 4 6 9
4 3 3 2 5 7 8

Merge - Pseudocode



Alg.: MERGE(A, p, q, r)

1. Compute n_1 and n_2
2. Copy the first n_1 elements into $L[1 \,.\,.\, n_1 + 1]$ and the next n_2 elements into $R[1 \,.\,.\, n_2 + 1]$
3. $L[n_1 + 1] \leftarrow \infty$; $R[n_2 + 1] \leftarrow \infty$
4. $i \leftarrow 1$; $j \leftarrow 1$
5. **for** $k \leftarrow p$ **to** r
6. **do if** $L[\,i\,] \leq R[\,j\,]$
7. **then** $A[k] \leftarrow L[\,i\,]$
8. $i \leftarrow i + 1$
9. **else** $A[k] \leftarrow R[\,j\,]$
10. $j \leftarrow j + 1$



Merge Sort - Discussion

- Running time insensitive of the input

- Advantages:
  – Guaranteed to run in $\Theta(n \lg n)$

- Disadvantage
  – Requires extra space $\approx N$

Sorting Challenge 1

Problem: Sort a file of huge records with tiny keys

Example application: Reorganize your MP-3 files

Which method to use?

A. merge sort, guaranteed to run in time ~NlgN
B. selection sort
C. bubble sort
D. a custom algorithm for huge records/tiny keys
E. insertion sort

Sorting Files with Huge Records and Small Keys

- Insertion sort or bubble sort?
  - NO, too many exchanges
- Selection sort?
  - YES, it takes linear time for exchanges
- Merge sort or custom method?
  - Probably not: selection sort simpler, does less swaps

Sorting Challenge 2

Problem: Sort a huge randomly-ordered file of small records

Application: Process transaction record for a phone company

Which sorting method to use?

A. Bubble sort
B. Selection sort
C. Mergesort guaranteed to run in time ~N*lg*N
D. Insertion sort

Sorting Huge, Randomly - Ordered Files

- Selection sort?
  - NO, always takes quadratic time
- Bubble sort?
  - NO, quadratic time for randomly-ordered keys
- Insertion sort?
  - NO, quadratic time for randomly-ordered keys
- Mergesort?
  - YES, it is designed for this problem

Sorting Challenge 3

Problem: sort a file that is already almost in order

Applications:

- Re-sort a huge database after a few changes
- Doublecheck that someone else sorted a file

Which sorting method to use?

A. Mergesort, guaranteed to run in time ~NlgN
B. Selection sort
C. Bubble sort
D. A custom algorithm for almost in-order files
E. Insertion sort

Sorting Files That are Almost in Order

- Selection sort?
  - NO, always takes quadratic time
- Bubble sort?
  - NO, bad for some definitions of "almost in order"
  - Ex: B C D E F G H I J K L M N O P Q R S T U V W X Y Z A
- Insertion sort?
  - YES, takes linear time for most definitions of "almost in order"
- Mergesort or custom method?
  - Probably not: insertion sort simpler and faster

Sorting Applications

Sorting algorithms are essential in a broad variety of applications

- Sort a list of names.
- Organize an MP3 library.
- Display Google PageRank results.
- List RSS news items in reverse chronological order.

- Find the median.
- Find the closest pair.
- Binary search in a database.
- Identify statistical outliers.
- Find duplicates in a mailing list

Now I know merge sort!