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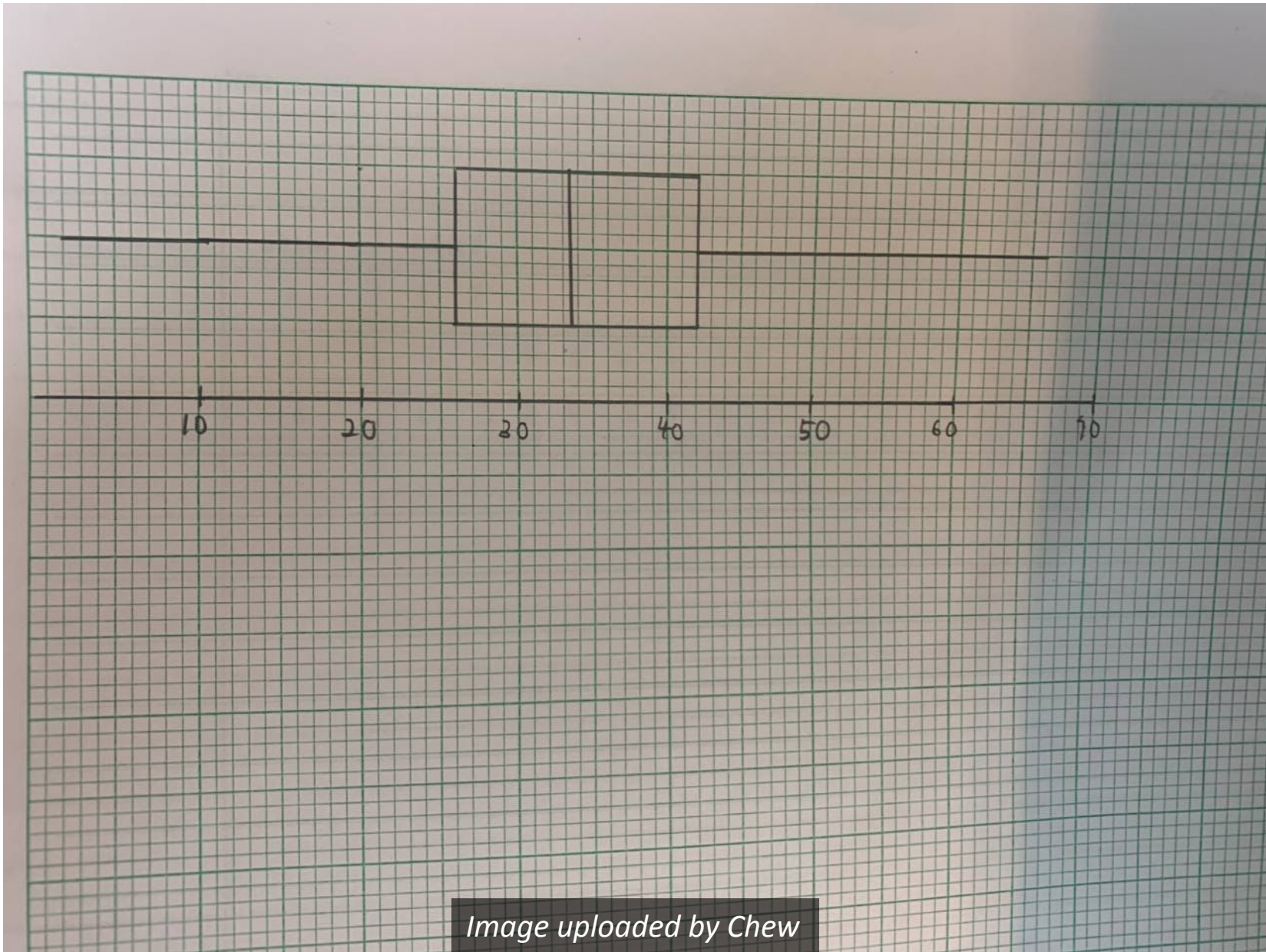


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$$(2) P(A \cup B) = P(A) + P(B)$$

$$= \frac{1}{5} + \frac{1}{10}$$

$$= \frac{3}{10}$$

$$P(A \cap C) = P(A) \times P(C)$$

$$P(A \cup C) = P(A) + P(C) - P(A)P(C)$$

$$\frac{7}{15} = \frac{1}{5} + P(C) - \frac{1}{5}P(C)$$

$$\frac{4}{5}P(C) = \frac{4}{15}$$

$$P(C) = \frac{1}{3}$$

$$P(B \cap C) = P(B) \times P(C)$$

$$P(B \cup C) = P(B) + P(C) - P(B \cap C)$$

$$\frac{23}{60} = \frac{1}{10} + \frac{1}{3} - P(B \cap C)$$

$$P(B \cap C) = \frac{1}{20}$$

~~It~~

$$P(B)P(C) = \frac{1}{30}$$

Since $\frac{1}{20} \neq \frac{1}{30}$, $P(B \cap C) \neq P(B)P(C)$, events B and C are not independent.

a)

Stem	Leaf
1	8 8
2	0 1 5 ⁵ 6 6 9
3	1 2 2 3 4 5
4	0 1 2 3 5 7
5	1 5

$$LF = 26 - 1.5(19) = -2.5$$

$$UF = 45 + 1.5(19) = 73.5$$

key: 511 means

b) $r = \frac{26}{2} = 13$

51

$$\text{median} = \frac{x_{13} + x_{14}}{2}$$

$$= \frac{23 + 34}{2}$$

$$= 33.5$$

$$Q_1: r = \frac{1}{4} \times 26 = 6.5$$

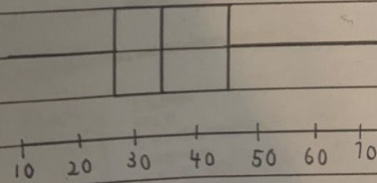
$$x_{[7]} = 26$$

$$Q_3: r = \frac{3}{4} \times 26 = 19.5$$

$$x_{[20]} = 45$$

$$IQR: Q_3 - Q_1 = 45 - 26 = 19$$

c)



d) $Q_3 - Q_2 = 45 - 33.5 = 11.5$

$$Q_2 - Q_1 = 33.5 - 26 = 7.5$$

$$\text{Since } Q_2 - Q_1 < Q_3 - Q_2$$

the distribution is negatively skewed

6) H_0 : There is no association between English and Mathematics results.

H_1 : There is an association between English and Mathematics

O_i	E_i	$\frac{(O_i - E_i)^2}{E_i}$
17	21.105	0.798
28	26.775	0.056
18	15.92	0.549
28	33.165	0.705
45	42.075	0.203
16	23.76	2.534
12	12.73	0.042
12	16.15	1.066
14	9.2	2.611

$\chi^2 = 8.564$

$\nu = 4$

$\chi^2_{(4)} = 9.488$

If $\chi^2 > 9.488$, H_0 is rejected

$\chi^2 = 8.564 < 9.488$, H_0 is not rejected.

There is insufficient evidence at 5% significance level, there is no association between English and Maths

(a)

$$7. \int_1^b \sqrt{\frac{n-1}{12}} dn = 1$$

$$\left[\frac{(n-1)^{\frac{3}{2}}}{\frac{3}{2}} \right]_1^b = \sqrt{12}$$

$$\frac{2}{3} [(b-1)^{\frac{3}{2}} - 2\sqrt{3}] = 2\sqrt{3}$$

$$b = 4$$

(b) $b=4$

$$f(n) = \begin{cases} \sqrt{\frac{n-1}{12}} & , 1 \leq n \leq 4 \\ 0 & \end{cases}$$

For $n < 1$

$$F(n) = P(X \leq n) = \int_{-\infty}^n 0 dy = 0$$

For $n \geq 4$

$$F(n) = P(X \leq n) = \int_1^4 \sqrt{\frac{n-1}{12}} dn + \int_4^n f(n) dn$$

$$= \frac{1}{\sqrt{12}} \int_1^4 (n-1)^{\frac{1}{2}} dn + 0$$

$$= \frac{1}{\sqrt{12}} \left[\frac{(n-1)^{\frac{3}{2}}}{\frac{3}{2}} \right]_1^4$$

$$= \frac{1}{3\sqrt{3}} (4-1)^{\frac{3}{2}}$$

$$= 1$$

For $1 \leq n < 4$

$$F(n) = P(X \leq n) = 0 + \int_1^n \sqrt{\frac{n-1}{12}} dn$$

$$= \frac{1}{\sqrt{12}} \int_1^n (n-1)^{\frac{1}{2}} dn$$

$$= \frac{1}{\sqrt{12}} \left[\frac{(n-1)^{\frac{3}{2}}}{\frac{3}{2}} \right]_1^n$$

$$= \frac{1}{3\sqrt{3}} (n-1)^{\frac{3}{2}}$$

$$F(n) = \begin{cases} 0 & , n < 1 \\ \frac{1}{3\sqrt{3}} (n-1)^{\frac{3}{2}} & , 1 \leq n < 4 \\ 1 & , n \geq 4 \end{cases}$$

(c) $E(X) = \int_1^4 n P(X=n)$

$$= \int_1^4 n \sqrt{\frac{n-1}{12}} dn$$

$$n = n-1 \quad n=1, n=0$$

$$dn = dn \quad n=4, n=3$$

$$= \int_0^3 (n+1) \sqrt{\frac{n}{12}} dn$$

$$= \frac{1}{\sqrt{12}} \int_0^3 (n^{\frac{3}{2}} + n^{\frac{1}{2}}) dn$$

$$= \frac{1}{2\sqrt{3}} \left(\frac{2}{5} 4^{\frac{5}{2}} + \frac{2}{3} 4^{\frac{3}{2}} \right)$$

$$= \frac{14}{5}$$

⑧ a) H_0 : The height of the students could be modelled by a normal distribution with mean 172 and standard deviation 6.
 H_1 : The height of the student could not be modelled by a normal distribution with mean 172 and standard deviation 6.

Height (cm), n	O_i	$E_i = P\left(\frac{a-172}{6} < Z < \frac{b-172}{6}\right) \times 200$
$n < 149.5$	0	$1 - P(-3.75) \times 200 = 0.018$
$149.5 \leq n < 154.5$	4	$P(-3.75 < Z < -2.917) \times 200 = 0.336$
$154.5 \leq n < 159.5$	6	$P(-2.917 < Z < -2.083) \times 200 = 3.372$
$159.5 \leq n < 164.5$	12	$P(-2.083 < Z < -1.25) \times 200 = 17.404$
$164.5 \leq n < 169.5$	30	$P(-1.25 < Z < -0.417) \times 200 = 46.538$
$169.5 \leq n < 174.5$	64	$P(-0.417 < Z < 0.417) \times 200 = 64.664$
$174.5 \leq n < 179.5$	52	$P(0.417 < Z < 1.25) \times 200 = 46.538$
$179.5 \leq n < 184.5$	18	$P(1.25 < Z < 2.083) \times 200 = 17.404$
$184.5 \leq n < 189.5$	10	$P(2.083 < Z < 2.917) \times 200 = 3.372$
$189.5 \leq n < 194.5$	4	$P(2.917 < Z < 3.75) \times 200 = 0.336$
$n \geq 194.5$	0	$P(3.75) \times 200 = 0.018$

n	O_i	E_i	$\frac{(O_i - E_i)^2}{E_i}$
$n < 149.5, 149.5 \leq n < 164.5$	22	21.130	0.6342
$164.5 \leq n < 169.5$	30	46.538	5.8770
$169.5 \leq n < 174.5$	64	64.664	0.0068
$174.5 \leq n < 179.5$	52	46.538	0.6411
$n \geq 179.5$	32	21.130	3.5919

$$\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i} = 12.151$$

Test at 5% significance level,
degree of freedom, $v = 5 - 1$
 $= 4$

$$\chi_{5\%}^2(4) = 9.488$$

Reject H_0 if $\chi^2 > 9.488$

\therefore Since $\chi^2 = 12.151 > 9.488$, H_0 is rejected,
There is sufficient evidence at 5% significance level
that the height of the students could not be modeled
by a normal distribution with mean 172 and standard
deviation 6.

b) If the mean and the standard deviation were unknown,
need to estimate the mean and variance.

The estimated mean is 173.15 while the estimated
variance is 58.22.

The degree of freedom is calculated by using 5 categories
subtract 3 restrictions. ($v = 5 - 3 = 2$)

$$4) \hat{\mu} = \frac{15310}{100}$$

$$= 153.1$$

$$\hat{\sigma}^2 = \frac{n}{n-1} s^2$$

$$= \frac{100}{100-1} \left[\frac{2870300}{100} - \left(\frac{15310}{100} \right)^2 \right]$$

$$= 266.051$$

$$95\% \text{ confidence} = \left[\bar{x} \pm Z_{\frac{\alpha}{2}} \sqrt{\frac{\sigma^2}{n}} \right]$$

$$\text{interval for } \mu = \left[153.1 \pm Z_{\frac{0.05}{2}} \sqrt{\frac{266.051}{100}} \right]$$

$$= \left[153.1 \pm 1.96 \sqrt{\frac{266.051}{100}} \right]$$

$$= (149.9, 156.3)$$

\therefore the 95% confidence interval for μ
is between $149.9 < \mu < 156.3$

of heads is a
 observed distribution shows evidence that the coins are unbi
 (a) Find and tabulate the frequencies of the number of defective items per
 having the same mean and total as the observed distribution
 (b) Test, at the 5% level, whether the observed distrib
 3. Analysis of goals scored per match
 Calculate

Number of defectives, x	0	1	2	3	4	5
Frequency, f	170	180	120	20	8	8

DATE:

3. $n = 5000$, $p = 0.001$

$X \sim N(5, 4.995)$

$P(X > 2) = P(\bar{X} > 2.5)$

$= P(Z > \frac{2.5 - 5}{\sqrt{4.995}})$

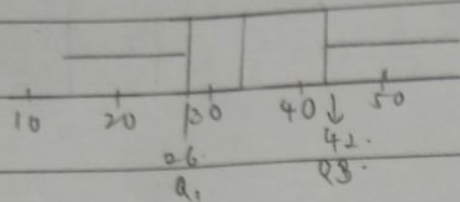
$= 0.8184$

4. $\hat{\mu} = \bar{x}$
 $= \frac{\sum x}{n}$

$\hat{\sigma}^2 = \frac{1}{n-1} S^2$
 $= \frac{100}{99} \left[\frac{2370360}{100} - (152.1)^2 \right]$

15.110
 266.05
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33.5
 (c) $mn = 18$ $max = 55$ $Median = 13$ $Q_1 = 26$ $Q_3 = 42$



(d) Skewed to the right because

$$Q_3 - M > M - Q_1$$

2) $P(A \cup B) = P(A) + P(B)$
 $= \frac{3}{10}$

$$P(A \cap C)$$

$$\frac{7}{15} = \frac{1}{5} + P(C) - P(A \cap C)$$

$$P(C) - P(A \cap C) = \frac{4}{15}$$

$$P(A) \times P(C) = \frac{1}{5} + P(C) - P(A \cup C)$$

$$\frac{1}{5} P(C) = \frac{1}{5} + P(C) - \frac{7}{15}$$

$$\frac{1}{5} P(C) = P(C) - \frac{4}{15}$$

$$\frac{1}{5} P(C) - P(C) = -\frac{4}{15}$$

$$-\frac{4}{5} P(C) = -\frac{4}{15}$$

$$P(C) = \frac{1}{3}$$

B and C are independent.

$$P(B \cap C)$$

$$\frac{23}{60} = \frac{1}{10} + P(C) - P(B \cap C)$$

$$\frac{23}{60} = \frac{1}{10} + \left(\frac{1}{3}\right) - P(B \cap C)$$

$$-P(B \cap C) + \frac{13}{30} = \frac{23}{60}$$

$$-P(B \cap C) = -\frac{1}{20}$$

$$P(B \cap C) = \frac{1}{20} *$$

3) Population proportion of defective bulbs:

$$X \sim N(5, 4.995) \text{ by CLT}$$

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$$\text{mean} = 0.001 (5000) = 5$$

$$= [153.10 \pm 1.040 \frac{-66.051}{1.04}]$$

$$= (149.9, 157.3)$$

5 population: X represents the amount of juice $X \sim N(60, 3^2)$

sample $n = 16$

μ represents mean amount of juice per carton

$H_0: \mu = 60$

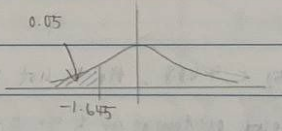
$H_1: \mu < 60$

If H_0 is true, then $\bar{X} \sim N(60, \frac{3^2}{16})$

(lower tailed test)

$\alpha = 0.05$

critical value $= -1.05$
 $= -1.645$



reject H_0 if $Z < -1.645$

$$Z = \frac{59.1 - 60}{\sqrt{\frac{3^2}{16}}}$$

$$= -1.2$$

Since $Z = -1.2 > -1.645$, H_0 is not rejected.

There is insufficient evidence at 5% significance level to uphold the complaints