

# APPLICATIONS OF MULTIVARIABLE CALCULUS

SOLVING A PROBLEM ON LOCAL  
LINEARIZATION

# What is Local Linearization?

- Local linearization generalizes the idea of tangent planes to any multivariable function.
- The idea is to approximate a function near one of its inputs with a simpler function that has the same value at that input, as well as the same partial derivative values.
- Written with vectors, here's what the approximation function looks like:  
$$L_f(\mathbf{x}) = f(\mathbf{x}_0) + \nabla f(\mathbf{x}_0)(\mathbf{x} - \mathbf{x}_0)$$

Where;

$f(\mathbf{x}_0)$  is a constant;  $\nabla f(\mathbf{x}_0)$  is the constant variable and  $\mathbf{x}$  is the variable

- This is called the local linearization of  $f$  near  $\mathbf{x}_0$ .

**Problem:** Suppose you are on a desert island without a calculator, and you need to estimate  $\sqrt{2.01} + \sqrt{0.99} + \sqrt{9.01}$ . How would you do it?

**Solution:**

We can view this problem as evaluating a certain three-variable function at the point  $(2.01, 0.99, 9.01)$ , namely

$$f(x, y, z) = \sqrt{x + \sqrt{y + \sqrt{z}}}$$

I don't know about you, but I'm not sure how to evaluate square roots by hand. If only this function was linear! Then working it out by hand would only involve adding and multiplying numbers. What we could do is find the local linearization at a nearby point where evaluating  $f$  is easier. Then we can get close to the right answer by evaluating the linearization at the point  $(2.01, 0.99, 9.01)$ .

The point we care about is very close to the much simpler point  $(2, 1, 9)$ , so we find the local linearization of  $f$  near that point. As before, we must find

- $f(2, 1, 9)$

- All partial derivatives of  $f$  at  $(2, 1, 9)$

The first of these is

$$\begin{aligned} f(2, 1, 9) &= \sqrt{2 + \sqrt{1 + \sqrt{9}}} \\ &= \sqrt{2 + \sqrt{1 + 3}} \\ &= \sqrt{2 + \sqrt{4}} \\ &= \sqrt{2 + 2} \\ &= \sqrt{4} \\ &= 2 \end{aligned}$$

Looks like someone chose a few convenient input values, eh?

On to the partial derivatives (heavy sigh). Since the square roots are abundant, let's write out for ourselves the derivative of  $\sqrt{x}$ .

$$\frac{d}{dx} \sqrt{x} = \frac{d}{dx} x^{\frac{1}{2}} = \frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$$

Okay, here we go. The simplest partial derivative is  $f_x$

$$f_x = \frac{\partial}{\partial x} \sqrt{x + \sqrt{y + \sqrt{z}}} = \frac{1}{2\sqrt{x + \sqrt{y + \sqrt{z}}}}$$

Since  $y$  is nestled in there,  $f_y$  requires some chain rule action:

$$f_y = \frac{\partial}{\partial y} \sqrt{x + \sqrt{y + \sqrt{z}}} = \frac{1}{2\sqrt{x + \sqrt{y + \sqrt{z}}}} \cdot \frac{1}{2\sqrt{y + \sqrt{z}}}$$

Nestled even deeper, that tricky  $z$  will require two iterations of the chain rule:

$$f_z = \frac{\partial}{\partial z} \sqrt{x + \sqrt{y + \sqrt{z}}} = \frac{1}{2\sqrt{x + \sqrt{y + \sqrt{z}}}} \cdot \frac{1}{2\sqrt{y + \sqrt{z}}} \cdot \frac{1}{2\sqrt{z}}$$

Next, evaluate each one of these at  $(2, 1, 9)$ . This might seem like a lot, but they are all made up of the same three basic components:

$$\frac{1}{2\sqrt{x + \sqrt{y + \sqrt{z}}}} = \frac{1}{2\sqrt{2 + \sqrt{1 + \sqrt{9}}}} = \frac{1}{2\sqrt{2 + 2}} = \frac{1}{4}$$

$$\frac{1}{2\sqrt{y+\sqrt{z}}} = \frac{1}{2\sqrt{1+\sqrt{9}}} = \frac{1}{2\sqrt{4}} = \frac{1}{4}$$

$$\frac{1}{2\sqrt{z}} = \frac{1}{2\sqrt{9}} = \frac{1}{6}$$

Plugging these values into our expressions for the partial derivatives, we have

$$f_x(2, 1, 9) = \frac{1}{4}$$

$$f_y(2, 1, 9) = \frac{1}{4} \cdot \frac{1}{4} = \frac{1}{16}$$

$$f_z(2, 1, 9) = \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{6} = \frac{1}{96}$$

Unraveling the formula for local linearization, we get

$$L_f(\mathbf{x}) = f(\mathbf{x}_0) + \nabla f(\mathbf{x}_0) \cdot (\mathbf{x} - \mathbf{x}_0)$$

$$= f(\mathbf{x}_0) + f_x(\mathbf{x}_0)(x - x_0) + f_y(\mathbf{x}_0)(y - y_0) + f_z(\mathbf{x}_0)(z - z_0)$$

$$= \boxed{2 + \frac{1}{4}(x - 2) + \frac{1}{16}(y - 1) + \frac{1}{96}(z - 9)}$$

Finally, after all this work, we can plug in  $(x, y, z) = (2.01, 0.99, 9.01)$  to compute our approximation

$$2 + \frac{1}{4}(2.01 - 2) + \frac{1}{16}(0.99 - 1) + \frac{1}{96}(9.01 - 9)$$

$$= 2 + \frac{0.01}{4} + \frac{-0.01}{16} + \frac{0.01}{96}$$

Calculating this by hand still isn't *easy*, but at least it's doable. When you work it out, the final answer is

2.001979

Had we just used a calculator, the answer is

$$\sqrt{2.01} + \sqrt{0.99} + \sqrt{9.01} \approx \boxed{2.001978}$$

So our approximation is pretty good!

**THANK YOU**

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# *APPLICATIONS OF MULTIVARIATE CALCULUS*

BY-SOHAIL CHHABRA

# *ARCHITECTURE*

- **An Architect Engineer uses calculus in determining the amount of the necessary materials to construct curved shape constructions (e.g. dome over a sports arena) and also to measure the weight of that structure. Calculus is used to improve the architecture not only of buildings but also of important infrastructures such as bridges.**
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- 1. **In designing structures, an architect must visualize the solutions to both functional and aesthetic problems – balancing art with science and math. There are architecture math requirements - calculus - so that an architect can understand the forces acting on the structures she designs. It also gives her a means of calculating factors such as heat loss over time. This understanding must be mathematically precise if the structures architects design are to be stable and safe for use.**
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- 1. **Various examples of calculus being used in famous architectural designs are-**
  - **The familiar hyperbolic curves in calculus class are just one example of how this branch of mathematics is important in architecture. Famous landmarks, such as the Gateway Arch in St. Louis and the dome of St Paul's Cathedral in London, both incorporate hyperbolic curves called catenaries into their design. You will recognize this curve if you hold a slack chain between two hands. A catenary arch can support the weight of a structure with a minimum amount of material.**
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# *WEATHER FORECASTING*

- history of calculus in weather-
- Before many years ago our scientists relied on temperature , pressure as well as on experience. Physical laws including conservation and thermodynamics can be used for this.
- British scientist Lewis Fry Richardson was the first person who predict this. The group of met
- eorologists at new jersy's institute for advanced study developed the first computer that was used to complete mathematical equation using calculus.

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- The understanding of calculus gives a forecaster a depth of understanding atmospheric processes more completely.
- Storms including Hurricanes, tornadoes, and even cyclones use calculus to predict their direction and intensity.
- equations used in weather forecasting -
- Primitive Equations-
- We can use primitive equation to approximate global atmospheric low.
- Three set of primitive equations are as below:
- 1: Conservation of momentum
- 2: Continuity equation
- 3: Thermal energy equations

#### VAIABLES USED ARE-

U is places for the zonal velocity

v is places for the *meridional velocity*

T is *temperature*

W is *water* that can form a precipitates

w is *vertical velocity*

### 1: Horizontal momentum equation

$$\frac{\partial u}{\partial t} = -u \frac{\partial u}{\partial x} - v \frac{\partial u}{\partial y} - w \frac{\partial u}{\partial z} + \frac{uv \tan \phi}{a} - \frac{uw}{a} - \frac{1}{p} \frac{\partial p}{\partial x} - 2\omega (w \cos \phi - v \sin \phi) + F_x$$

$$\frac{\partial v}{\partial t} = -u \frac{\partial v}{\partial x} - v \frac{\partial v}{\partial y} - w \frac{\partial v}{\partial z} - \frac{u^2 \tan \phi}{a} - \frac{vw}{a} - \frac{1}{p} \frac{\partial p}{\partial y} - 2\omega u \sin \phi + F_y$$

### 2: Vertical momentum equation

$$\frac{\partial w}{\partial t} = -u \frac{\partial w}{\partial x} - v \frac{\partial w}{\partial y} - w \frac{\partial w}{\partial z} - \frac{u^2 + v^2}{a} - \frac{uw}{a} - \frac{1}{p} \frac{\partial p}{\partial z} - 2\omega u \cos \phi - g + F_z$$

### 3: Thermodynamic equation

$$\frac{\partial T}{\partial t} = -u \frac{\partial T}{\partial x} - v \frac{\partial T}{\partial y} + w (\gamma - \gamma_d) + \frac{1}{C_p} \frac{\partial H}{\partial t}$$

$$\frac{\partial T}{\partial t} = -u \frac{\partial T}{\partial x} - v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} - w \gamma_d + \frac{Q}{C_p}$$

### 3: Continuity equation

$$\frac{\partial p}{\partial t} = -u \frac{\partial p}{\partial x} - v \frac{\partial p}{\partial y} - w \frac{\partial p}{\partial z} - p \frac{\partial u}{\partial x} - p \frac{\partial v}{\partial y} - p \frac{\partial w}{\partial z}$$

$$\frac{\partial q_v}{\partial t} = -u \frac{\partial q_v}{\partial x} - v \frac{\partial q_v}{\partial y} - w \frac{\partial q_v}{\partial z} + Q_v$$

# Applications of Multivariate Calculus in Image Processing

A project by Srinidhee Shastry

# Image Processing

- Definition: Image Processing are operations which take an image as an input, and produce an image as an output.
- Every black and white image is a function  $f(x,y)$ .
- Processing an image typically means converting the image with function  $f$  to a new image  $f'$ .  
(Where  $f'$  is a function of  $f$ )

*Different versions of an image based on different functions.*



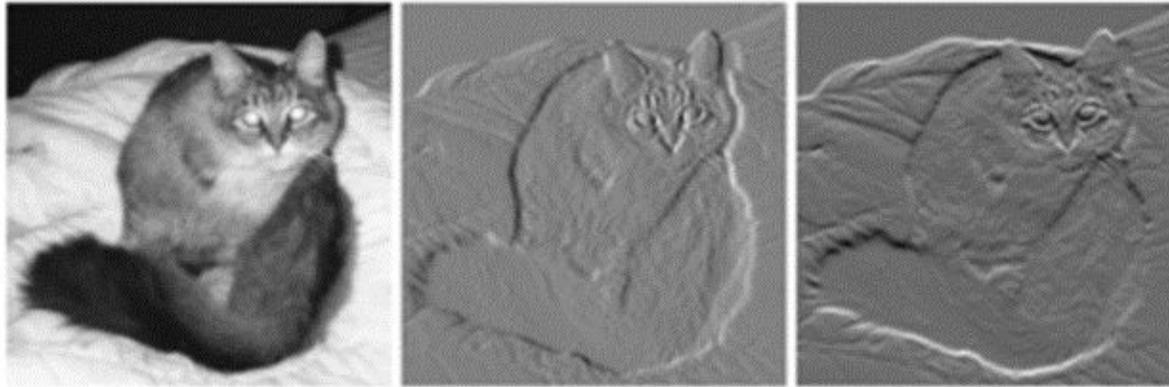
# Multivariate Calculus in Image Processing

- Pixel to Pixel transformation: The easiest form of image processing is to transform each pixel in isolation. These transformations can be written as:  $g_{xy} = t(f(x, y))$
- Convolution: It is a common method of filtering an image. This process is conducted by combining two functions:

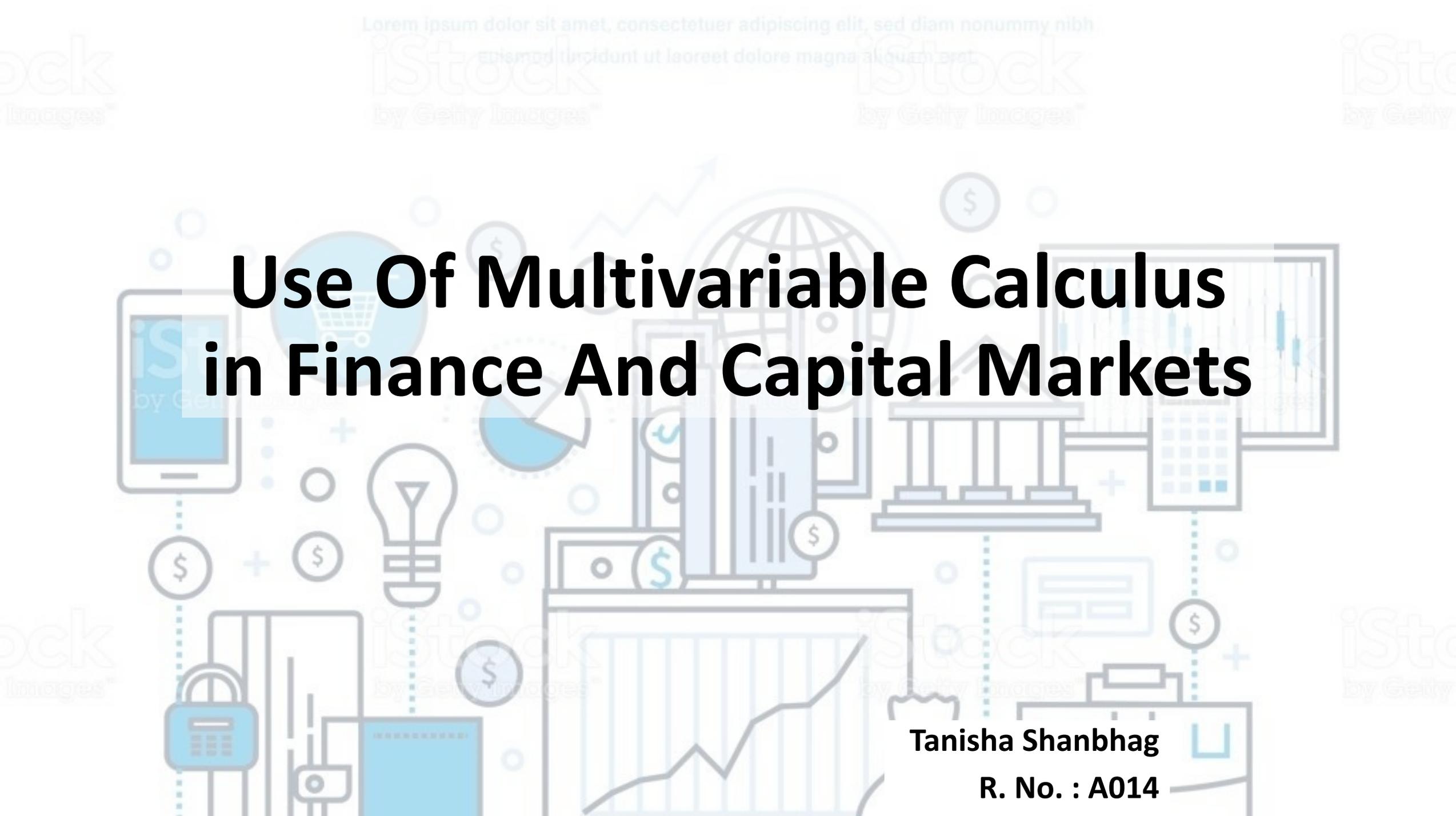
$$h(x, y) = f(x, y) * g(x, y)$$

# Multivariate Calculus in Image Processing

- Gradient: An image gradient is a directional change in the intensity or colour in an image.



- The Gradient of a **2D Image** is:  $\left[ \frac{\partial f(x, y)}{\partial x}, \frac{\partial f(x, y)}{\partial y} \right]$ .



# Use Of Multivariable Calculus in Finance And Capital Markets

Tanisha Shanbhag

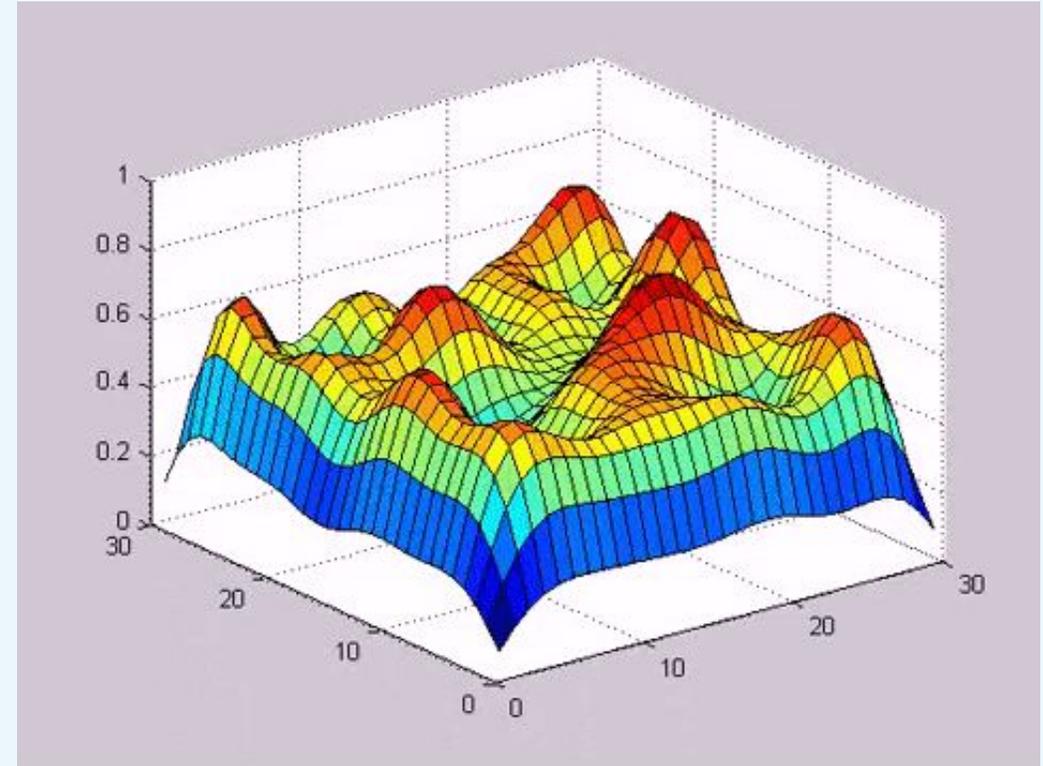
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# What is a Partial Differential Equation?

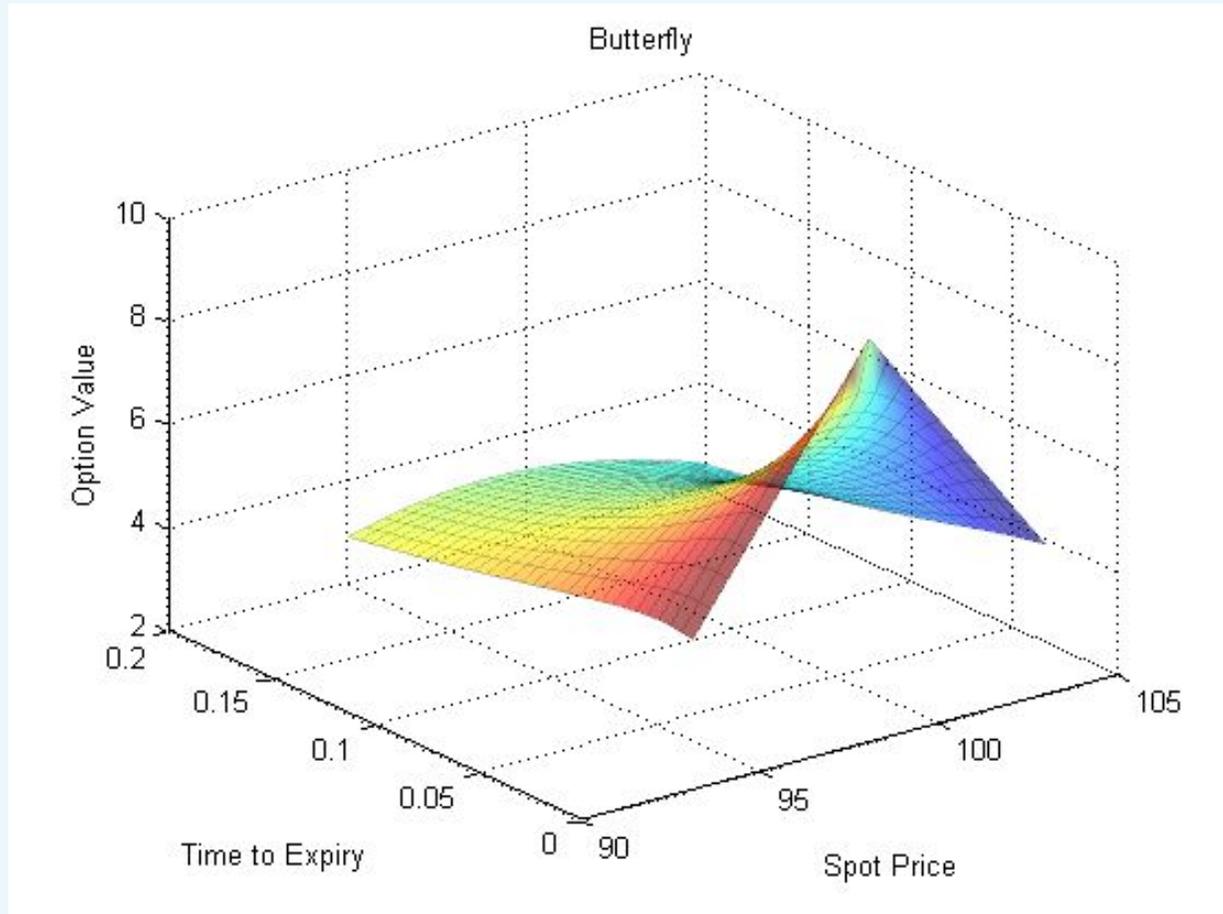
- In mathematics, a partial Differential Equation (PDE) is an equation which imposes relations between the various partial derivatives of a multivariable function.
- E.g.,

$$\frac{dU}{dx_1} + \frac{dU}{dx_2} = 0 \text{ is linear.}$$

$$\frac{dU}{dx_1} + \left(\frac{dU}{dx_2}\right)^2 = 0 \text{ is non-linear.}$$



# What is the Black-Scholes Equation?



- In mathematical Finance, it is a partial differential Equation(PDE) governing the price evolution of a European call or put under the Black-Scholes model.
- The term may refer to a similar PDE that can be derived for a variety of options, or more generally derivatives.

Black Scholes Model based returns the fair price of a European Stock Option given a few variables:

1. Volatility (Standard Deviation of Log Returns)
2. Option Type
3. Underlying Price (Price of the underlying Stock)
4. Strike Price (Price at which the option is being traded at)
5. Time (Time to expire)
6. Risk – free Interest Rate

## *The Equation?*

- For a European call or put on an underlying stock paying no dividends, the equation in simple terms will be:

$$\frac{dV}{dt} + \frac{1}{2} \sigma^2 S^2 \left( \frac{d^2V}{dS^2} \right) + rS \left( \frac{dV}{dS} \right) - rV = 0$$

where  $V$  is the price of the option as a function of stock price  $S$  and time  $t$ ,  $r$  is the risk-free interest rate, and  $\sigma$  is the volatility of the stock.

## *How does this equation help us in real life?*

- Under the model assumption of a frictionless market, one can perfectly hedge the option by buying and selling the underlying asset in just the right way and consequently, “eliminate risk”.
- Used as a base for more complicated hedging strategies by investment banks.
- Used in derivative pricing and risk management.
- Calculators based on this model are available for convenience.

### Black Scholes Option Calculator

Call  
 Put  
 All

strike price

stock price

time (days)

volatilitiy (%)

risk free interest rate (%)

**Call: \$7.2997**

## *Financial Interpretation?*

$$\frac{dV}{dt} + \frac{1}{2} \sigma^2 S^2 \left( \frac{d^2V}{dS^2} \right)$$

$$-rS \left( \frac{dV}{dS} \right) + rV$$

- The left-hand side consists of a "time decay" term, the change in derivative value with respect to time, called *theta*, and a term involving the second spatial derivative *gamma*, the convexity of the derivative value with respect to the underlying value.
- The right-hand side is the riskless return from a long position in the derivative and a short position consisting of  $dV/dS$  shares of the underlying.

- For an option, theta is typically negative, reflecting the loss in value due to having less time for exercising the option (for a European call on an underlying without dividends, it is always negative).
- Gamma is typically positive and so the gamma term reflects the gains in holding the option. The equation states that over any infinitesimal time interval the loss from theta and the gain from the gamma term must offset each other so that the result is a return at the riskless rate.
- From the viewpoint of the option issuer, e.g. an investment bank, the gamma term is the cost of hedging the option.

*Thank you!*

# Applications of Multivariate Calculus

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By Tanya

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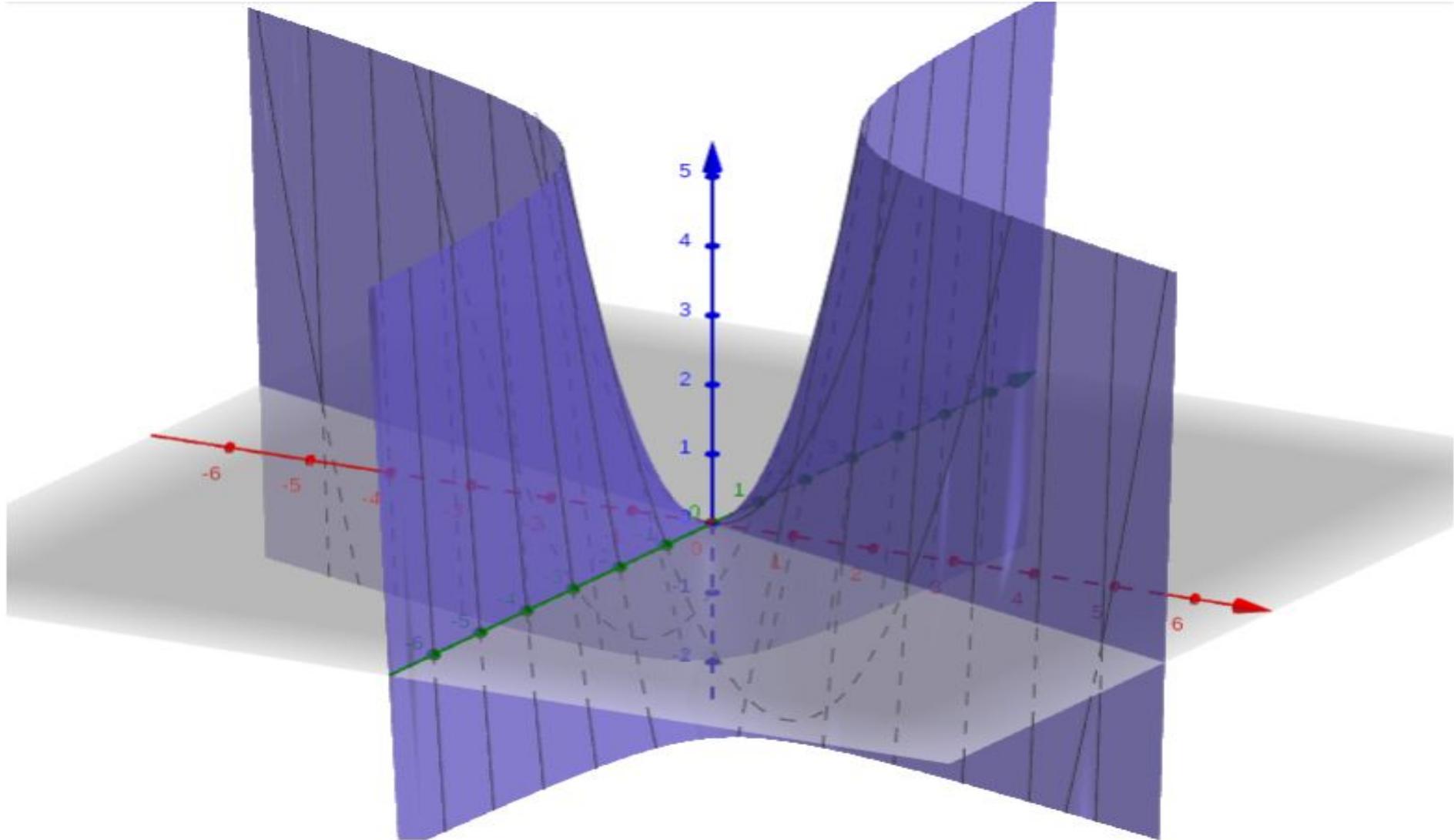
# Applications of partial derivatives in daily life.

- The technique of differentiating multivariable function is known as Partial Differentiation and the resulting derivative is called Partial Derivative. We differentiate one of the variables while keeping the other variable fixed. The equations involving partial derivatives are known as partial differential equations or simply PDEs. The use of Partial Derivatives in real world is very common. Partial Derivatives are used in basic laws of Physics for example Newton's Law of Linear Motion, Maxwell's equations of Electromagnetism and Einstein's equation in General Relativity. In economics we use Partial Derivative to check what happens to other variables while keeping one variable constant.
- They measure rates of change.
- They are used in approximation formulas.
- They help identify local maxima and minima.

# Partial Differentiation

- Partial differentiation is used when we take one of the tangent lines of the graph of the given function and obtaining its slope.
- Example:
- Suppose that  $f$  is a function of more than one variable such that,
- $f = x^2 + 3xy$
- The graph of  $z = x^2 + 3xy$  is given below:

# Graph of the function $z=x^2+3xy$



# Explanation of the Graph

- The graph of this function represents a surface in Euclidean space. To each point on this surface, there is an infinite number of tangent lines. Now, we can apply the partial differentiation for choosing one of these lines and finding its slope.

To find the tangent line's slope to the function at one point, say  $P(1,1)$  and parallel to the  $xz$ -plane, we treat  $y$  as a constant.

By finding the derivative of the equation taking  $y$  as a constant, we can get the slope of the given function  $f$  at the point  $(x, y)$ . This can be done as follows.

$$\frac{\partial f}{\partial x} = \left(\frac{\partial}{\partial x}\right) (x^2 + 3xy) = 2x + 3y$$

The value of  $\frac{\partial f}{\partial x}$  at  $(1, 1)$  is:

$$2(1) + 3(1) = 5$$

That means the slope is 5.

Therefore,  $\frac{\partial f}{\partial x} = 5$  at  $(1, 1)$ .

# Applications of Partial Derivatives in the Real World

- **Application of Partial Derivative in Economics:**

- In economics the demand of quantity and quantity supplied are affected by several factors such as selling price, consumer buying power and taxation which means there are multivariable factors that affect the demand and supply. In Economics Marginal Analysis is used to find out or evaluate the change in value of a function resulting from 1-unit increase in one of its variables. For example Partial derivative is used in Marginal Demand to obtain condition for determining whether two goods are substitute or complementary.

- **Application of Partial Derivative in Engineering:**

- In image processing edge detection algorithm is used which uses partial derivatives to improve edge detection.

**Thank You!**