Emphasizing Conceptual Understanding Using **Maple**:

A Paradigm Shift in Teaching Undergraduate Physics

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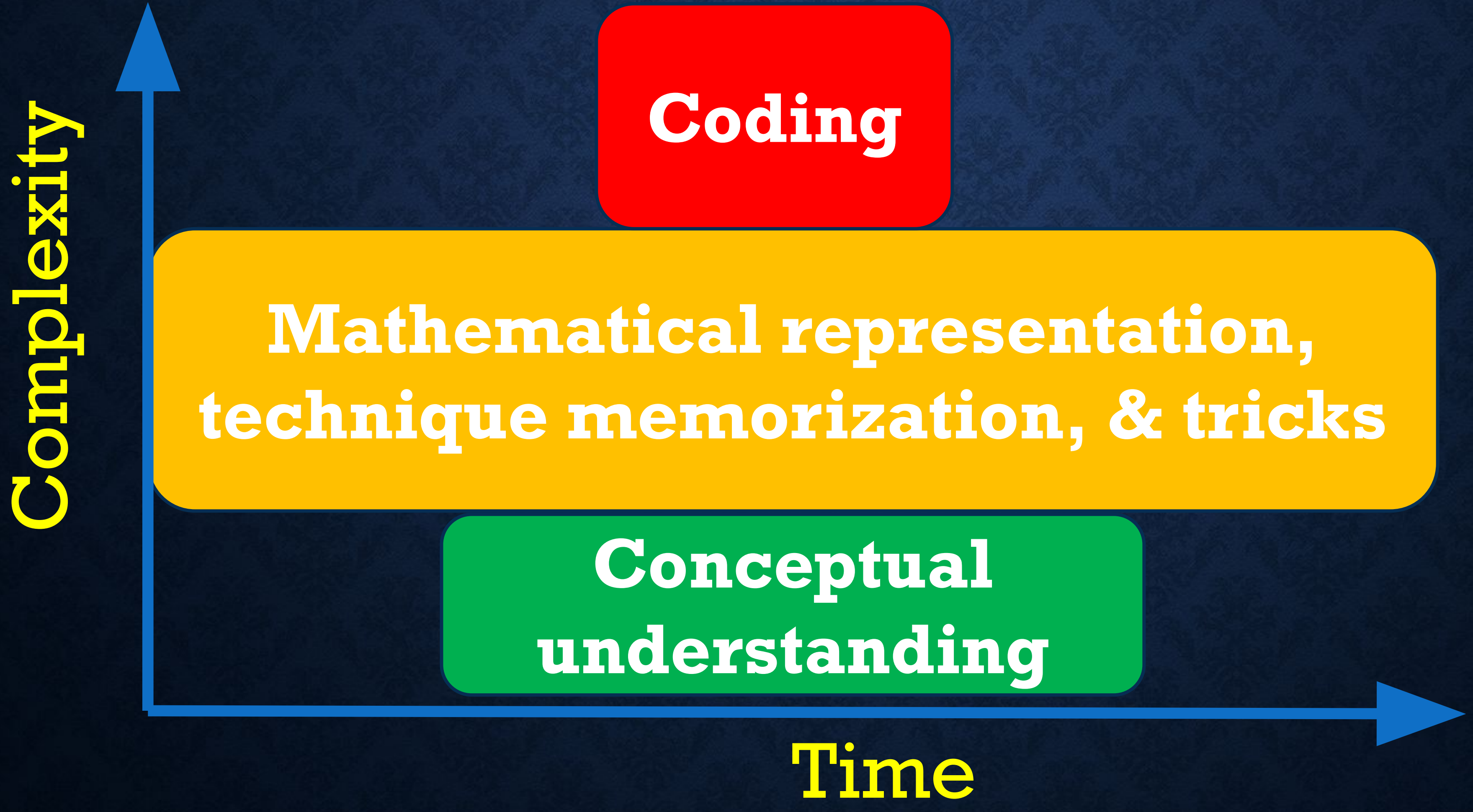
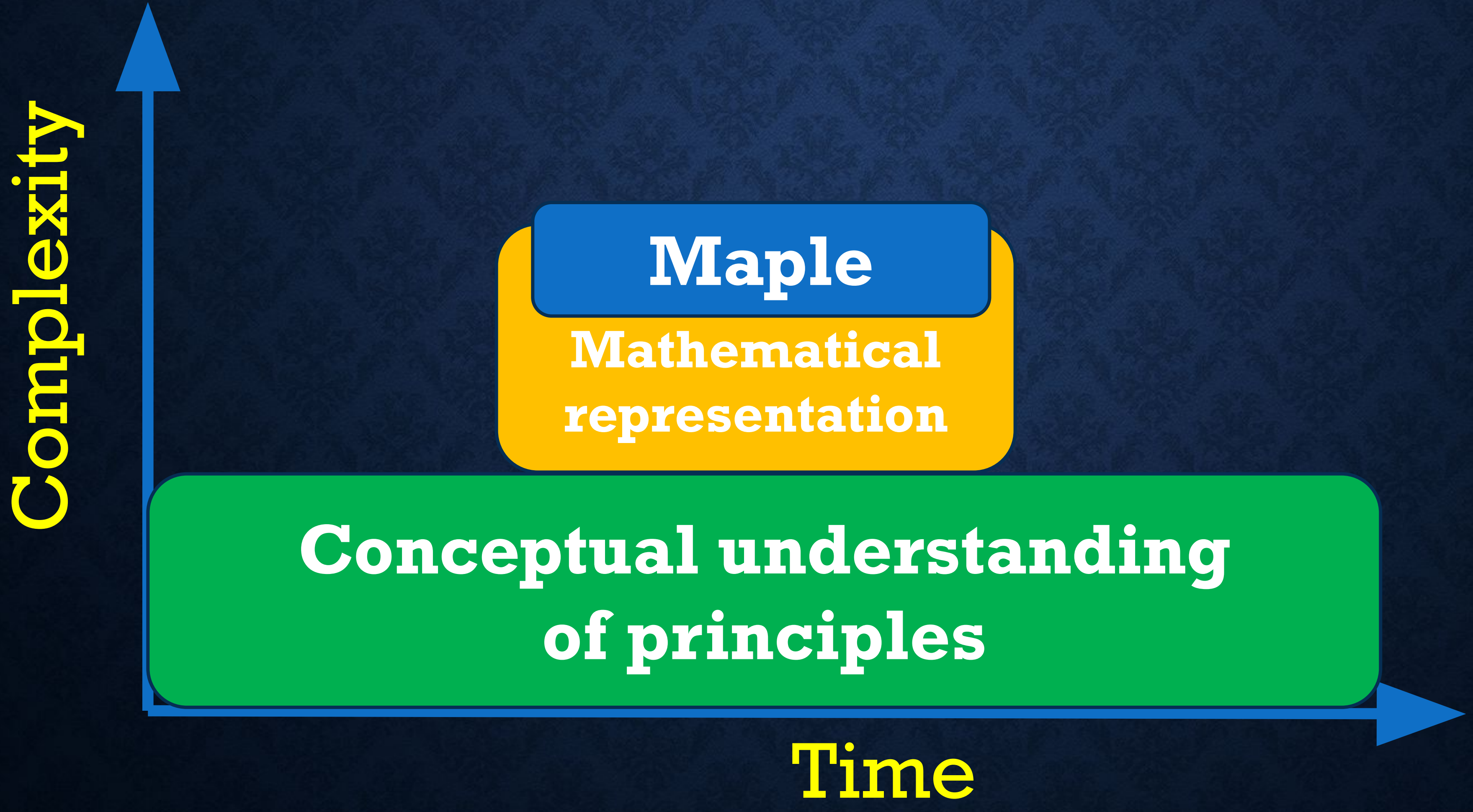
History: Adding computational problem-solving to introductory physics courses

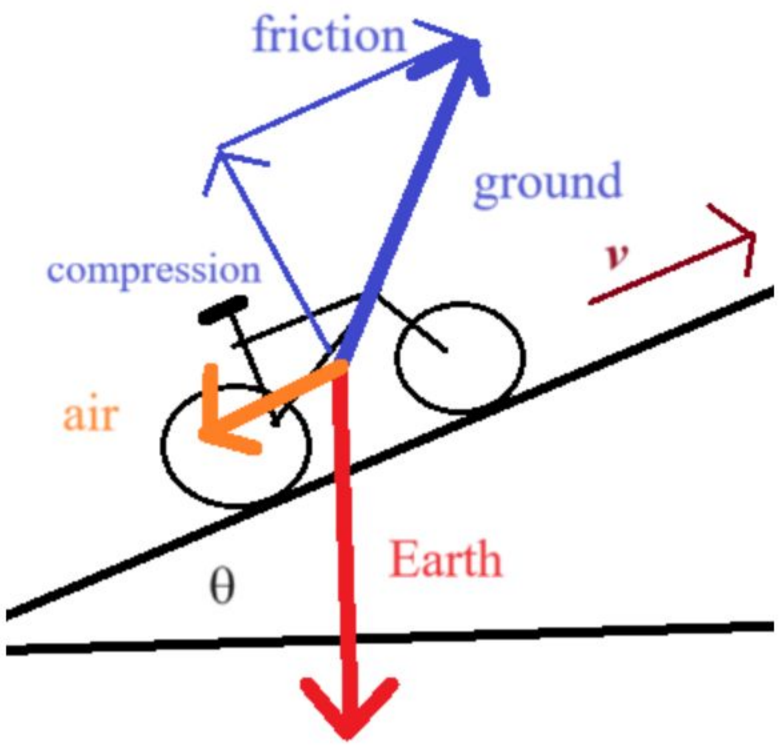


Adding **Maple** to an introductory physics course



Physics: Top-down problem-solving

a) Riding a bike ***uphill*** at a constant velocity. Given that the speed, mass and power of the rider can be measured, derive the expression for the drag coefficient constant of the air. $|\vec{F}_{air}| = C\, v^2$.

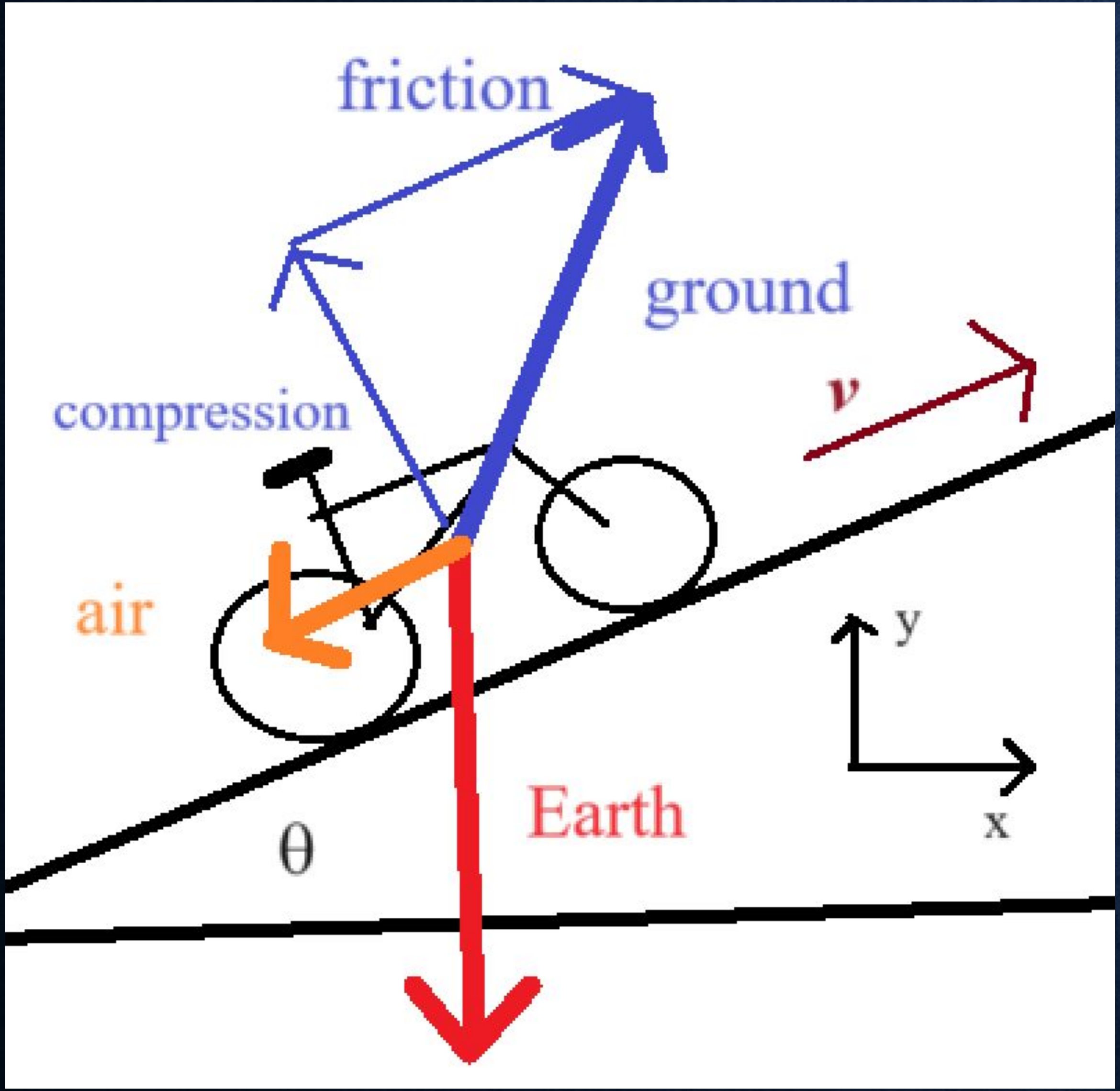


$$> Force\_Equation := \vec{F}_{net} = m \cdot \vec{a} :$$

$$\vec{F}_{net} := \vec{F}_{Earth} + \vec{F}_{ground} + \vec{F}_{air} :$$

□ **Maple code**

Maple: Top-down problem-solving



$$> \vec{F}_{Earth} := \overrightarrow{v_{2d}}\left(m \cdot g, -\frac{\pi}{2}\right) :$$

$$\vec{F}_{ground} := \vec{F}_{g,\, friction} + \vec{F}_{g,\, compression} :$$

$$\vec{F}_{g,\, friction} := \overrightarrow{v_{2d}}\left(F_{scot}, \theta\right) : \quad F_{scot} := \frac{P}{v} :$$

$$\vec{F}_{g,\, compression} := \overrightarrow{v_{2d}}\left(F_c, \theta + \frac{\pi}{2}\right) :$$

$$\vec{F}_{air} := \overrightarrow{v_{2d}}\left(C \cdot v^2, \theta + \pi\right) :$$

$$> \overrightarrow{v_{2d}}(v, \theta) := \langle\, v \cdot \cos(\theta), v \cdot \sin(\theta) \,\rangle :$$

$$\vec{a} := \langle\, 0, 0 \,\rangle :$$

Maple: Minimize mathematical minutia & coding

> *Force_Equation*

$$\begin{bmatrix} \frac{P\cos(\theta)}{v} - F_c\sin(\theta) - C v^2\cos(\theta) \\ -m g + \frac{P\sin(\theta)}{v} + F_c\cos(\theta) - C v^2\sin(\theta) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Solve for the unknown variables, C, F_c

> $solutions := solve(Force\_Equation, \{C, F_c\})$

$$solutions := \left\{ C = \frac{-\sin(\theta)\, m g v + \cos(\theta)^2 P + \sin(\theta)^2 P}{v^3 \left(\cos(\theta)^2 + \sin(\theta)^2\right)}, F_c = \frac{\cos(\theta)\, m g}{\cos(\theta)^2 + \sin(\theta)^2} \right\}$$

> $C_{sol} := simplify(eval(C, solutions));$

$$C_{sol} := \frac{-\sin(\theta)\, m g v + P}{v^3}$$

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Minimal # of procedures to learn

Calculate the value given a speed of 18 kph climbing at 2 degrees where the power is 180 W.

$$> \quad C_{value} := eval\left(C_{sol}, \left\{m = 110, g = 9.8, P = 180, \theta = 0.02 \cdot \frac{\pi}{2.}, v = 5\right\}\right);$$

$$C_{value} := 0.08557$$

Use the value for C to calculate the maximimum angle given the maximum short-term power of the rider.

$$> \quad \theta_{max} := solve\left(C_{sol} = C, \theta\right)$$

$$\theta_{max} := -\arcsin\left(\frac{C\, v^3 - P}{g\, m\, v}\right)$$

$$> \quad \theta_{max\ value} := eval\left(\theta_{max}, \left\{C = C_{value}, m = 110, g = 9.8, P = 400, v = 1.7\right\}\right)$$

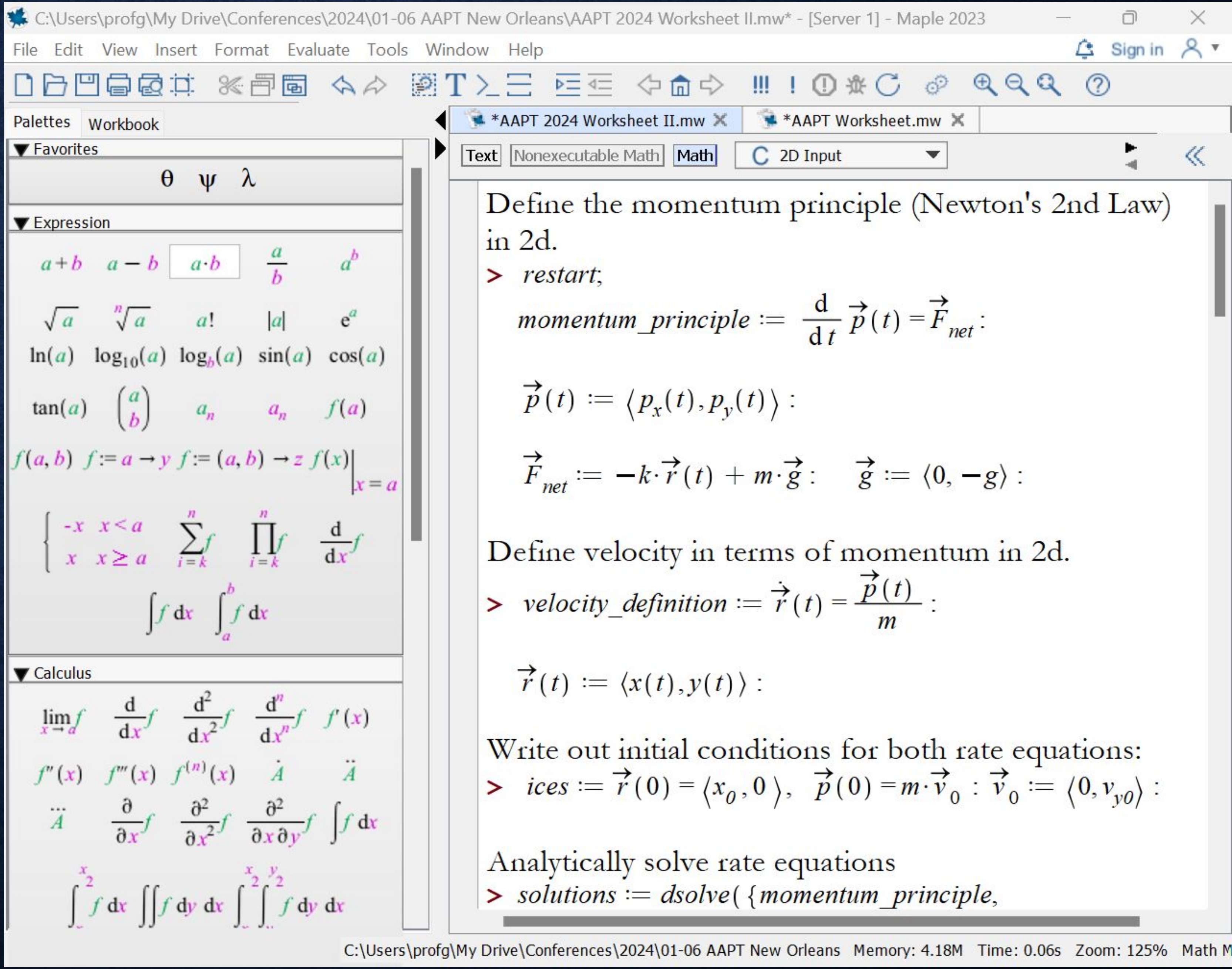
$$\theta_{max\ value} := 0.21981$$

$$> \quad \theta_{degrees} := \theta_{max\ value} \cdot \left(\frac{2}{\pi}\right) \cdot 100$$

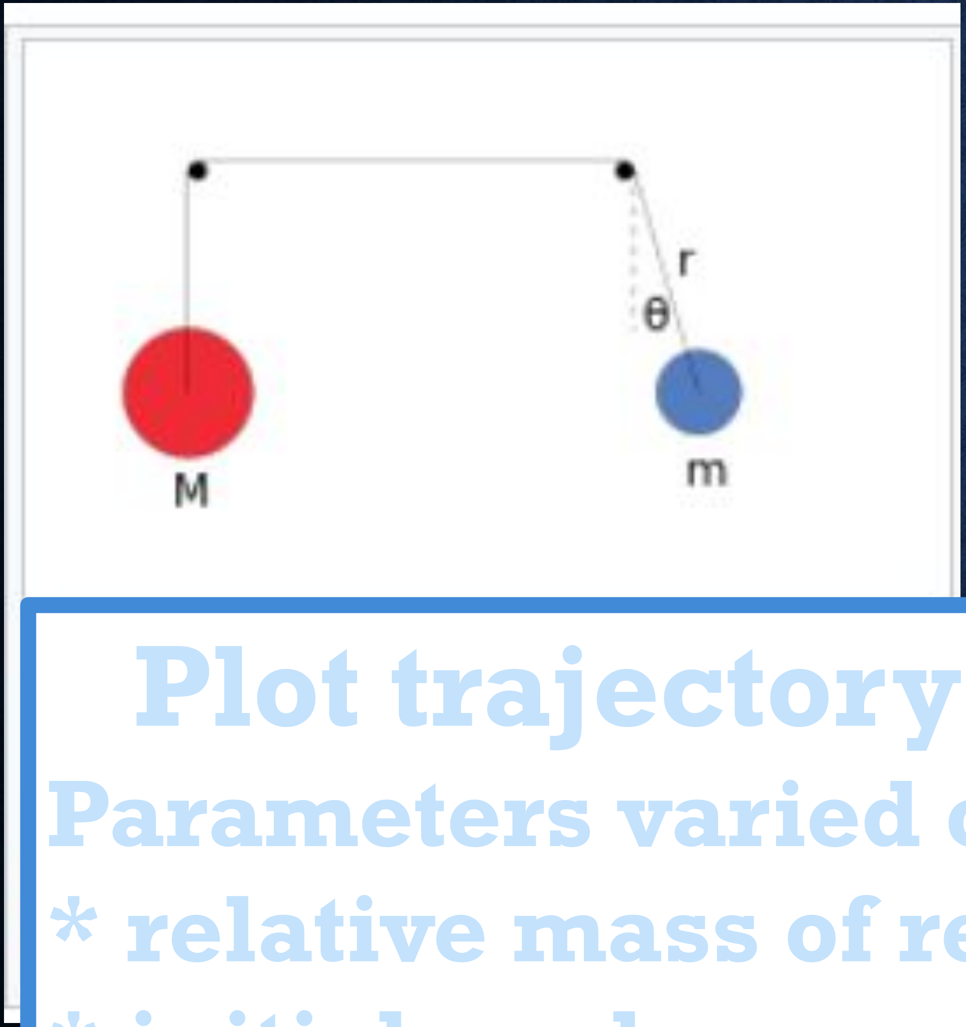
$$\theta_{degrees} := 13.99325$$

Student buy-in? Yes!

- Low barrier to generate content
- Maple math = written math
- Symbolic calculations
- Minimal coding □ *solve, eval, plot*
- Immediate feedback
- Not a black box sim.
- DEs in intro



"What if" example



Derive the Lagrangian and initial condition equations

> $odes := \quad (M+m)\cdot\ddot{r}(t) = m\cdot r(t)\cdot\dot{\theta}(t)^2 - M\cdot g + m\cdot\cos(\theta(t)),$

$$r(t)^2\cdot\ddot{\theta}(t) + 2\cdot r(t)\cdot\dot{r}(t)\cdot\dot{\theta}(t) = -g\cdot r(t)\cdot\sin(\theta(t)) :$$

$ices := \quad r(0) = r_0, \quad \dot{r}(0) = 0, \quad \theta(0) = \theta_0, \quad \dot{\theta}(0) = 0 :$

Numerical problem: all constants need a value unless on-the-fly parameters.

> $M := \alpha\cdot m : \quad m := 1.0 : \quad r_0 := 1.0 : \quad g := 9.8 :$

Solve differential equations numerically and extract solutions:

> $solutions := dsolve(\{odes, ices\}, numeric,$

$s = [\alpha, \theta_0], output = listprocedure) :$

$= eval(\theta(t), solutions) :$

arameters, 2) plots trajectory of blue ball.

$]) :$

$\cdot\cos(\theta(t)), t = 0..10],$

view = [..., 1.3 ..1.3], *title* = "Trajectory of Small Mass");

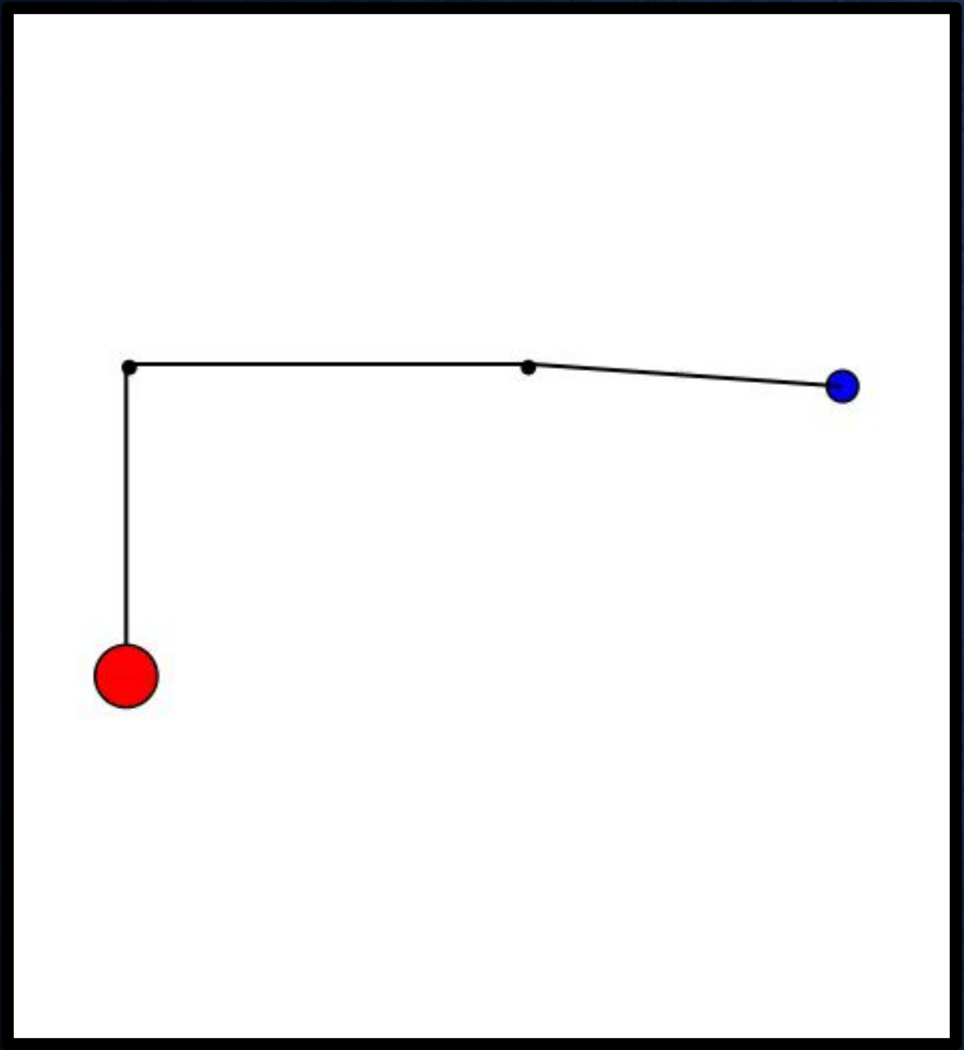
end proc:

Plot trajectory of blue ball

Parameters varied on demand

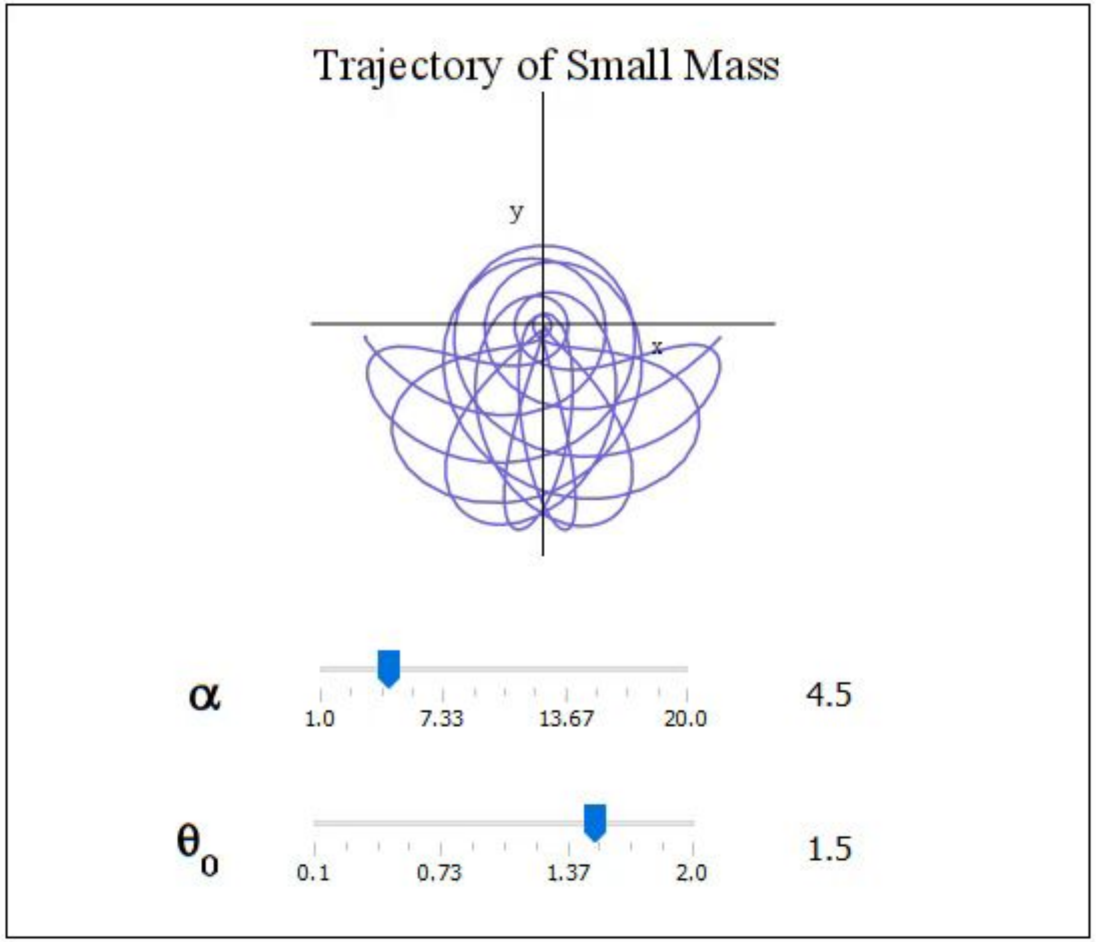
*** relative mass of red ball &**

*** initial angle**



Explore the procedure with parameters on sliders:

```
> Explore(Trajectory(α, θ_0),
        parameters = [α = 1 ..20.0, θ_0 = 0.1 ..2.0],
        initialvalues = [α = 4.5, θ_0 = 1.5]);
```



Maple Immersion:

- Use Maple from day 1. In class, walk through calculations, line by line, building confidence.
- Present all calculations/complex derivations in Maple.
- Assign problems that cannot be solved by hand.
- Emphasize creating graphics & "What-if apps."
- **Homework & exam submissions: Maple worksheet *ONLY***
- Rely on the **Learning Maple Textbook / Video series** to teach and remind students how to use Maple.

Learning Maple: Max Productivity-Min Coding

Maple Fundamentals:

- 1: Setting Up Maple and Finding Help (document)
- 2: Maple as a Calculator (document)
- 3: Writing Symbolic Expressions (document)
- 4: Solving Symbolic Equations (document)
- 5: Solving Numeric Equations (document)
- 6: User-generated Functions (document)
- 7: 2d Plotting (document)
- 8: Document Enhancement (document)

Mathematics:

- **Complex Numbers 1**: Fundamentals (document)
- **Vectors 1**: Cartesian Coordinates (document)
- **Vectors 2**: Vector Products (document)
- **Statistics 1**: Descriptive Statistics (document)
- **Statistics 2**: Curve Fitting (document)
- **Calculus 1**: Limits & Differentiation (document)
- **Calculus 2**: Integration (document)
- **Calculus 3**: Summation & Series (document)

Learning Maple: Max Productivity-Min Coding

Maple Instructional Videos/Documents for Science and Engineering

https://**gould.prof** *or* https://**YouTube/@MapleProf**

- Each video is limited to**12 minutes**
- Minimal number of procedures to learn
- Practice problems from the physics undergraduate curriculum
- Embedded Maple coding instruction where appropriate
- Documents: additional problems & Troubleshooting

Advantages of teaching Physics using Maple – concepts, concepts, concepts

- Principles: emphasized through mathematics, not black box
- Top-down problem-solving approach – like physics
- Reduction of math minutia and teaching tricks
- Interpretative interface near-immediate feedback.
- Solutions are symbolic & beyond the spherical cow
- Transferable principles and skills ($149 – student version)

Learning Physics Using Maple

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Now seeking collaborators

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Advanced Mathematics:

- **Ordinary Differential Equations 1:** Symbolic (document)
- **Ordinary Differential Equations 2:** Numeric (document)
- **Ordinary Differential Equations 3:** Systems of ODEs (document)
- **Ordinary Differential Equations Topic:** Boundary Value Problems (document)

- **Linear Algebra 1:** Matrices Arithmetic (document)
- **Linear Algebra 2:** Eigenvalues & Eigenvectors (document)
- **Linear Algebra Topic:** Linear Transformations (document)

- **Advanced Mathematics Topic:** Fourier Series (document)
- **Advanced Mathematics Topic:** Transformations (document)
- **Advanced Mathematics Topic:** Dirac delta function (document)

- **Partial Differential Equations 1:** Basics (document)
- **Partial Differential Equations Topic:** Heat Equation (document)

- **Vector Calculus 1:** Div, Grad, Curl (document)
- **Vector Calculus 2:** Integrals (document)
- **Vector Calculus 3:** Fundamental Theorems (document)

Useful Maple Procedures:

- **Evaluate expressions - *eval*:** (document)
- **Sequence generator - *seq*:** (document)
- **Conditional procedures - *ifelse, piecewise, Heaviside* :** (document)

- **Random numbers - *rand, randomize*:** (document)
- **Extrema - *minimize, maximize*:** (document)

- **Animation - *plots:-Animate* :** (document) (worksheet)
- **Exploration application generator - *Explore*:**