Inference for two independent means

Randomization for the difference in means

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Inference for comparing two independent means

- Confidence intervals and hypothesis tests to differences in population means that come from two groups,
  - Group 1 and
  - Group 2

Randomization test for the difference in means

- An instructor decided to run two slight variations of the same exam: A & B
- We like to evaluate whether the difference observed in the groups is so large that it provides convincing evidence that Version B was more difficult (on average) than Version A

Difference: 3.1

Group	n	Mean	SD	Min	Max
A	58	75.1	13.9	44	100
B	55	72.0	13.8	38	100

Table 20.1: Summary statistics of scores for each exam version.

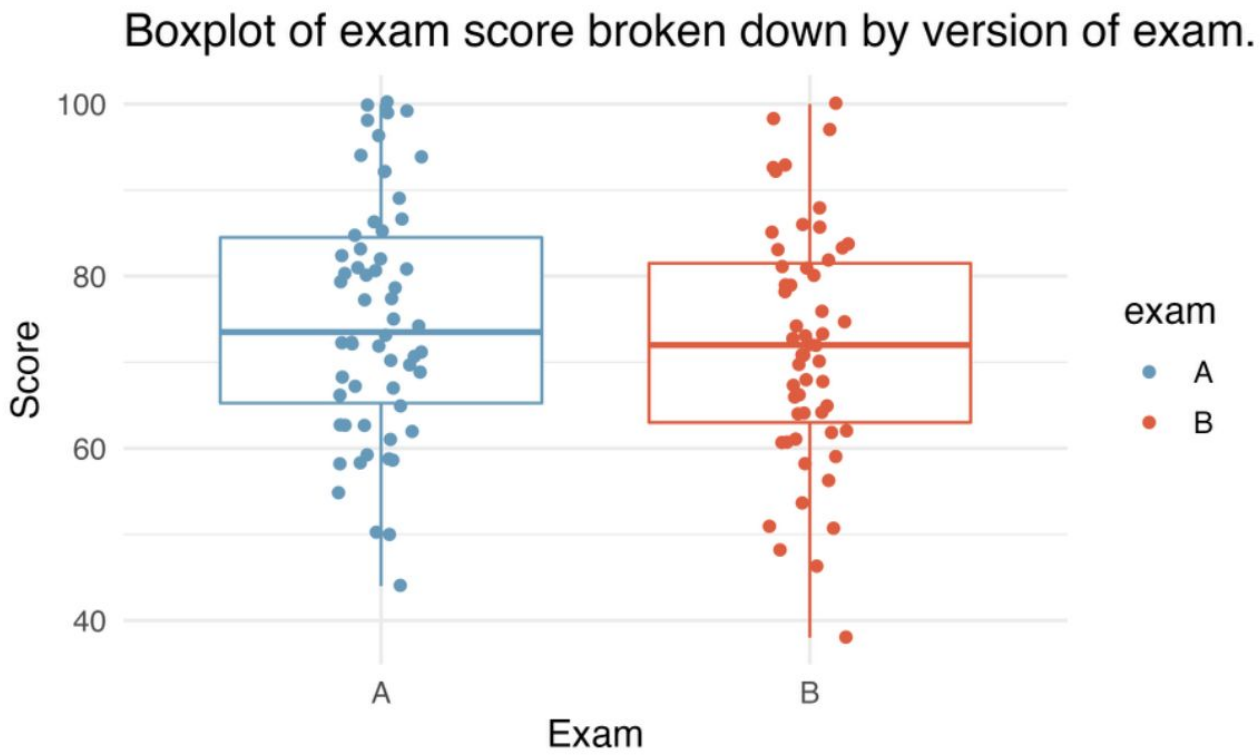


Figure 20.1: Exam scores for students given one of three different exams.

- Construct **hypotheses** to evaluate whether the observed difference in sample means, $\mathbf{x_A - x_B = 3.1}$
- Is likely to have happened due to chance, if the null hypothesis is true.

- We will later evaluate these hypotheses using **α=0.01**

Technical conditions

- Before moving on to evaluate the hypotheses
- Think carefully about the dataset.
    - Are the observations across the two groups independent?
    - Are there any concerns about outliers?

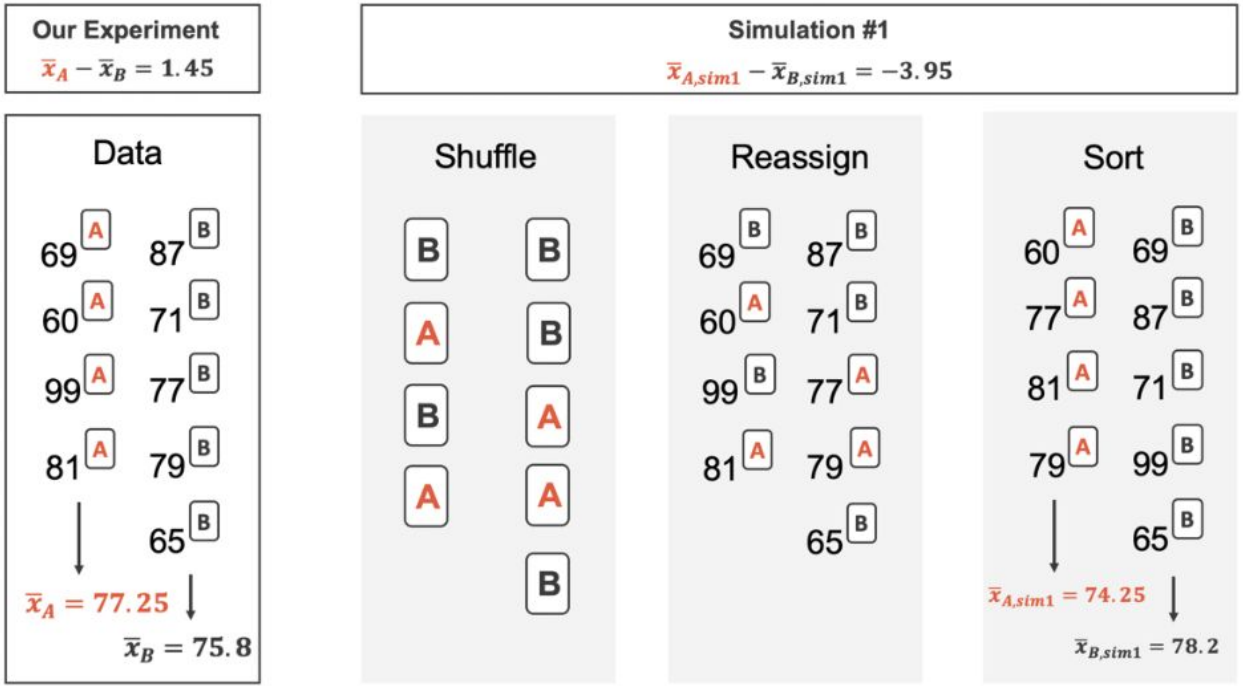


Figure 20.2: The version of the test (A or B) is randomly allocated to the test scores, under the null assumption that the tests are equally difficult.

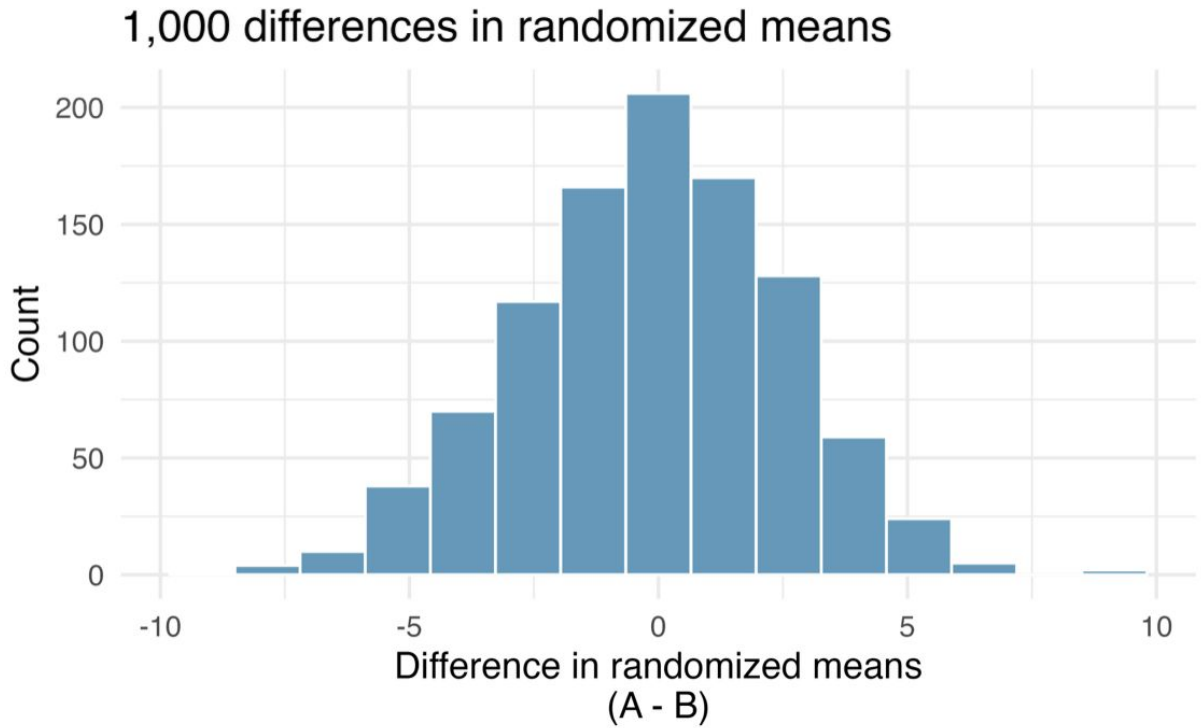


Figure 20.3: Histogram of differences in means, calculated from 1,000 different randomizations of the exam types.

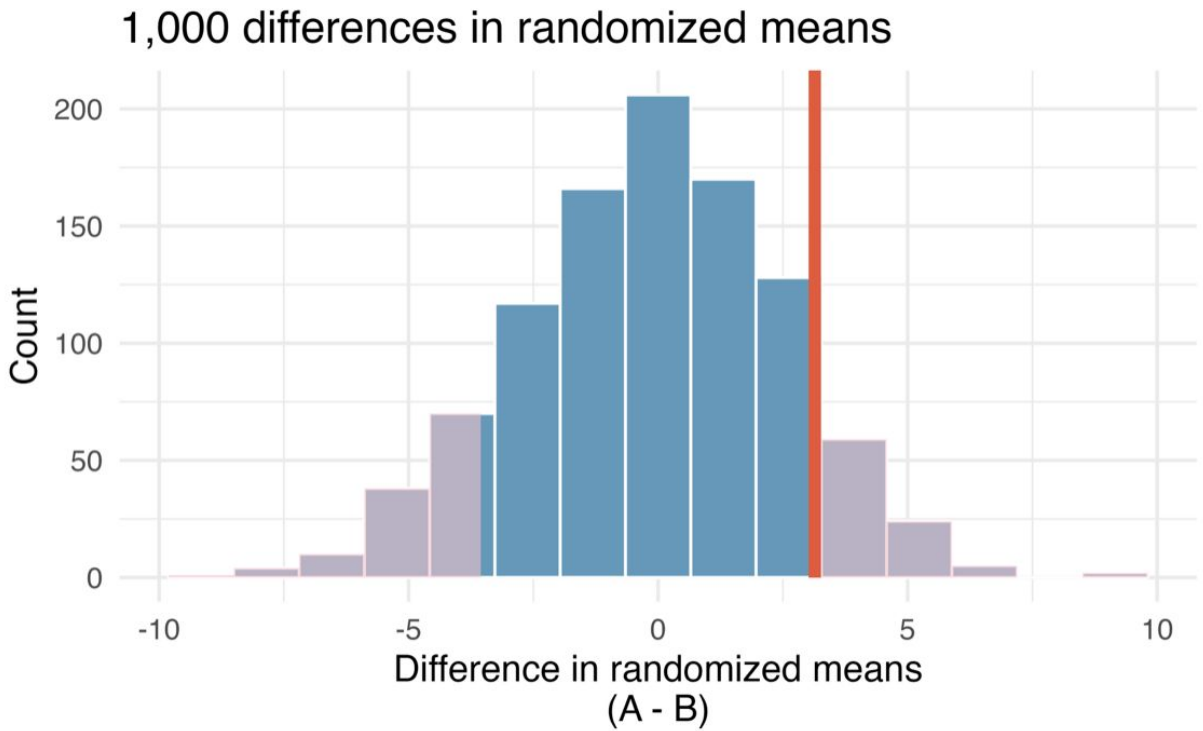
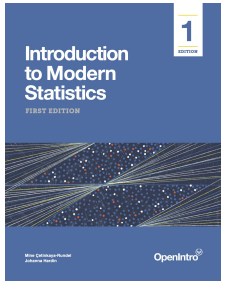


Figure 20.4: Histogram of differences in means, calculated from 1,000 different randomizations of the exam types. The observed difference of 3.1 points is plotted as a vertical line, and the area more extreme than 3.1 is shaded to represent the p-value.

Resources



The content of this presentation is mainly based on the excellent book “Introduction to Modern Statistics” by Mine Çetinkaya-Rundel and Johanna Hardin (2021).

The online version of the book can be accessed for free:

https://openintro-ims.netlify.app/index.html