YEAR 12 - MATHEMATICS

HSC Topic 12 – FURTHER CALCULUS SKILLS

MATHEMATICS EXTENSION 1

LEARNING PLAN			
Learning Intentions Student is able to:	**Learning Experiences** Implications, considerations and implementations:	**Success Criteria** I can:	**Resources**
Integrate a function using a given substitution	The given substitution should reduce a complicated function to a standard form.		
(i)Indefinite Integral	The final integral must be rewritten in terms of the variable in the question. e.g. $\int x\sqrt{x^2+1}\,dx$ let $u=x^2+1$ e.g. $\int \frac{t}{t^2+1}\,dt$ let $u=t^2+4$ e.g. $\int \frac{t}{\sqrt{t+1}}\,dt$ let $u^2=t+1$	Find the indefinite integral of a function using a given substitution.	

(ii)Definite Integral	The limits of integration need to be recalculated in terms of the new variable. $\int_1^7 x\sqrt{x+2}\,dx$ let $u=x+2$	Integrate a function given the substitute involving limits.	
(iii) Evaluate definite integrals with given trigonometric substitutions.	e.g. Evaluate $\int_0^1 \sqrt{4-x^2}\,dx$ let $u=2\sin\theta$ note. The substitution should be more correctly considered as $\theta=\sin^{-1}\left(\frac{x}{2}\right)$ which will give a unique θ for each *x*. e.g. Evaluate $\int_0^3 \frac{1}{\sqrt{9+x^2}}\,dx$ let $x=3\tan\theta$	Use given trigonometric substitutions to reduce integrals to a standard form and then using the new endpoints evaluate the integral.	Mixed Questions
prove and use the identities $sin^2 nx=\frac{1}{2}(1-\cos 2nx)$ and $cos^2 nx=\frac{1}{2}(1+\cos 2nx)$ to solve problems	Prove the identities including $sin^2 nx=\frac{1}{2}(1-\cos 2nx)$ and $cos^2 nx=\frac{1}{2}(1+\cos 2nx)$	I can recall the double angle results to prove given identity.	
solve problems involving $\int sin^2 nx\,dx$ and $\int cos^2 nx\,dx$	Use the above identities to integrate expressions involving perfect squares which would otherwise be difficult such as: Eg. $\int \sin^2 x\,dx$, $\int \sin^2 2x\,dx$, $\int \sin^2 \frac{x}{2}\,dx$ $\int \tan^2 2x\,dx$ Find area and Volume using Trig Functions	Use the double angle formulas to help find the integrals of squares of trigonometric functions. Find areas and volumes that involve Trigonometric functions.	Integration of Trig Fns Integration using trig identities of trig substitution

	(e) e.g. Volume formed when the region bounded by x-axis and $y=\sin x$, for $0 \le x \le \pi$ is rotated about the x-axis. e.g. Find area bounded by $y=\sin x$, $y=\cos x$ between $x=\frac{\pi}{4}$ and $x=\frac{3\pi}{4}$.		NEAP Trig Fns Test
complete integrals that link differentiation and integration as inverse operations	e.g. Differentiate $x\tan x$ and hence evaluate $\int_0^{\frac{\pi}{4}} x\sec^2 x\,dx$	Find and use the derivatives of trigonometric functions to help to find more difficult integrals.	
find derivatives of inverse functions by using the relationship $\frac{dy}{dx}=\frac{1}{\frac{dx}{dy}}$	Use these results of standard derivatives to differentiate expressions such as $y=2\sin^{-1}3x$, $y=\pi\cos^{-1}\pi x$, $y=2\tan^{-1}\left(\frac{x}{4}\right)$, $y=\frac{\pi}{2}+2\sin^{-1}\left(\frac{x}{2}\right)$ $\tan^{-1}\left(\frac{2x}{3}\right)$ and $\sin^{-1}(x)+\cos^{-1}(x)$	Differentiate examples such as $x\tan^{-1}\left(x^2\right)$, $\sin^{-1}\sqrt{1-x^2}$, $\sin^{-1}\left(\frac{x}{a}\right)$, $\cos^{-1}\left(\frac{x}{a}\right)$, $\tan^{-1}\left(\frac{x}{a}\right)$	
solve problems involving the derivatives of **inverse trigonometric functions**	**18 12c** Let $f(x)=\sin^{-1}x+\cos^{-1}x$. (i) Show that $f'(x)=0$.	Apply differentiation to a range of questions including tangents and normals, max min and curve sketching curve sketching	`

(i) Equations of tangents and normal.	(ii) Hence, or otherwise, prove that $\sin^{-1}x+\cos^{-1}x=\frac{\pi}{2}$. (iii) Hence, sketch $f(x)=\sin^{-1}x+\cos^{-1}x$.		
(ii) Curve sketching	e.g. Use calculus and basic features to graph e.g. $y=\sin^{-1}x+\cos^{-1}x$, $y=\tan^{-1}x+\tan^{-1}\left(\frac{1}{x}\right)$ where $x\neq 0$, $y=2\sin^{-1}\sqrt{x}+\cos^{-1}(2x-1)$.		
	10 5b Solut Let $f(x)=\tan^{-1}x+\tan^{-1}\left(\frac{1}{x}\right)$ for $x\neq 0$. (i) By differentiating, or otherwise, show that $f(x)=\frac{\pi}{2}$ for $x>0$. **3** (ii) Given that $f(x)$ is odd, sketch the graph of $y=f(x)$ for $x\neq 0$. **1**		

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<td>Complete maximum and minimum value problems involving inverse trigonometry.</td>
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e.g. A flag pole 5 metres high stands on a building which is 50 metres high. The building stands on horizontal ground. Find the distance from the foot of the building to the point where the flag subtends the greatest possible angle.

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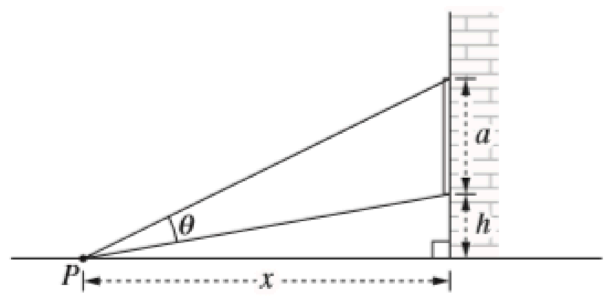
A billboard of height a metres is mounted on the side of a buildi
its bottom edge h metres above street level. The billboard subtends an
at the point P, x metres from the building.



(i) Use the identity $\tan(A-B)=\dfrac{\tan A-\tan B}{1+\tan A\tan B}$ to show that

$$\theta=\tan^{-1}\left[\frac{ax}{x^2+h(a+h)}\right].$$

(ii) The maximum value of θ occurs when $\dfrac{d\theta}{dx}=0$ and x is positive

Find the value of x for which θ is a maximum.

Solution

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integrate expressions of the form $\int \frac{1}{\sqrt{a^2-x^2}}\,dx$ or $\int \frac{a}{a^2+x^2}\,dx$	e.g. 1. $\int_{-\frac{3}{2}}^{\frac{\sqrt{3}}{2}} \frac{1}{\sqrt{9-x^2}}\,dx$, 2. $\int \frac{1}{x^2+4}\,dx$ Find integrals which first need to be split 3. $\int \frac{x^2+1}{x^2+4}\,dx = \int 1-\frac{3}{x^2+4}\,dx$		
	15 7 What is the value of k such that $\int_0^k \frac{1}{\sqrt{4-x^2}}\,dx = \frac{\pi}{3}$? (A) 1 (B) $\sqrt{3}$ (C) 2 (D) $2\sqrt{3}$	**1**	Solution

	e.g. (a) find area bounded by $y = 3\cos^{-1} 2x$ and the coordinate axes. (b) find the area bounded by $y = 4\sin^{-1}\frac{x}{3}$, the positive x-axis for $0 \le x \le 1\frac{1}{2}$.

Established Goals(Syllabus Outcomes): ME12-1, ME12-4, ME12-6, ME12-7
Estimated Time: 1 week
Reading Strategies: Annotate the Question, Vocabulary Pre-Teach + Morphology, Graph/Diagram Decode Routine, Skim–Scan–Select Routine