

Geometry

NUMBERS

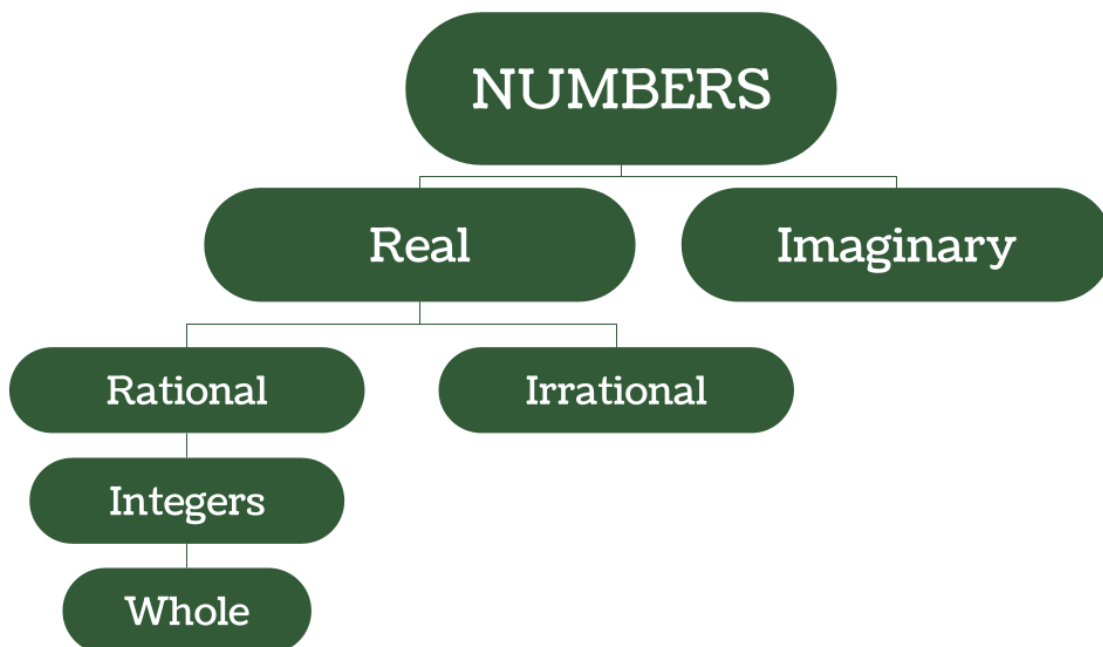
Welcome!

Hiya! This class introduces the fundamentals of geometry. But here's the catch - you kind of have to know a little bit of math before we get started. This chapter is a quick review of numbers and some fundamental math to help prepare you for the rest of the course. Most of this should feel like a review but there might be some gems in here that you don't know!

Definitions

You are probably familiar with different types of numbers (for instance, whole numbers, integers, decimals, fractions, and maybe even imaginary numbers). We have different needs for different types of numbers, but for the most part we all can get by knowing just **integers** (positive and negative **whole numbers**) and decimals.

But there are more nuanced classifications of numbers. Here's the deal - you don't need to commit these definitions to memory, but you should be aware of each of these categories:



- **Whole numbers** are what we typically think of. They are not decimals and they are not negative. The whole numbers start at 0 and go up from there:

$$\{0, 1, 2, 3, \dots\}$$

- **Integers** are whole numbers but include negative numbers as well.

$$\{\dots -3, -2, -1, 0, 1, 2, 3, \dots\}$$

- **Rational numbers** include fractions (don't forget that every fraction can also be expressed as a decimal, but not every decimal has a fractional equivalent)

$$\{\dots -\frac{2}{3} \dots 0 \dots \frac{7}{8} \dots 10 \dots\}$$

- **Irrational numbers** are decimals that *cannot* be written as a fraction. This is because it is a decimal that goes on forever and doesn't repeat. The most famous example is *pi* (π) since the number goes on forever:

$$3.141592653$$

Note that some fractions *come close* to approximating *pi*, but are not quite *pi*. After all, an irrational number by definition isn't a fraction!

$$\pi = 3.141592653$$

$$22/7 = 3.142857142$$

$$335/113 = 3.141592920$$

- **Real numbers** include both rational and irrational numbers (that means real numbers also include integers). Basically, most of the math we do in our lives only uses real numbers.
- **Imaginary numbers** are real wonky - those are square roots of negative numbers. Yeah, like really crazy. We won't be using them in this class.

Operations

Just as a reminder, there are four major operations in math (plus a bonus fifth operation!). We'll be using the four major ones in this class, but let's check them all out right now.

Addition +

This is just adding two numbers. You know it. You love it. You've been doing this your whole life.

$$3 + 4 = 7$$

$$-6 + 2 = -4$$

Subtraction -

This is just subtracting two numbers. You know it. You love it. You've also been doing this your whole life. The only crazy part is that when you subtract a negative number, you are really adding it.

$$4 - 1 = 3$$

$$7 - -2 = 9$$

Multiplication \times , $$, \bullet , \circ*

This is the fun one - multiplying two numbers. Just be careful of negative signs. A negative times a negative is a positive!

$$4 * 1 = 4$$

$$-3 * -6 = 18$$

$$-5 * 4 = -20$$

Division $/$, \div

Same rules as multiplication - you'll get a positive answer (**quotient!**) if both numbers are positive or both numbers are negative.

$$5 / 2 = 2.5$$

$$21 / -3 = -7$$

$$-52 / -4 = 13$$

Modulo %

This one might be new to you - **modulo** is the *remainder* when you divide two numbers.

$$100 \% 4 = 0$$

$$10 \% 3 = 1$$

$$55 \% 7 = 6$$

Distributing

The notion of **distributing** is pretty neat. In the real world, we tend to use the word *distributing* to mean divide things up, sometimes evenly. But in math it's a bit different. It means take one number and apply it to a whole bunch of other numbers (through multiplying). I know that sounds like a mouthful, so let's look at a pretty easy example where we distribute the "2" to all the numbers in parentheses by multiplying:

$$\begin{aligned}
 &2(3 + 4 - 8) \\
 &= 2 \bullet 3 + 2 \bullet 4 - 2 \bullet 8 \\
 &= 6 + 8 - 16 \\
 &= -2
 \end{aligned}$$

Let's try another example where we distribute the 3 across all the numbers in the parentheses:

$$\begin{aligned}
 &3(-3 + 14 - 8) \\
 &= 3 \bullet (-3) + 3 \bullet 14 - 3 \bullet 8 \\
 &= -9 + 42 - 24 \\
 &= 9
 \end{aligned}$$

Order of Operations

One of the most critical skills in computations is knowing the **order of operations**. Not all math operations are created equally - some take **precedence** over others (that means you have to do them first). There's a good chance you remember the mnemonic for order of operations - **PEMDAS** - as "Please Excuse My Dear Aunt Sally". PEMDAS encodes the proper order of operations in a computation.

1. Parentheses
2. Exponents
3. Multiplication & Division
4. Addition & Subtraction

It's a bit confusing if you haven't done this in a while, but basically it means to scan an expression and search out parentheses. If you encounter any parentheses, do the operations in there first. Once you are done (or if there are no parentheses), then move on to exponents. After you've handled all the exponents (or if there aren't any), move on to multiplication and division *in the order you encounter them*. This is important - PEMDAS does *not* mean "do multiplication first and then division" - what it *really* means is that multiplication and division should be done from left to right. Lastly, handle any addition and subtraction *in the order you encounter them left to right*.

It may have been a while since you've done the order of operations, so let's look at a few examples. It'll probably come back pretty quickly.

$$\begin{aligned}
 &3 + 4 * 12 \\
 &3 + 48 \\
 &51
 \end{aligned}$$

In this previous example, we did the multiplication first. There were no parentheses or exponents, so we looked

for multiplication and division.

$$8 / 4 * 3 - 2 * 4$$

$$2 * 3 - 8$$

$$6 - 8$$

$$-2$$

In this case, we did the division first *since multiplication and division are done in the order you encounter them*. At the same time, we multiplied 2 and 4. Next we just multiplied 2 and 3, and then we did the subtraction.

$$3 (4 + 6)$$

$$3 (10)$$

$$30$$

Parentheses first!

$$4 (5 - 2)^3$$

$$4 (3)^3$$

$$4 * 27$$

$$108$$

Another set of parentheses - let's first do the math in there (that's the five minus two). Then we handle the exponents, then just multiply.

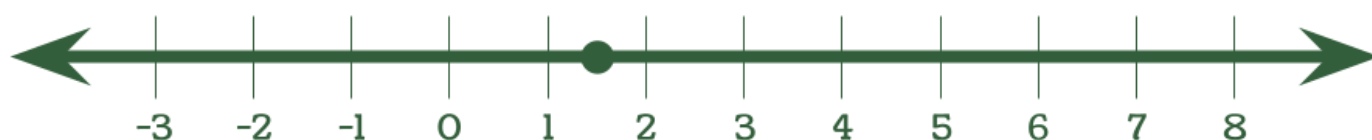
Phew! That's a lot of math (in fact, that's probably the most arithmetic we'll do in this course). But it is critical that you have a math refresher before moving on. Don't worry - at the end of every chapter you'll have an opportunity to practice your skills.

DECIMALS

Definitions

The term **decimal** is based on the Latin prefix *deci*, meaning ten. We tend to think of decimals as the numbers between whole numbers (for instance, 6.78 is between 6 and 7), though the number 6.0 and 7.0 are also decimals.

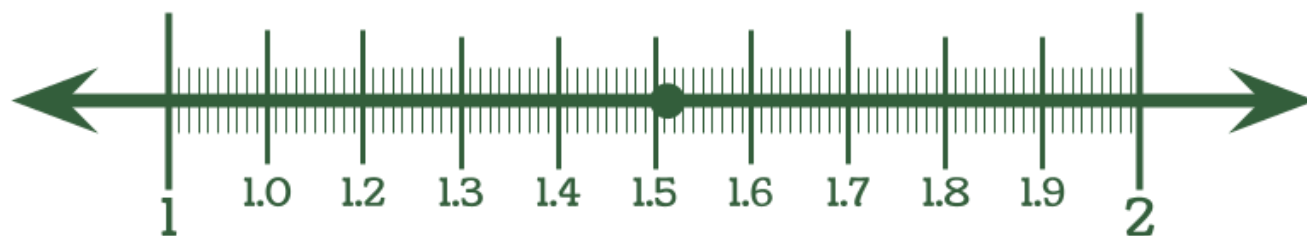
Let's consider the number line below. The point resides somewhere between 1 and 2, and it looks to be just about halfway between so we would probably estimate this to be right around 1.5.



Now let's zoom in a little bit:



Now that we can see a bit more, it looks like we are just slightly past the 1.5 mark. Let's call it 1.51. As we zoom in, we can get more information from the little marks between the two numbers. It does indeed look like the point resides just a hair past the 1.5 mark. But let's zoom in even more:



It looks like the point is actually between 1.51 and 1.52, but it's a bit closer to 1.51. If the point were directly between 1.51 and 1.52, we'd say it is probably 1.515. But since it's a tiny bit closer to 1.51, let's call it 1.513.

This is an example of measuring, estimating, and precision. The better our measuring equipment is, the more precise we can be with the measurement.

Math with Decimals

When it comes to doing math with a decimal, the best thing to do is to use a calculator. But there will be times when you might need to do some estimation or comparison, so let's just refresh our decimal skills. One thing to recognize is that you should always have a **leading zero** when dealing with decimals (that means that it's better to write .25 as 0.25).

Comparing decimals

If you can look at two decimals and figure out which one is bigger, then you're golden. It is pretty straightforward. You can start comparing two numbers, digit by digit, and if they are the same you keep going. When you get to a point where the two digits are not the same, whichever one is larger belongs to the larger number:

$$\begin{array}{r} 2.89721 \\ 2.89657 \end{array}$$

In this case, the two numbers looked to be the same at "2", then at "8", then at "9". But the top number wins out because "7" is greater than the corresponding "6" in the bottom number.

Adding and subtracting decimals

The only rule you need to remember is to line up the decimal points; all the other addition and subtraction rules are still maintained:

$$\begin{array}{r} 1.589 \\ + 0.45 \\ \hline 2.039 \end{array}$$

Sure, it looks a little wonky (like, there's nothing on the second line under the 9 - but that's okay because we can just treat that blank spot as a zero).

Multiplying decimals

You should count the number of spots to the right of the decimal point in both numbers before doing the math and that is the number of spots you should have in the answer:

$$0.15 * 1.887$$

In this example, 0.15 has two spots to the right of the decimal point and 1.887 has three spots to the right of the decimal point meaning we have five spots to the right of the decimal point in our answer. We can just multiply 15 by 1887 and when we get an answer, we'll move the decimal point five spots:

$$15 * 1887 = 28305$$

Now we just have to move the decimal five spots to the left:

$$0.28305$$

Dividing decimals

This is pretty much the same thing as multiplication, but now we move the decimal point to the right:

$$4 / .2$$

I count one number after the decimal point in .2, and there is no decimal point in 4. So it looks like when we are done, we'll just move the decimal over one spot to the right.

Let's do the math of $4 / 2$

$$4 / 2 = 2$$

Now move that decimal point over:

$$20$$

Honestly, just use a calculator 😊

FRACTIONS

Definitions

A **fraction** is the **ratio** of two different numbers (the **numerator** is on top and the **denominator** is on the bottom). So the fraction $\frac{1}{4}$ means that we are considering the number 1 as it relates to 4 (or, if you prefer, eating one piece of pie out of a pie that has four pieces).

We tend to like nice fractions; that is, fractions where the top number is smaller than the bottom number. I think that's because we are brainwashed into thinking of fractions as pieces of pie (yes, I just perpetuated that thought). If we do think of pie as an example, then the fraction $\frac{4}{3}$ doesn't really make sense for two reasons. First, who cuts a pie into three pieces? Seriously? Second, how can someone eat four pieces from a pie that only has three pieces in it?

But try to abstract that thought - a fraction is nothing more than a number on the top and a number on the bottom. If the number on top is bigger than the number on the bottom, then we call it an **improper fraction**. That's it. Sometimes, we'll encounter a **mixed fraction** (that's a whole number and a fraction). So if you consider the number $2\frac{1}{7}$, in the parlance of pies that means we have two full pies and 1 piece (of 7 pieces) of a third pie. But again, who really cuts pies into seven pieces? How would you even do that? Regardless, sometimes you'll see mixed fractions written as improper fractions (so $2\frac{1}{7} = \frac{15}{7}$).

Math with Fractions

This class is more about geometry than arithmetic and algebra, so math with fractions is a bit out of scope for this class. But one thing that you probably remember is that when you are adding or subtracting fractions, you'll need a common denominator. And when you're multiplying fractions, you just multiply across the top and multiply across the bottom. Oh yeah - and if you are dividing fractions, flip the second fraction (we call that the **reciprocal**) and then multiply. If you need refreshers on dealing with fractions, [check out this video on adding and subtracting fractions](#). For [multiplying](#), [this video is neat](#) and for [dividing](#), [be sure to watch this video](#).

For example, the reciprocal of $\frac{2}{5}$ is $\frac{5}{2}$. This will be important later on 😊.

Repeating decimal

We also like to favor fractions that are easy to understand. Like $\frac{1}{4}$ is easy to understand because we deal with currency and we know one quarter is 25% of something. Like, a quarter coin is worth twenty-five cents or 0.25 - but some numbers cannot be written so elegantly. Let's consider the fraction $\frac{1}{3}$. When we convert that to a decimal (by dividing 1 by 3 on our calculator) we get 0.333333333 and it goes on forever. That's no good! I

mean, it's not the end of the world but it isn't clean and nice.

Irrational Numbers

But it gets worse! There are some numbers that can't be written as fractions! This is because not only do they go on forever (like 0.333333...) but they also *don't repeat*! You probably already know the most famous one - pi! The number pi, or π , cannot be written as a fraction. We know pi is roughly:

$$3.14159265358979323846...$$

But it goes on forever. Just recently, [100 trillion digits of pi](#) were calculated! We use pi when dealing with circles (which we'll do later in this class!) However, there is no fraction that approximates pi. It can't be done. Sure, we can consider the following two as *close enough*:

$$\frac{22}{7} = 3.14285714286$$

$$\frac{355}{113} = 3.14159292035$$

Note that neither one of those fractions is the value of pi (but they are close enough for most things). The other number you might be familiar with is $\sqrt{2}$. This number, in decimal form, is:

$$1.41421356237...$$

There is no fraction that can express this (though $\frac{99}{70}$ comes close).

The last "famous" irrational number is *e*, or *Euler's number*. Similar to pi, *e* is a decimal that does not repeat nor does it ever end:

$$2.71828182845904523536...$$

This number shows up in a few different realms of math, perhaps most notably with natural logarithms.

Comparing Fractions

Probably the only thing you really need to take away from this chapter (other than a healthy appreciation for irrational numbers) is *how to compare fractions*. I don't deal much with fractions because they are harder to conceptualize than decimals. Consider the following two questions:

1. Which of the following is the larger value $\frac{4}{9}$ or $\frac{13}{30}$?
2. What is a fraction that is larger than $\frac{1}{4}$ but less than $\frac{1}{3}$?

Honestly, I don't know the answer to either. The first question would be easier to answer if the denominators were the same, but they aren't. In the second example, I don't even know where to start.

But.

But I know that if I punch in $4 / 9$ on my calculator I get 0.4444 and if I punch in $13 / 30$ I get .0433333, so I *do know* that $\frac{4}{9}$ is greater than $\frac{13}{30}$!

I also know that if I type $1 / 4$ on my calculator I get .25 and $1 / 3$ gives me .333333, so I just have to pick a number in between those two numbers (I dunno... maybe .3?) and then I get an answer. I also know that .3 is equivalent to $\frac{3}{10}$, so I was able to find a fraction between $\frac{1}{4}$ or $\frac{1}{3}$!