Engineering Assignment: Energy from a Water Dam

Student Learning Outcomes

3.3.6	Combine the differentiation rules to find the derivative of a polynomial or rational function
3.4.3	Apply rates of change to displacement, velocity, and acceleration of an object moving along a straight line
3.8.1	Recognize the difference between an implicit function and an explicit function
3.8.2	Apply rates of change to displacement, velocity, and acceleration of an object moving along a straight line
4.1.3	Use the chain rule to find the rate of change of one quantity that depends on the rate of change of other quantities

1. The oldest operational dam is the Quatinah Barrage or Lake Homs Dam, located in Syria, which was built around 284–305 AD[1]. The dam is 7 meters high and 2000 meters wide, with a capacity of 90 cubic meters.

a. As the water flows from the top of the dam, assuming the initial velocity is 0 m/s, write a formula for the position h, in meters, with respect to time, t in seconds.

Recall: $h(t) = -\frac{1}{2}a_0t^2 + v_0t + h_0$, where a_0 is the gravitational acceleration $g = 9.8m/s^2$.

b. How long does it take the body of water from top of the dam to drop to the bottom?

[1] *Schnitter, Niklaus (1978), "Römische Talsperren"; Smith, Norman (1971), A History of Dams*

c. Using your answers in the previous parts, find the velocity of the water when it hits the bottom of the dam.
Hint: You will need to find the velocity function $v(t)$ through differentiation.

2. Dams can generate power using the conversion from potential energy from top of the dam to kinetic energy at the bottom of the dam. The kinetic energy can be calculated as $KE = \frac{1}{2}m \cdot v^2$, where m is the mass of water in kilograms, and $v$is the velocity of water in m/s. The unit for energy is Joules, where 1 J = 1 $kg \cdot m^2/s^2$

a. Find the kinetic energy generated in the Lake Homs Dam, assuming the mass of water is 14000 kg.

b. If the velocity stays constant, how fast does the kinetic energy change with respect to mass m? Write your final answer in a complete sentence, including units.

c. If the mass stays the same, find the rate of change of the kinetic energy with respect to velocity v.Write your final answer in a complete sentence, including units.

d. Given the answers in previous parts, how can the dam be remodeled to generate more energy?

3. The potential energy $PE = m \cdot g \cdot h$. If we assume there is no loss of energy as the water flows from top to the bottom of the dam, we can get

$$\frac{1}{2}m \cdot v^2 = m \cdot g \cdot h$$

$$\text{Therefore, } \frac{1}{2}v^2 = g \cdot h$$

a. Find the velocity of water as it hits the bottom of the dam using this formula. It should match your answer in part 1c.

b. Find the $\frac{dv}{dh}$. Interpret your answer in the given context, including units.