Lecture 1. Part 2

Lecture notes. Instructor Igor V. Baryakhtar

How to Solve It

What is the best way to organize mathematical calculations?

Follow the method suggested by George Pólya in the small book “How to Solve It”

Written for graduate students this book widely used to teach and learn math everywhere from community colleges to Ivy league universities.

A perennial bestseller by eminent mathematician G. Polya, ***How to Solve It*** *will show anyone in any field how to think straight. In lucid and appealing prose, Polya reveals how the mathematical method of demonstrating a proof or finding an unknown can be of help in attacking any problem that can be "reasoned" out—from building a bridge to winning a game of anagrams. Generations of readers have relished Polya's deft—indeed, brilliant—instructions on stripping away irrelevancies and going straight to the heart of the problem.*

From Princeton University Press

https://press.princeton.edu/books/paperback/9780691164076/how-to-solve-it

How To Solve It

A New Aspect of Mathematical Method

G. POLYA
Stanford University

SECOND EDITION

Doubleday Anchor Books
Doubleday & Company, Inc.
Garden City, New York

HOW TO SOLVE IT

UNDERSTANDING THE PROBLEM

First.

You have to *understand* the problem.

What is the unknown? What are the data? What is the condition?

Is it possible to satisfy the condition? Is the condition sufficient to determine the unknown? Or is it insufficient? Or redundant? Or contradictory?

Draw a figure. Introduce suitable notation.

Separate the various parts of the condition. Can you write them down?

DEVISING A PLAN

Second.

Find the connection between the data and the unknown. You may be obliged to consider auxiliary problems if an immediate connection cannot be found. You should obtain eventually a *plan* of the solution.

Have you seen it before? Or have you seen the same problem in a slightly different form?

Do you know a related problem? Do you know a theorem that could be useful?

Look at the unknown! And try to think of a familiar problem having the same or a similar unknown.

Here is a problem related to yours and solved before. Could you use it? Could you use its result? Could you use its method? Should you introduce some auxiliary element in order to make its use possible?

Could you restate the problem? Could you restate it still differently? Go back to definitions.

If you cannot solve the proposed problem try to solve first some related problem. Could you imagine a more accessible related problem? A more general problem? A more special problem? An analogous problem? Could you solve a part of the problem? Keep only a part of the condition, drop the other part; how far is the unknown then determined, how can it vary? Could you derive something useful from the data? Could you think of other data appropriate to determine the unknown? Could you change the unknown or the data, or both if necessary, so that the new unknown and the new data are nearer to each other?

Did you use all the data? Did you use the whole condition? Have you taken into account all essential notions involved in the problem?

CARRYING OUT THE PLAN

Third.

Carry out your plan.

Carrying out your plan of the solution, *check each step*. Can you see clearly that the step is correct? Can you prove that it is correct?

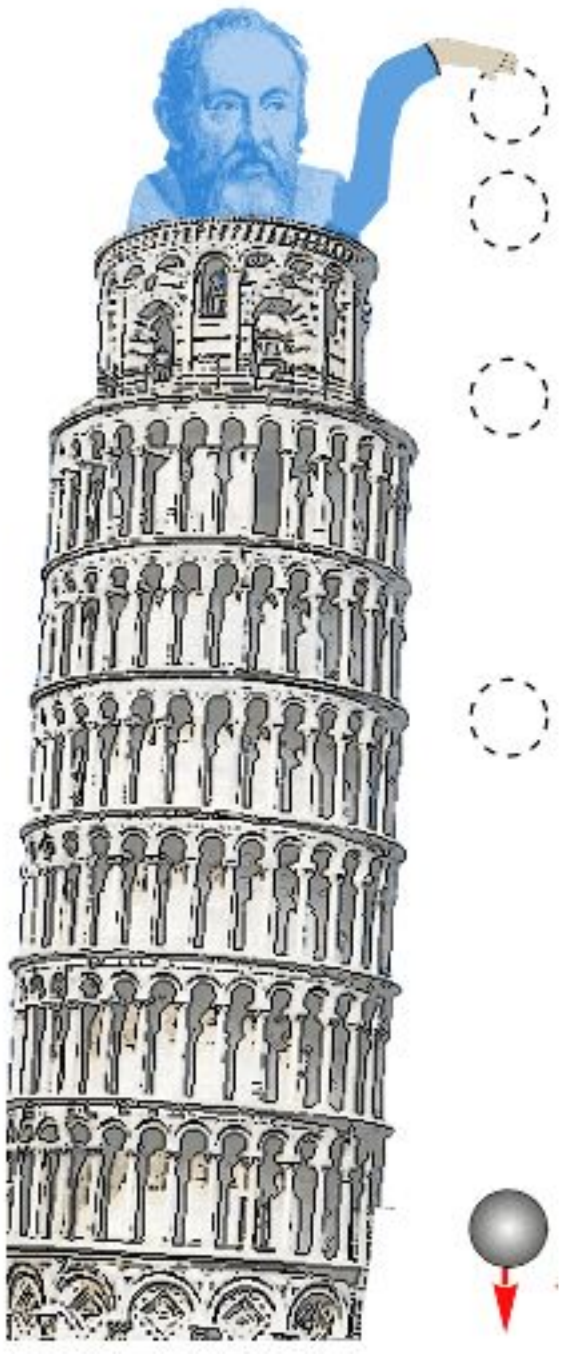
LOOKING BACK

Fourth.

Examine the solution obtained.

Can you *check the result?* Can you check the argument?
Can you derive the result differently? Can you see it at a glance?
Can you use the result, or the method, for some other problem?

Example: The Motion of Falling body



https://www.famousscientists.org/

$$x = \frac{1}{2}gt^2$$

Here g is the gravity acceleration

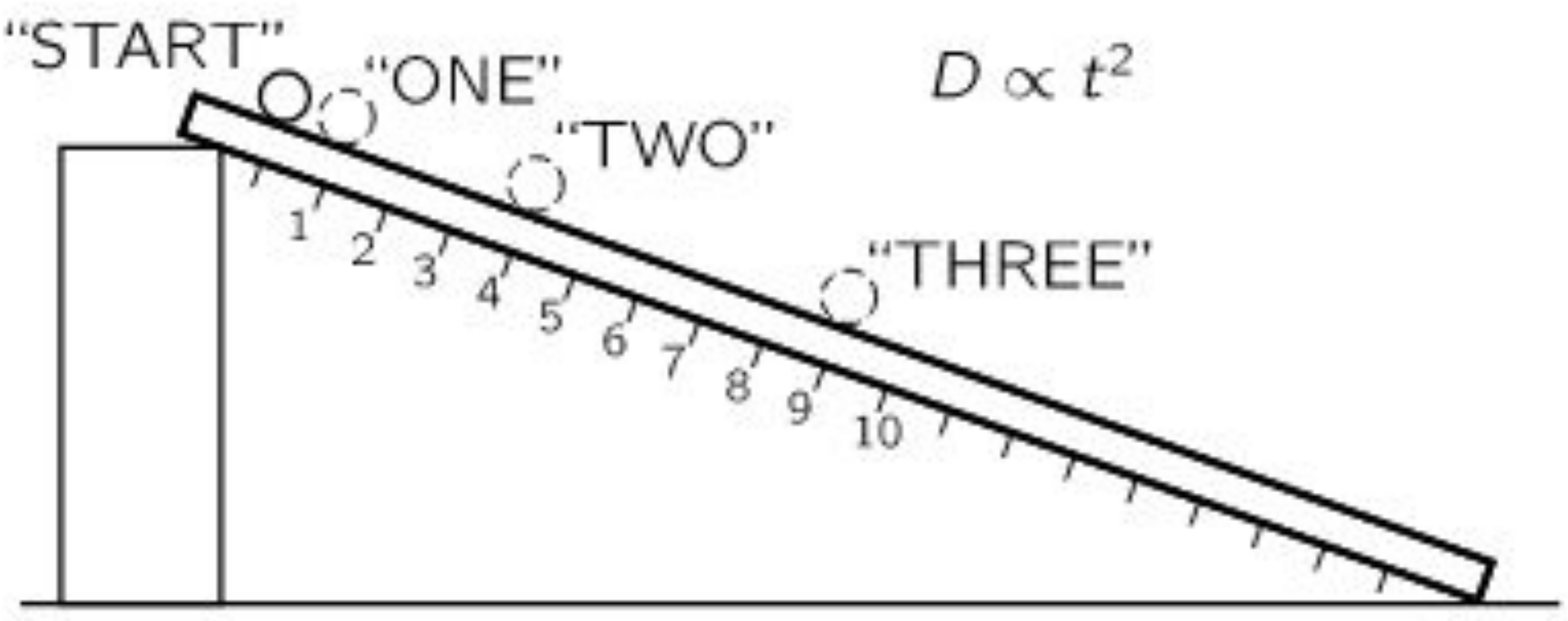
$g = 32\ ft/s^2$

The Law of Fall says that the distance traveled by a falling body is directly proportional to the square of the time it takes to fall.

Galileo did roll balls down inclined planes about 1610. That incline insured the balls rolled at much lower speeds, making their acceleration easier to measure. The balls were similar in size, but were made of different materials, therefore their masses were different. He found that the balls traveled at the same acceleration and the total distance, x*, traveled by the object is proportional to the time traveled squared. This fact allowed Galileo to find the value of of the gravitational acceleration* g *.*

Galileo's inclined plane experiment. Fresco by G. Bezzuoli.

$$D \propto t^2.$$



The Feynman Lectures on Physics, Volume I

http://www.feynmanlectures.caltech.edu/I_05.html

Galileo believed that **mathematics is the language of the world around us**: whether it is the behavior of planets and pendulums, or the fundamentals of music and mechanics – all could be understood using mathematics.

Leaning Tower of Pisa

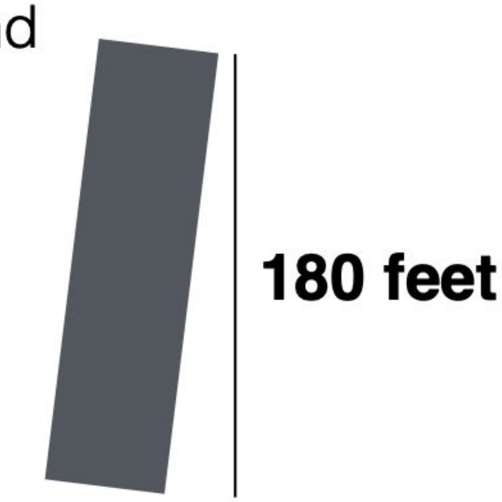
If a tourist accidentally drops a binoculars from the observation deck on the top of a Tower, how long will it take the binoculars to fall to the ground?

What is given? What is necessary to find?

1. Familiarize, any picture helps!!

The height of the tower is 183.27 feet from the ground on the low side or approximately 180 feet.

given:	**unknown:**
The height of the tower=180 feet gravity acceleration g=32 ft/s^2	**time**



2. Translate

the height of the tower -> x

time -> t

x=180 feet g=32 ft/s²

Set up of the problem: apply the right formula

$$x = \frac{1}{2} g t^2$$

Substituting x=180 ft and g =32 ft/s^2:

$$180 = \frac{1}{2} \cdot 32 \cdot t^2$$

3. Solve

$$180 = 16 \cdot t^2$$

or

$$\frac{180}{16} = \frac{16}{16}t^2$$

or

$$11.25 = t^2$$

Applying the square root property:

$$\sqrt{11.25} = \pm\sqrt{t^2}$$

The solution is:

$$t = \pm 3.35$$

4. Check

Since time cannot be negative, $t = -3.35$ cannot be a solution.

Apply units: $t = 3.35$ seconds

Plug into the original formula:

$$x = \frac{1}{2} \cdot 32(ft/s^2) \cdot (3.35s)^2 = 179.56ft \approx 180ft$$

5. State

It takes about $t = 3.35 \approx 3.4$ seconds for a binoculars to fall to the ground from the top of the Tower of Pisa.