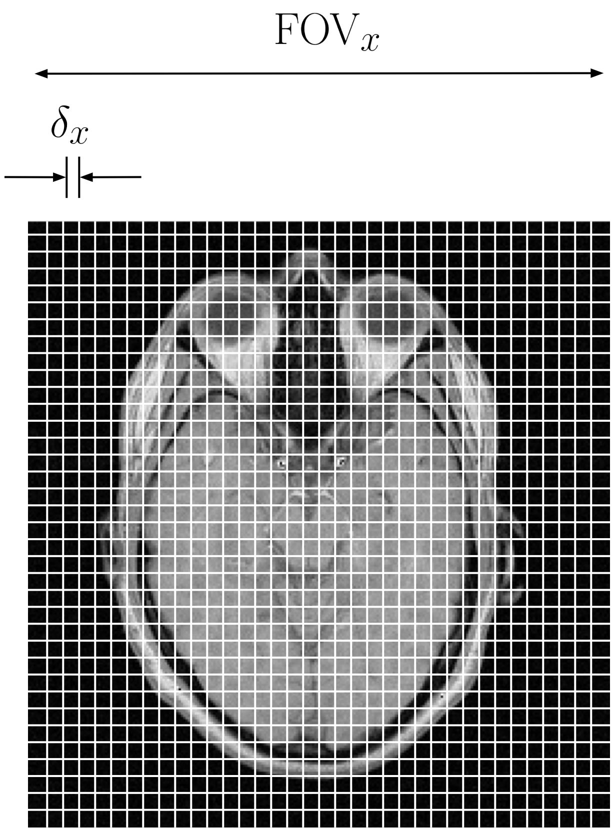
Week 4 Section

Sampling and Parallel Imaging

Nyquist Review

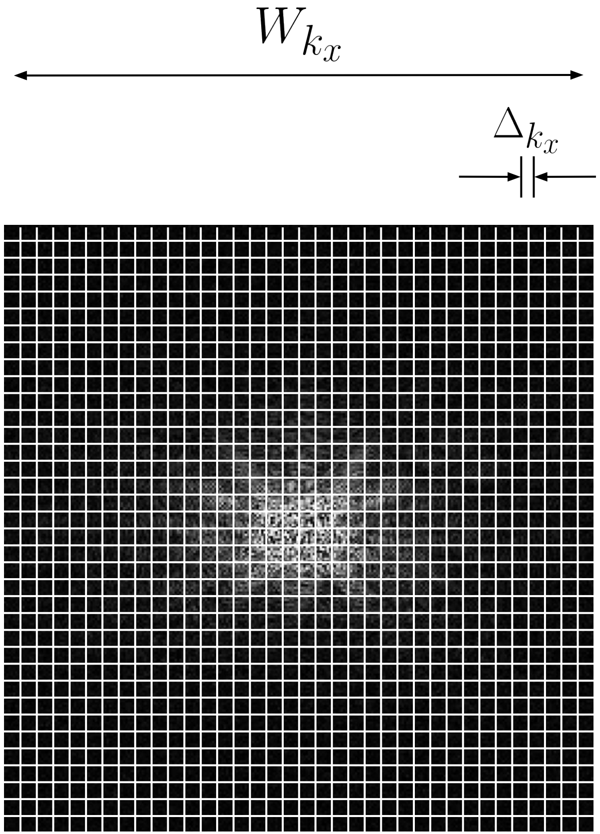


$$W_{k_x} \geq \frac{1}{\delta_x}$$

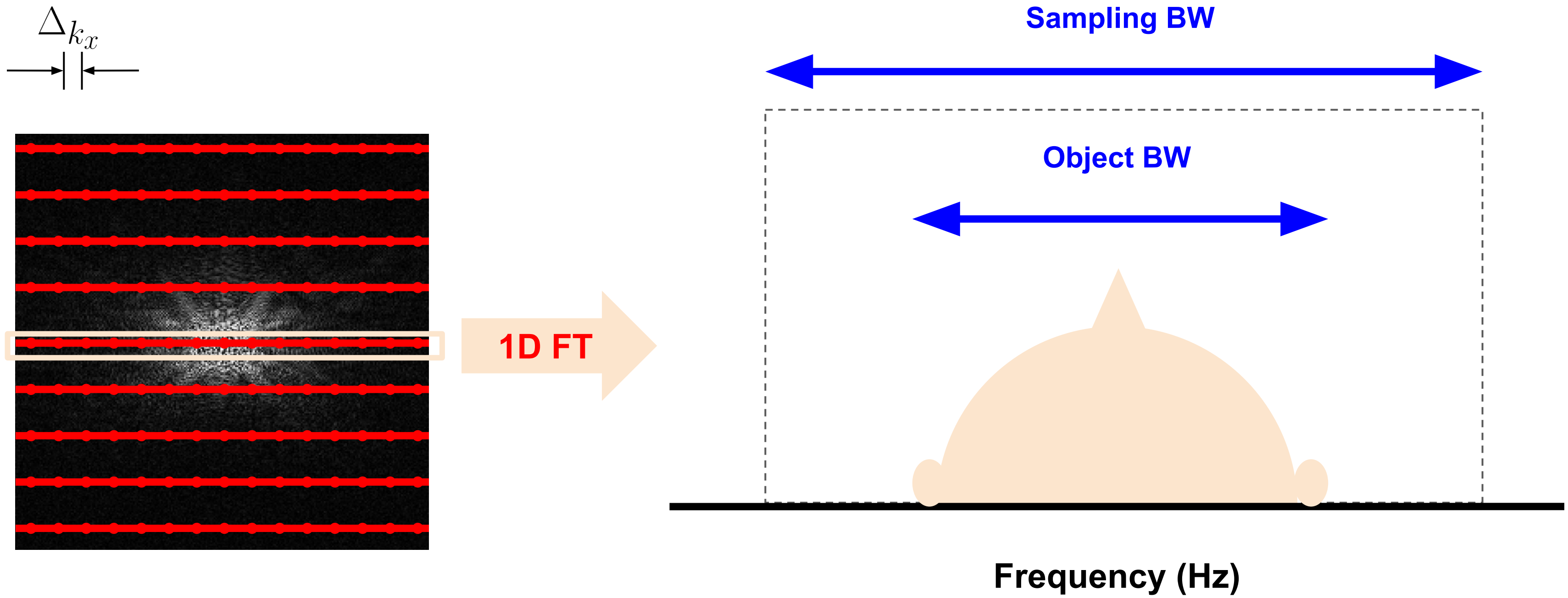
$$\Delta_{k_x} \leq \frac{1}{\mathrm{FOV}_x}$$

****Same Holds for Y-direction****

2DFT

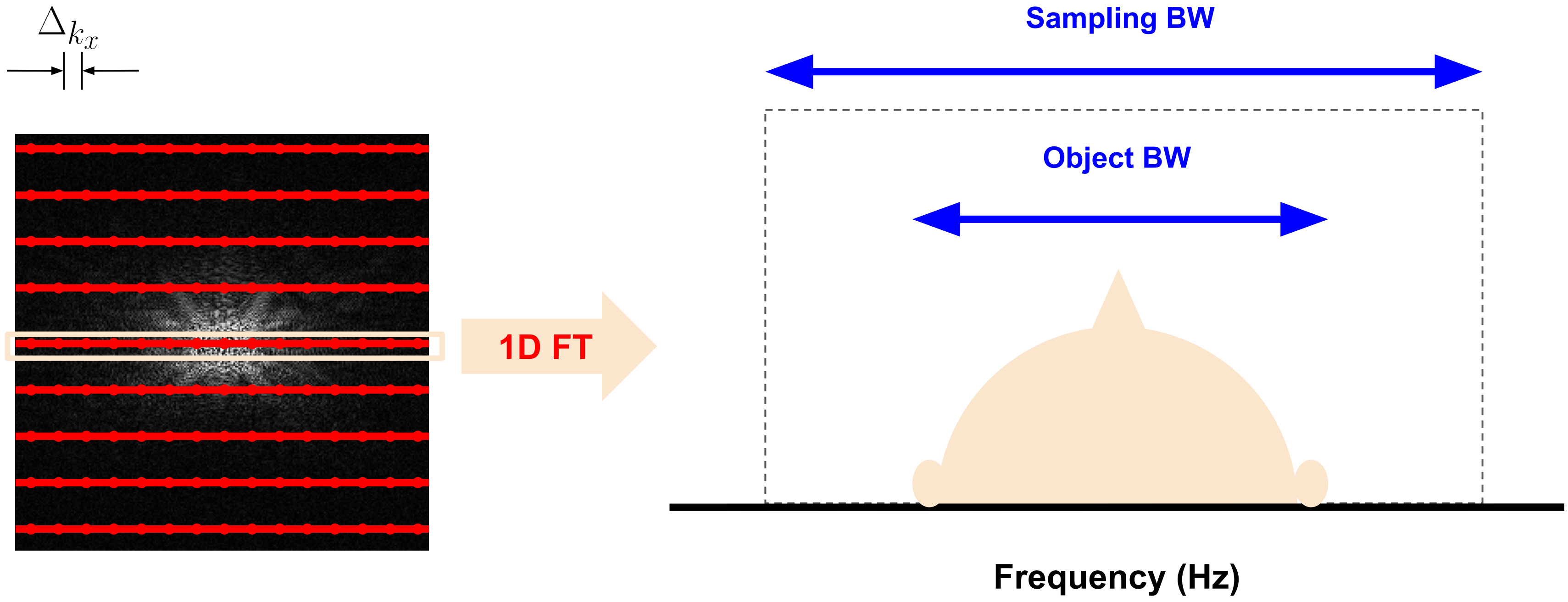


Readout Sampling Example: 2DFT



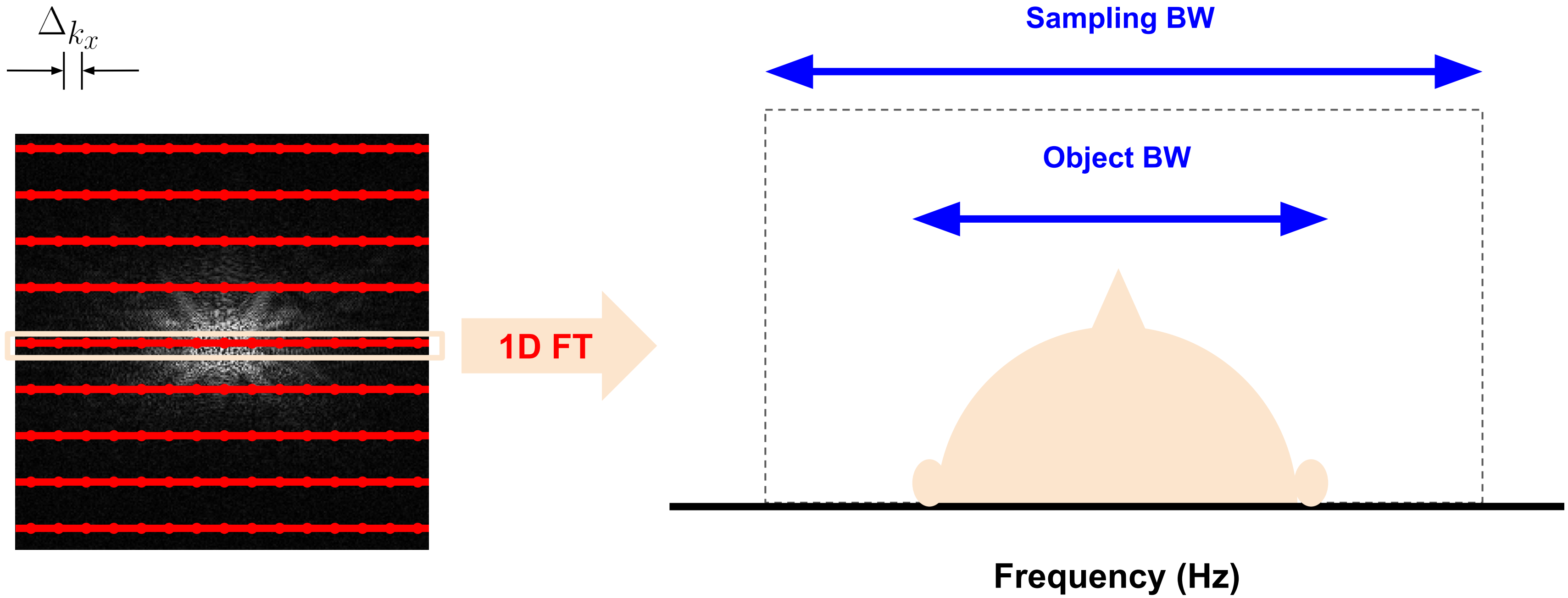
What determines the *object* bandwidth?

Readout Sampling Example: 2DFT



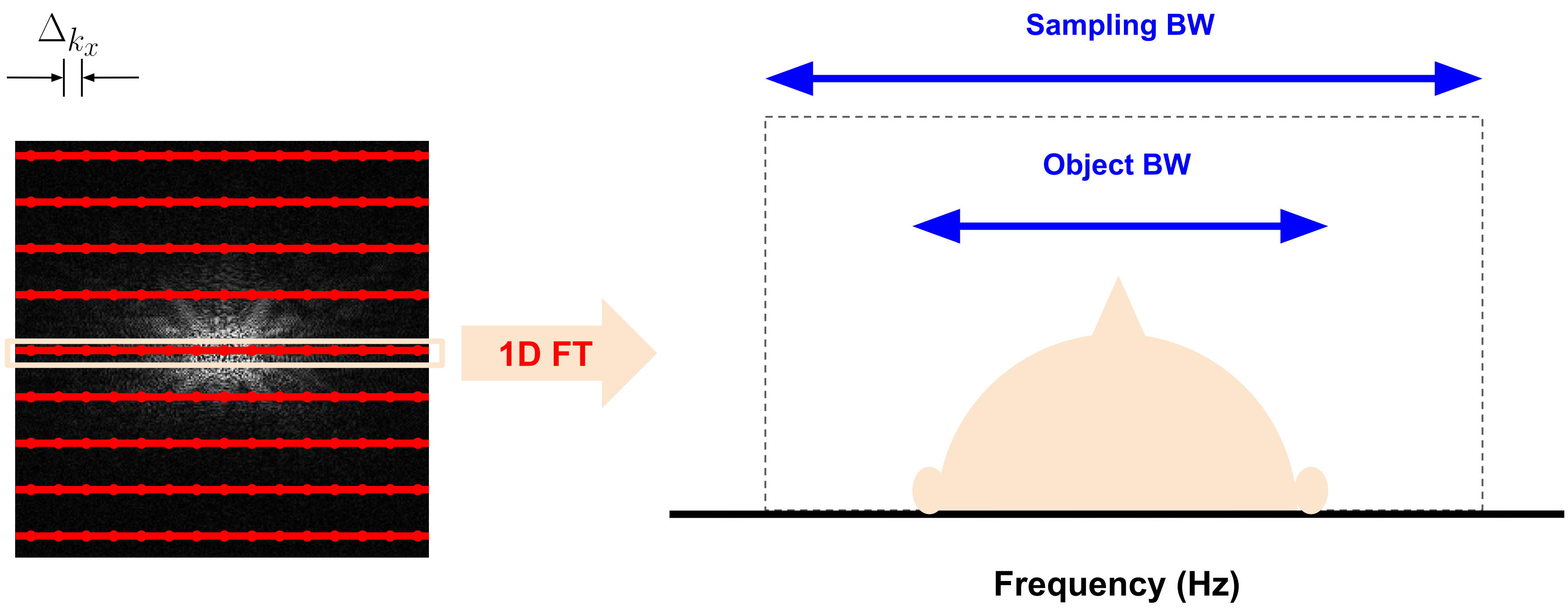
$$\mathrm{BW}_{\mathrm{object}} = \frac{\gamma}{2\pi} G_x \mathrm{FOV}_x$$

Readout Sampling Example: 2DFT



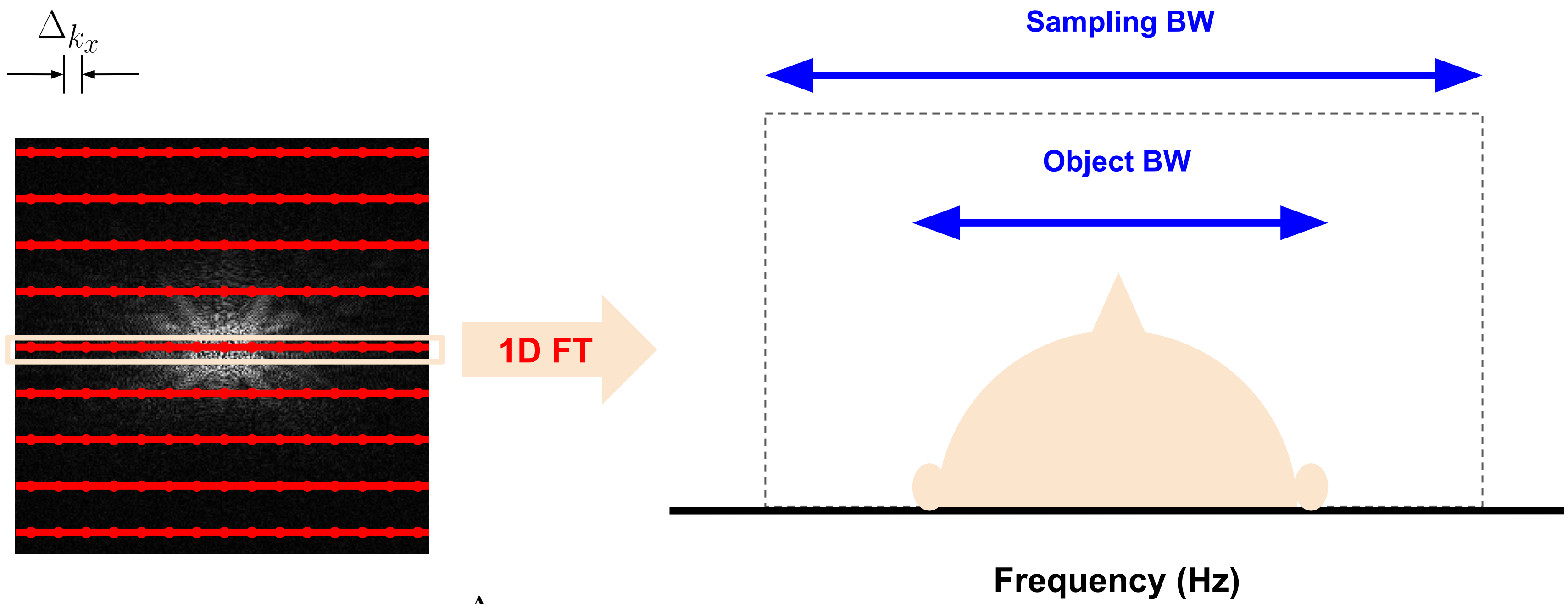
What is the *sampling* bandwidth?

Readout Sampling Example: 2DFT



The rate at which your ADC receives samples: $\frac{1}{\Delta t}$

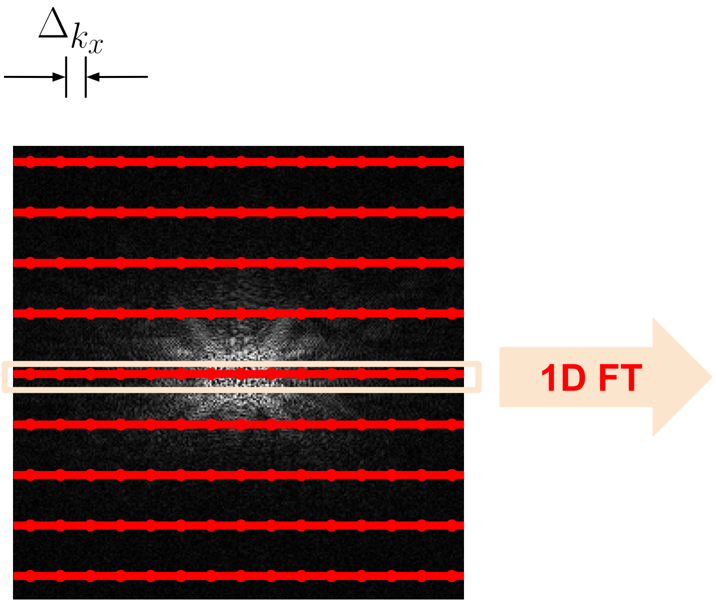
Readout Sampling Example: 2DFT

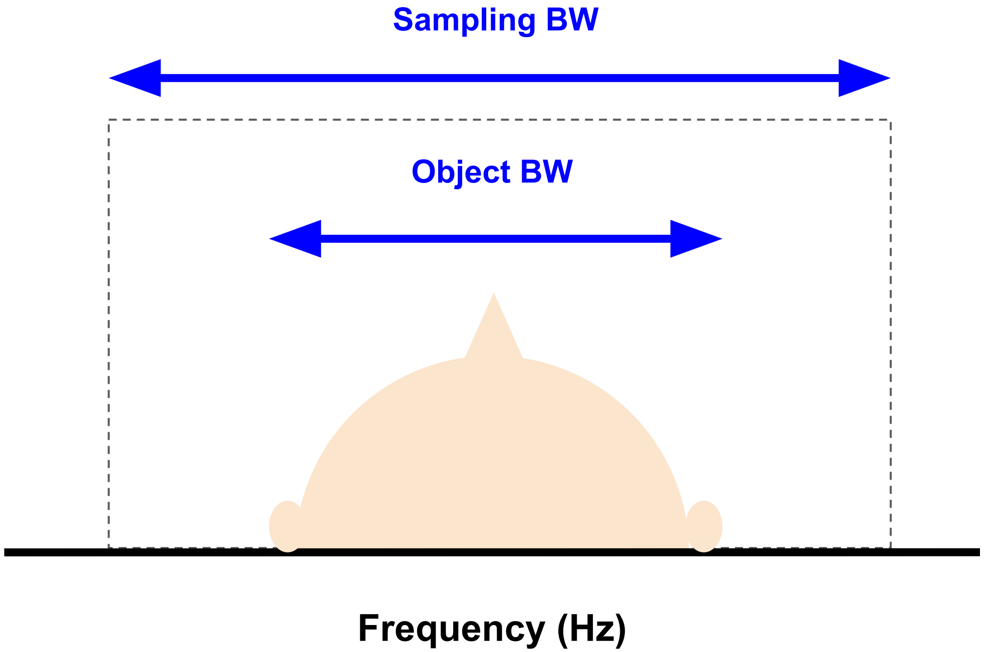


How is Δ_{k_x} related to the sampling bandwidth/rate?

Hint: $k_x(t) = \frac{\gamma}{2\pi}\int_0^t G_x(\tau)d\tau$

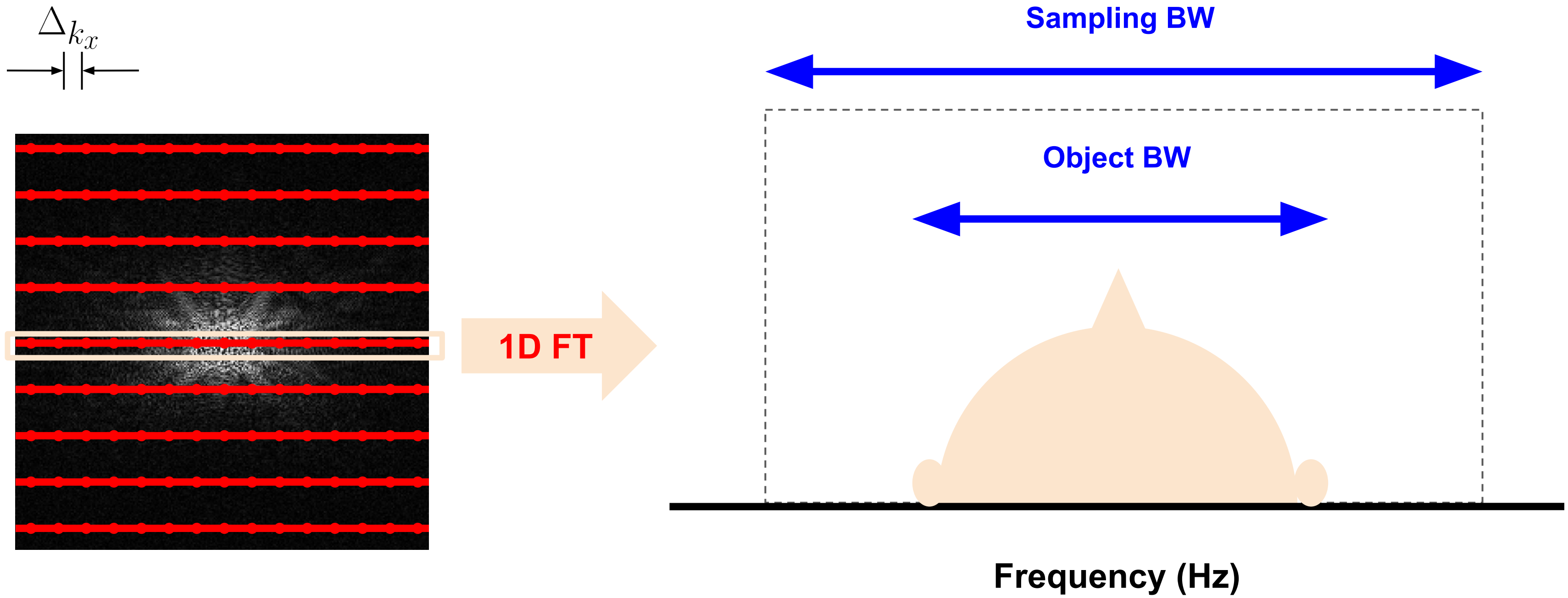
Readout Sampling Example: 2DFT





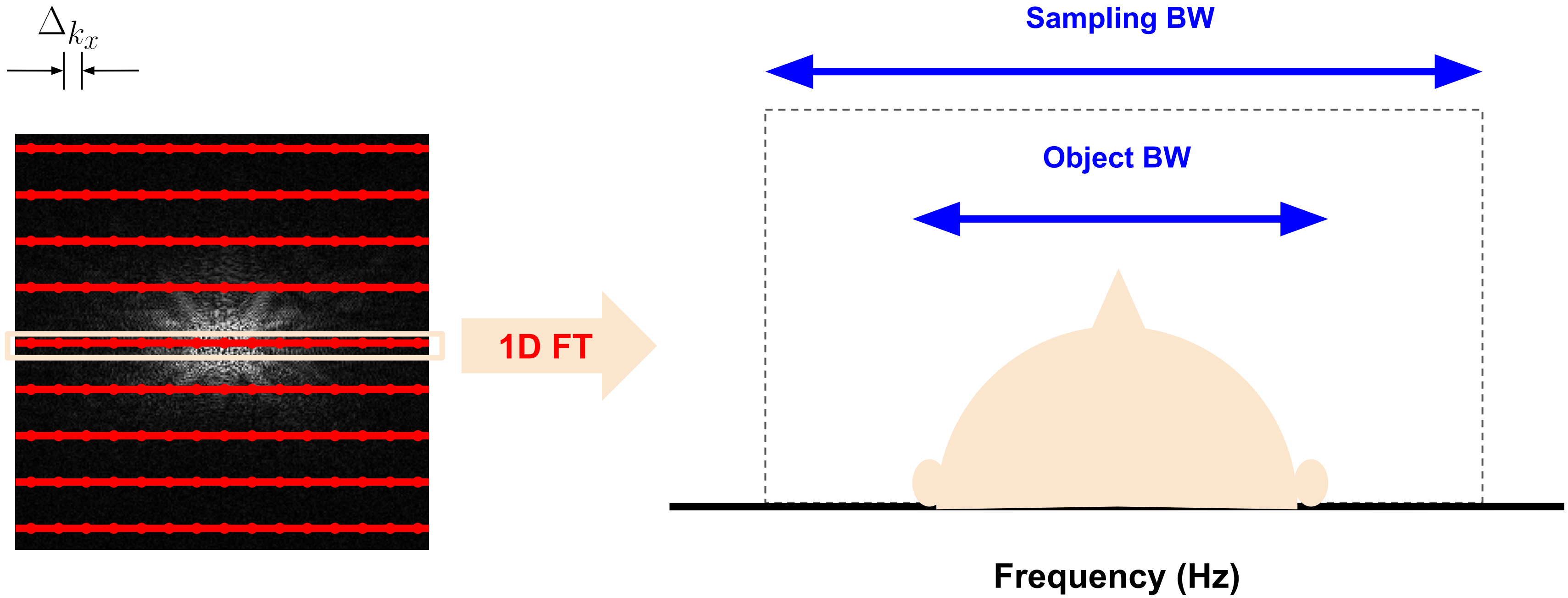
$$\Delta_{k_x} = \frac{\gamma G_x \Delta t}{2\pi} = \frac{\gamma G_x}{2\pi \mathrm{BW_{samp}}}$$

Readout Sampling Example: 2DFT



$$\Delta_{k_x} \leq \frac{1}{\mathrm{FOV}_x} \implies \mathrm{BW}_{\mathrm{samp}} \geq \mathrm{BW}_{\mathrm{object}}$$

Readout Sampling Example: 2DFT

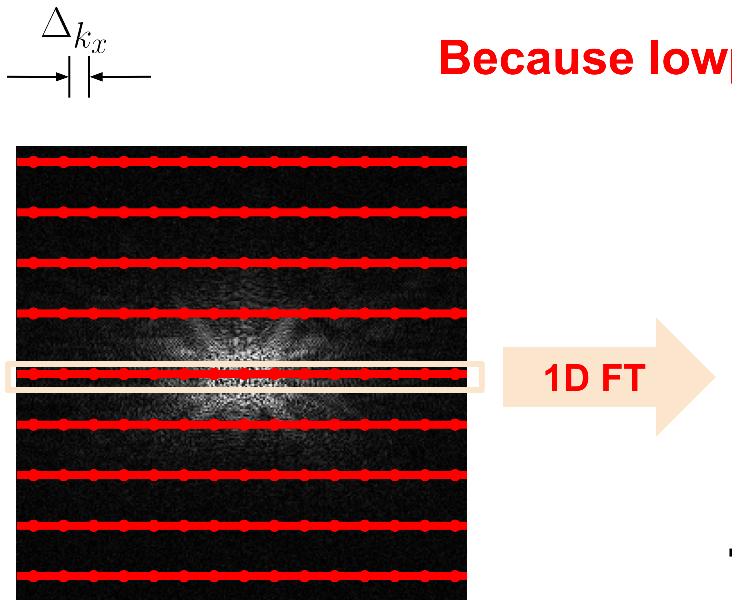


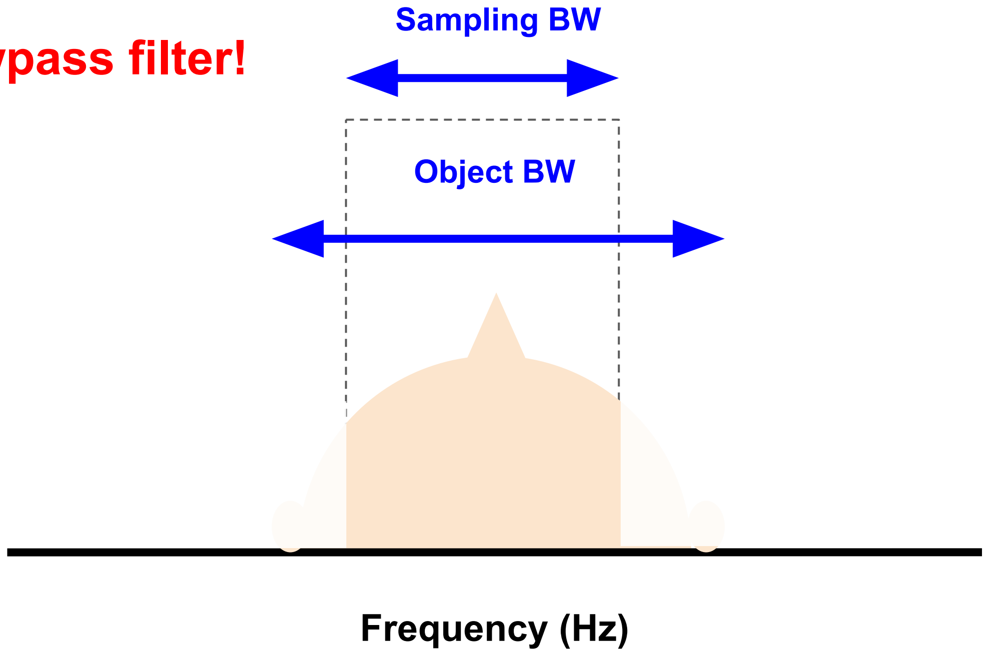
What happens if $\mathrm{BW}_{\mathrm{samp}} \leq \mathrm{BW}_{\mathrm{object}}$ **?**

a.) Aliasing along x **b.) image cut off along x** **c.) nothing**

Readout Sampling Example: 2DFT

Because lowpass filter!

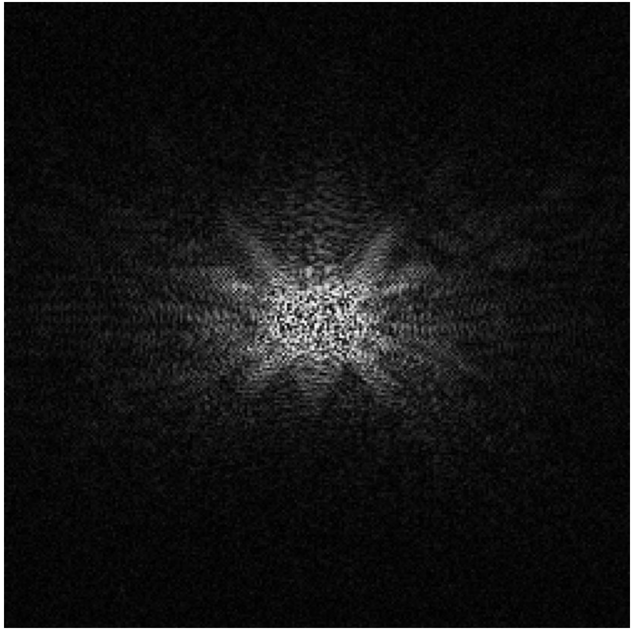




What happens if $\mathrm{BW}_{\mathrm{samp}} \leq \mathrm{BW}_{\mathrm{object}}$?

a.) Aliasing along x **b.) image cut off along x** c.) nothing

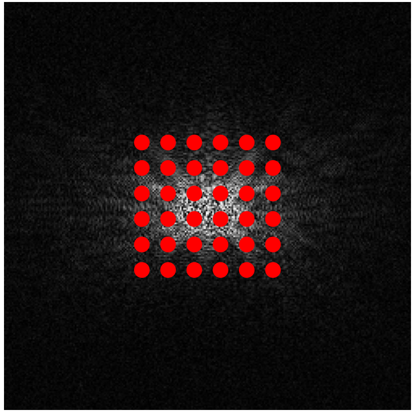
Readout Sampling Example: Spiral



When designing **complicated** trajectories, need to make sure that $\Delta_k \leq \frac{1}{\mathrm{FOV}_{\mathrm{radial}}}$ is satisfied **everywhere**

FOV and Resolution Model

$$\hat{M}(k_x, k_y) = \frac{1}{\Delta_{k_x}\Delta_{k_y}} \text{III}^2\left(\frac{k_x}{\Delta_{k_x}}, \frac{k_y}{\Delta_{k_y}}\right) \sqcap^2\left(\frac{k_x}{W_x}, \frac{k_y}{W_y}\right) M(k_x, k_y)$$



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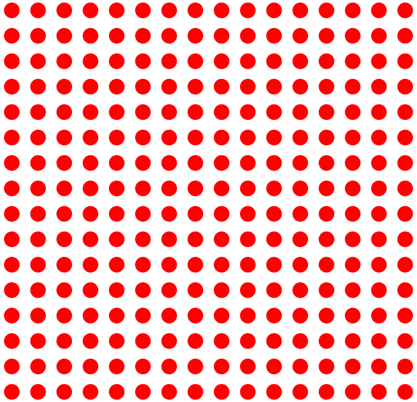
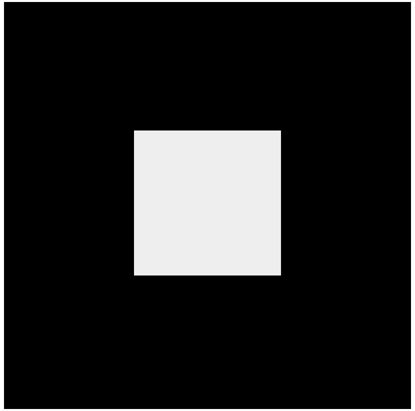
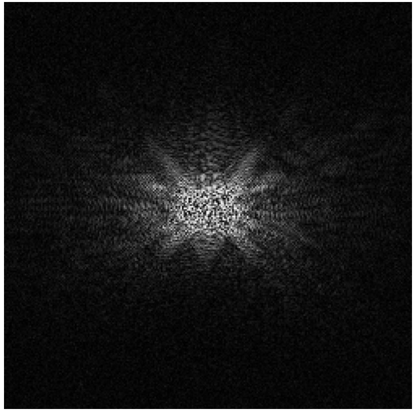






FOV and Resolution Model

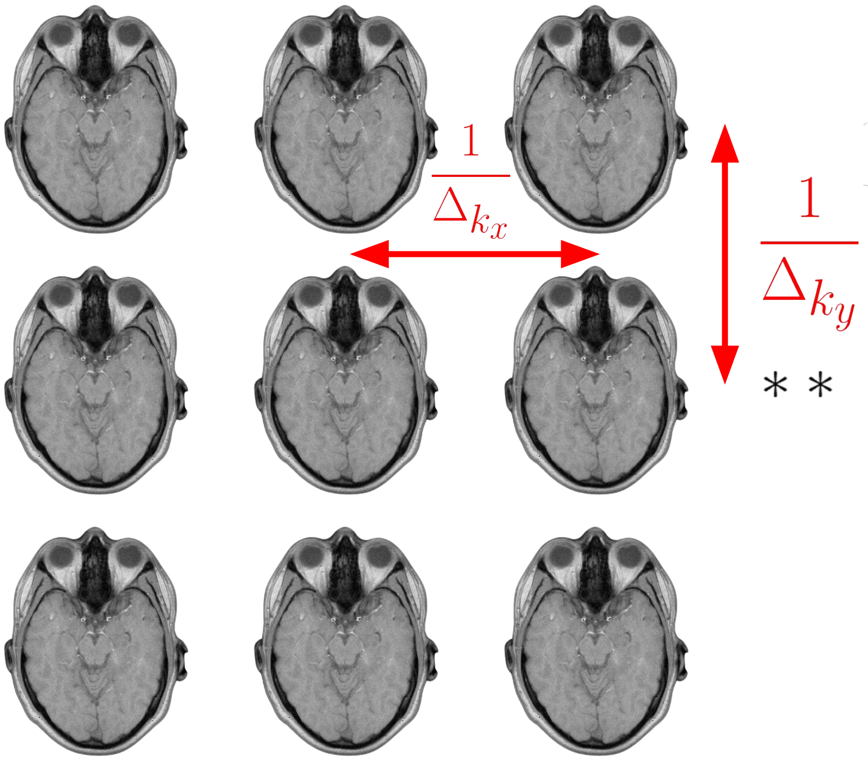
$$\hat{M}(k_x, k_y) = \frac{1}{\Delta_{k_x}\Delta_{k_y}} \text{Ш}^2\left(\frac{k_x}{\Delta_{k_x}}, \frac{k_y}{\Delta_{k_y}}\right) \quad \sqcap^2\left(\frac{k_x}{W_x}, \frac{k_y}{W_y}\right) \quad M(k_x, k_y)$$

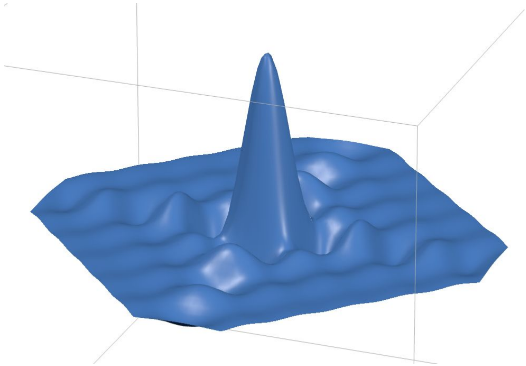
FT

$$\hat{m}(x, y) = \text{Ш}(\Delta_{k_x} x, \Delta_{k_y} y) ** W_{k_x} W_{k_y} \mathrm{sinc}(W_x x)\mathrm{sinc}(W_y y) ** m(x, y)$$

FOV and Resolution Model

$$\hat{m}(x,y) =$$





$\mathrm{FWHM}_y \propto W_{k_y}$

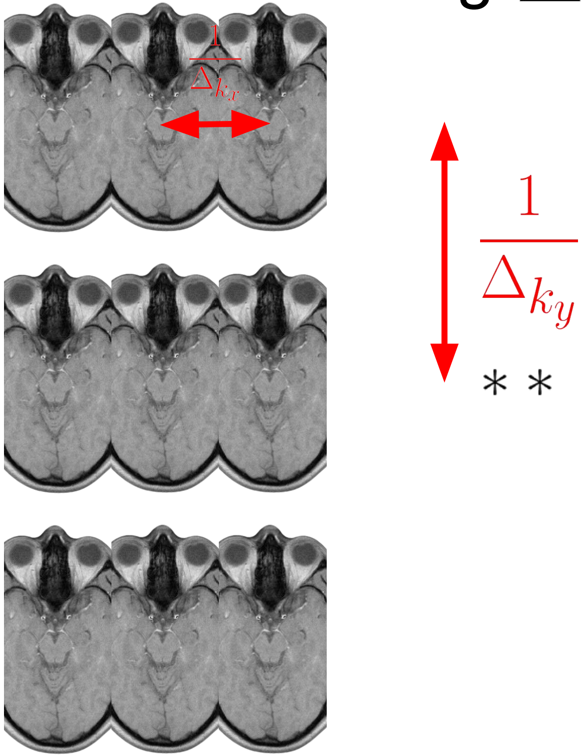
$\mathrm{FWHM}_x \propto W_{k_x}$

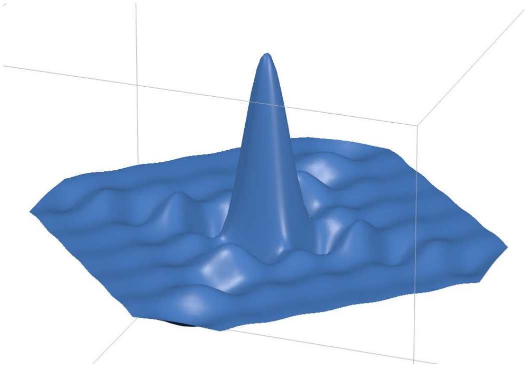
$$\hat{m}(x,y) = \text{Ш}(\Delta_{k_x} x, \Delta_{k_y} y) ** W_{k_x} W_{k_y} \mathrm{sinc}(W_x x)\mathrm{sinc}(W_y y) ** m(x,y)$$

FOV and Resolution Model: Increasing Δ_{k_x}

only if x is readout direction

$$\hat{m}(x,y) =$$





$\text{FWHM}_y \propto W_{k_y}$

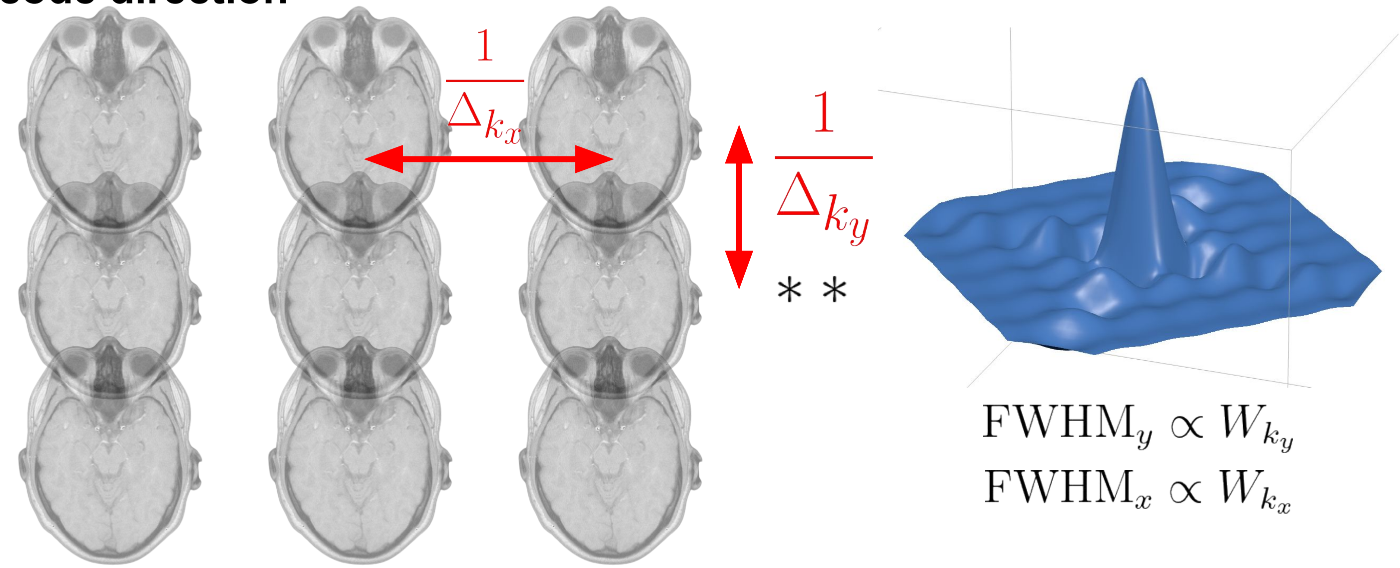
$\text{FWHM}_x \propto W_{k_x}$

$$\hat{m}(x,y) = \text{Ш}(\Delta_{k_x}x, \Delta_{k_y}y) ** W_{k_x}W_{k_y}\text{sinc}(W_x x)\text{sinc}(W_y y) ** m(x,y)$$

FOV and Resolution Model: Increasing Δ_{k_y}

only if y is phase encode direction

$$\hat{m}(x,y) =$$



$\text{FWHM}_y \propto W_{k_y}$

$\text{FWHM}_x \propto W_{k_x}$

$$\hat{m}(x,y) = \text{III}(\Delta_{k_x}x, \Delta_{k_y}y) ** W_{k_x}W_{k_y}\text{sinc}(W_x x)\text{sinc}(W_y y) ** m(x,y)$$

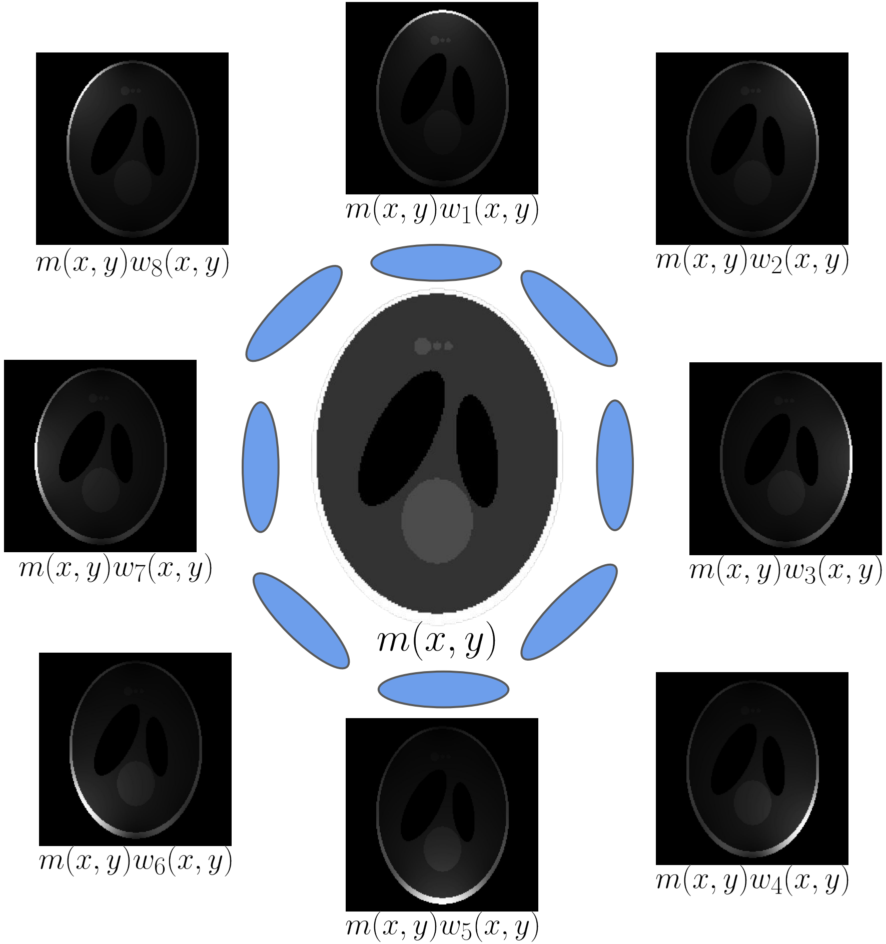
Parallel Imaging

Multi-Coil Receive

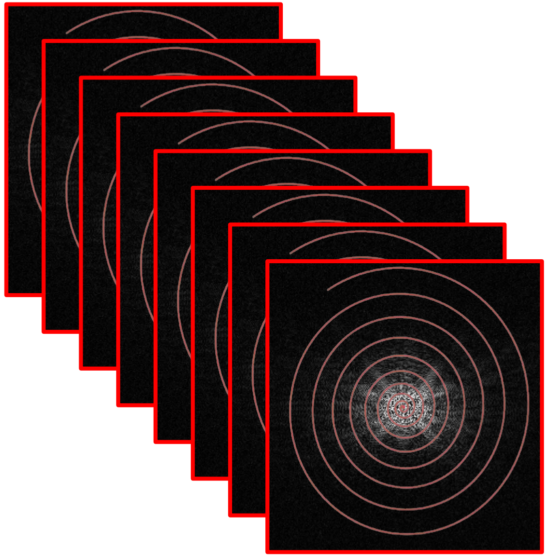
- MRIs have multiple coils at different positions around the subject
- These coils will see slightly different images due to proximity from the signal source (subject)
- The signal recorded from coil c is:

$$s_c(t) = M_c(k_x(t), k_y(t))$$
$$M_c(k_x, k_y) = \mathcal{F}\{w_c(x,y)m(x,y)\}$$

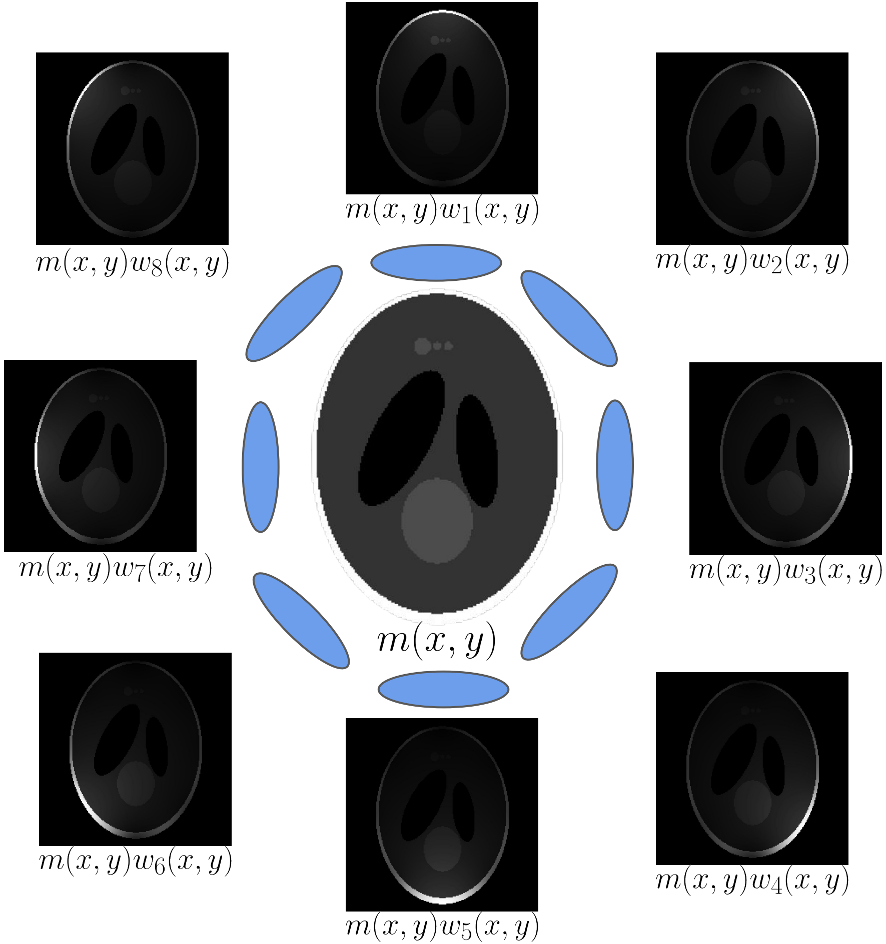
- Where $w_c(x,y)$ is the coil's weighting function (sensitivity map)



Multi-Coil Receive



Each coil reads unique data,
but same trajectory



Multi-Coil Receive: Linear Inverse Model

$$\underbrace{\begin{bmatrix} b_1 \\ \vdots \\ b_C \end{bmatrix}}_{b} = \underbrace{\begin{bmatrix} FS_1 \\ \vdots \\ FS_C \end{bmatrix}}_{A} x$$

$$\min_x ||Ax - b||_2^2 + \mathcal{R}(x)$$

We won't be covering this in much detail, check out EE369C for more!