Fluid Mechanics MATLAB

San Jose State University, Charles W. Davidson College of Engineering
BME117 Biomedical Transport Phenomena Term Project

Smriti Bhalerao, Rahul Srinivas, Kayalvili Vivekanandan

Section 1 - Group 6
Alessandro Bellofiore

May 16, 2021

CONTENTS

1. INTRODUCTION

Over the last few years, mathematical models and computer simulations have seen great growth in biomedical engineering to examine biological phenomena at a wide range of scales, ranging from nanoscale devices to large life support systems. One of the important physiological systems that are simulated is the cardiovascular system, where medicine and fluid mechanics work together to study blood's behavior in the human body.

In this project, we attempt to explore the analytical solutions of the phenomenon mentioned in three parts: simplified versions of the real phenomenon with reduced dimensionality and complexity, using MATLAB tools to generate pathlines, streamlines, and track blood cells. The first part deals with defining the velocity profile and steady flow rate in a converging blood vessel; the second part deals with finding analytical expression for the position of the stagnation point in the steady flow of opposite impinging streams; the third part deals with finding the analytical expression for the time-dependent location of the stagnation point in the unsteady flow of opposite impinging streams.

Part one focuses on the application of a one-dimensional blood flow model in the blood vessels to study the effects of cardiovascular diseases such as atherosclerosis that causes narrowing of blood vessels (*Atherosclerosis | NHLBI, NIH*, n.d.). Blockages by an accumulation of plaques lead to the narrowing of walls, affecting the blood dynamics, causing many complications and diseases such as ischemic heart disease (*Atherosclerosis | NHLBI, NIH*, n.d.). Studying the effects of fluid flow in a converging blood vessel can help better understand the effects of cardiovascular diseases such as atherosclerosis on the blood flow mechanics.

Part two involves finding the location of the stagnation point in the steady flow of opposite impinging streams. This concept is widely applied in microfluidic medical devices with simple cross-slot shapes where two streams flow at a steady rate in opposite directions of the impinging streams and form a region where the local velocity is zero (Brimmo & Qasaimeh, 2017 and Jayamohan et al., 2012). Microfluidic has many applications in developing lab-on-chip devices, where stagnation point offers a better resolution of micro-particles (Brimmo & Qasaimeh, 2017 and Jayamohan et al., 2012).

Part three is very similar to part two but is for the unsteady flow of streams (Brimmo & Qasaimeh, 2017). This too holds multiple applications in microfluidic devices that help in the development of lab-on-chip devices.

Studying the steady and unsteady flows of opposite impinging streams using basic models discussed in this term paper can offer great insight into understanding the functioning of microfluidic medical devices that are widely used for the diagnosis of cardiac-related conditions (Institute of Medicine et al., 2010).

2. MODELS AND ANALYSIS

2.1 Steady Flow in a converging blood vessel (Part I)

2.1.1 Problem Statement

In Part I, we are given a rigid circular blood vessel with a converging geometry where blood is flowing through it. Information given is that the length of the vessel is 12 cm, and the initial radius and the final radius are 1.5 cm and 1 cm respectively. A figure is illustrated below (Figure 1).

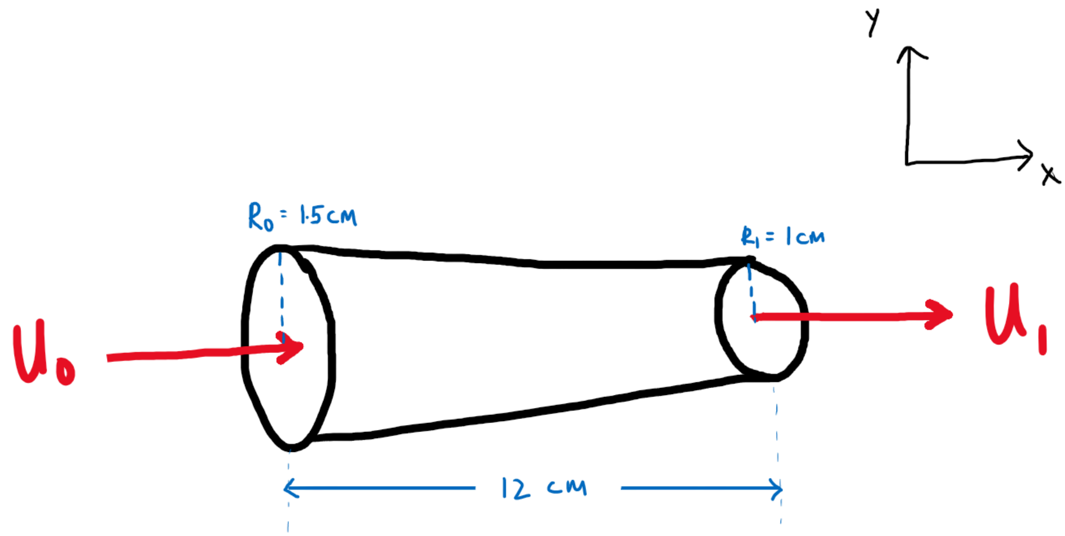


Figure 1: Blood vessel

The blood flow that is converging is given with the following equations:

$$u = U_o\left(\frac{ax}{L} + 1\right)$$

$$v = -by$$

where U_o is the average of the three last two-digit numbers of the group members SJSU ID and x is the horizontal coordinate and both a and b are constants.

<u>Last 2 digits of each students ID number:</u> Rahul: 78; Kayal: 99; Smriti: 12

$$U_o = \frac{78 + 99 + 12}{3}$$

$$U_o = 63\ cm/s$$

2.1.1.1 Finding the values of a and b that satisfy the conservation principles

<u>To find the values of a and b, the following assumptions are made:</u>

(a) The flow is steady

(b) The density, ρ, is constant as there is only one fluid present, the flow is incompressible

(c) 2-D flow (x and y component present)

Firstly to find a and b, we have to find the outlet velocity with the information that has been given to us

$$\rho\, A_0 U_0 \;=\; \rho\, A_1 U_1$$

Since the fluid is incompressible, ρ*can be crossed out. Doing so, we obtain,*

$$A_0 U_0 \;=\; A_1 U_1$$

$$\pi r_0^{\,2} U_0 = \pi r_1^{\,2} U_1$$

$$\pi\,(1.5\;cm)^2\,(63\;cm/s) \;=\; \pi\,(1\;cm)^2\,U_1$$

$$\therefore U_1 \;=\; 141.75\;cm/s$$

By finding the exit velocity, we can substitute that in the first equation, $u = U_o\,(\frac{ax}{L} \;+\; 1)$ to find a.

$$u \;=\; U_0\,(\frac{ax}{L} + 1)$$

$$141.75\frac{cm}{s} \;=\; 63\frac{cm}{s}\,(\frac{a\,(12cm\,)}{(12cm)} + 1)$$

$$\frac{141.75\;cm/s}{63\;cm/s} = a\,+\,1$$

$$\therefore a \;=\; 1.25$$

To find values of b, we will be using the Navier Stokes equation, which is a derivation of the Conservation of Mass:

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} + \frac{\partial \rho w}{\partial z} \;=\; 0$$

Taking into consideration the previously mentioned assumptions, $\frac{\partial \rho w}{\partial z}$can be eliminated since there is no z-component, and $\frac{\partial \rho}{\partial t} \;=\; 0$, as it is in steady state. Doing so, we obtain the following:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\frac{\partial}{\partial x}\,(U_0\frac{ax}{L} + 1) \;+\; \frac{\partial}{\partial y}\,(-\,by) \;=\; 0$$

$$U_0\frac{a}{L} - b = 0$$

$$b \;=\; U_0\frac{a}{L}$$

$$b \;=\frac{63(1.25)}{12} = 6.5625$$

$$\therefore b \cong 6.56$$

Therefore, a = 1.25 and b = 6.5625.

2.1.1.2 Finding the velocity and flow rate at the exit of the converging section

2.1.1.2.1 Velocity

To find the velocity of the outlet, we will use the equation of the magnitude of the velocity

$$U_1 = \sqrt{u^2 + v^2}$$

Substituting u and v, we get:

$$U_1 = \sqrt{[U_0(\frac{ax}{L} + 1)]^2 + (-\ by)^2}$$

We know that the flow only moves horizontally, so y = 0, and since this is the exit, x = 12

$$U_1 = \sqrt{[63(\frac{1.25(12)}{(12)} + 1)]^2 + [(-\ 6.56)(0)]^2}$$

$$U_1 = \sqrt{[63(2.25)]^2}$$

$$\therefore U_1 = 141.75\frac{cm}{s}$$

Therefore, we get a value of the exit velocity of 141.75 cm/s (as we have previously calculated).

2.1.1.2.2 Flow rate

To find the flow rate, we have to find the inlet velocity as well. Following the same method, we used to find the outlet velocity, except this time x will be 0 as we are entering the system.

$$U_0 = \sqrt{[63(\frac{1.25(0)}{(12)} + 1)]^2 + [(-\ 6.56)(0)]^2}$$

$$U_0 = \sqrt{[63(1)]^2}$$

$$\therefore U_0 = 63\frac{cm}{s}$$

To find the mass flow rate we will use the following equation:

$$\rho A_0 U_0 = \rho A_1 U_1$$

$$A_0 U_0 = A_1 U_1 \quad (\because \text{ Fluid is incompressible})$$

Since we only want to find the mass flow rate for the outlet, we can rewrite the equation and solve for it:

$$Q = A_0 U_0$$

$$Q = [pi\,(1.5cm)^2]\,[63\frac{cm}{s}]$$

$$\therefore Q = 445.32\frac{cm^3}{s}$$

Therefore, the mass flow rate for the outlet is 445.32 cm^3/s

2.1.1.3 Finding the pressure drop across the whole blood vessel

To find the pressure drop, we will have to use Bernoulli's equation. Before that three assumptions should be made:

(a) Viscosity is constant; inviscid flow
(b) The density of blood is 1060 kg/m^3
(c) There is no y-component in this flow, therefore the height is a constant

$$P_0 + \frac{1}{2}\rho {v_0}^2 + \rho g h_0 = P_1 + \frac{1}{2}\rho {v_1}^2 + \rho g h_1$$

$$\because \rho g h_0 \ and\ \rho g h_1 \ are\ constants,\ the\ equation\ transforms\ to\ the\ following$$

$$P_0 + \frac{1}{2}\rho {v_0}^2 = P_1 + \frac{1}{2}\rho {v_1}^2$$

$$P_0 - P_1 = \frac{1}{2}\rho({v_1}^2 - {v_0}^2)$$

$$P_0 - P_1 = \frac{1}{2}\rho({u_1}^2 - {u_0}^2)$$

$$P_0 - P_1 = \frac{1}{2}\ 1060\frac{kg}{m^3}\ [(1.4175\ \frac{m}{s})^2 - (0.63\frac{m}{s})^2]$$

$$P_0 - P_1 = 854.57\ \frac{kg}{ms^2}$$

$$P_0 - P_1 = 854.57\text{Pa}$$

Therefore, the pressure difference is 854.57 Pa.

2.1.1.4 Finding the function that should be used for the streamline

$$\int \frac{1}{(6.5625x + 63)}dx = \int \frac{1}{-6.5625y}dy \qquad (1)$$

Consider LHS: $\int \frac{1}{6.5625x + 63)}dx$

Let, $u = 6.5625x + 63$ So, $du = 6.5625\ dx$

$$\int \frac{1}{6.5625x + 63}\ dx = \int \frac{1}{u} * \frac{1}{6.5625}du$$

$$\int \frac{1}{6.5625x + 63}\ dx = \frac{1}{6.5625}ln\ |u| + C_1$$

$$\int \frac{1}{6.5625x + 63}\, dx = \frac{1}{6.5625} ln\,|6.5625x + 63| + C_1 \qquad (2)$$

Consider RHS: $\int \frac{1}{-6.5625y} dy$

Let, $v = -6.5625y$ So, $dv = -6.5625\, dy$

$$\int \frac{1}{-6.5625y} dy = \int \frac{1}{v} * \frac{-1}{6.5625} dv$$

$$\int \frac{1}{-6.5625y}\, dy = -\frac{1}{6.5625} ln\,|v| + C_2$$

$$\int \frac{1}{-6.5625}\, dy = -\frac{1}{6.5625} ln\,|-6.5625y| + C_2 \quad (3)$$

Substituting equations (2) and (3) into equation (1), we get

$$\frac{1}{6.5625} ln\,|6.5625x + 63| + C_1 = -\frac{1}{6.5625} ln\,|-6.5625y| + C_2$$

$$\frac{1}{6.5625} ln\,|6.5625x + 63| + \frac{1}{6.5625} ln\,|-6.5625y| = C$$

2.2 Steady Flow of opposite impinging streams (Part II)

2.2.1 Problem Statement

In Part two of this report, we have been given a mixing system where the flows of the fluid inside a stream flow in opposite directions until they both impinge onto one another in a certain plane (Figure 2).

<u>We are given a few assumptions:</u>

(a) The driving pressure gradient of both the strains present are <u>constants</u> with respect to time.
(b) The flow is steady
(c) The density, ρ, is constant as there is only one fluid present, the flow is incompressible
(d) 2-D flow (x and y component present)

The two equations for the flow are given such that:

$$u = 2.0 + 1.2x$$
$$v = 1.0 - 1.2y$$

The domain of the fluid in interest is

$$x \in (-4, 4) \text{ and } y \in (-4, 4)$$

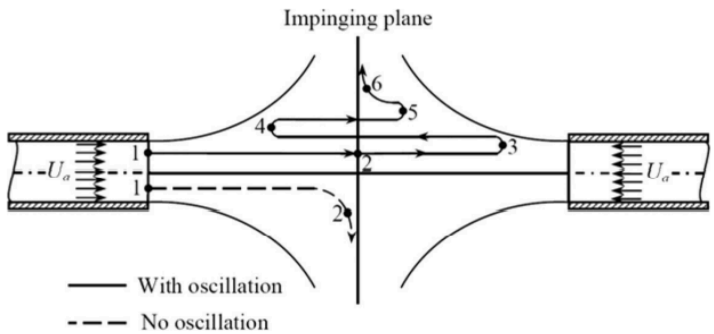


Figure 2: Impinging jets (no defined axes)

2.2.1.1 Finding the location of the stagnation point in the flow field

In order to find the stagnation point, we must first use the equation of a velocity field. A stagnation point is a certain point in the flow field where the velocity of the fluid is zero.

$$\overline{V} = ui + vj + wk$$
$$Since\ we\ don't\ have\ any\ z - component\ in\ our\ system,\ wk = 0$$
$$\overline{V} = ui + vj$$

Using the above equation and canceling out the value of wk as there is no z-component in this system. We can then substitute the equations given to us in the problem statement into the velocity field equation.

$$\overline{V} = (2.0 + 1.2x)\ \hat{i} + (1.0 - 1.2y)\ \hat{j}$$

To get both stagnation points, we set the velocity field to be zero. This means that, u = 0 and v = 0 as this is a steady flow.

$$\overline{V} = 0\ makes\ u = 0\ and\ v = 0$$

$$u = 2.0 + 1.2x \qquad\qquad v = 1.0 - 1.2y$$
$$0 = 2.0 + 1.2x \qquad\qquad 0 = 1.0 - 1.2y$$
$$x = \frac{-2.0}{1.2} \qquad\qquad y = \frac{1}{1.2}$$
$$x = -1.667 \qquad\qquad y = 0.833$$

Therefore, the location of the stagnation point in the flow field is (-1.667, 0.833)

2.2.2 Finding the function that should be used for the streamline

$$\int \frac{1}{(2+1.2x)}\ dx = \int \frac{1}{(1-1.2y)}\ dy \qquad (1)$$

Consider LHS: $\int \frac{1}{2+1.2x} dx$

Let, $u = 2 + 1.2x$ So, $du = 1.2\, dx$

$$\int \frac{1}{2+1.2x} dx = \int \frac{1}{u} \cdot \frac{1}{1.2} du$$

$$\int \frac{1}{2+1.2x} dx = \frac{1}{1.2} ln\,|u| + C_1$$

$$\int \frac{1}{2+1.2x} dx = \frac{1}{1.2} ln\,|2 + 1.2x| + C_1 \qquad (2)$$

Consider RHS: $\int \frac{1}{1-1.2y} dy$

Let, $v = 1 - 1.2y$ So, $dv = -1.2\, dy$

$$\int \frac{1}{1-1.2y} dy = \int \frac{1}{v} \cdot \frac{-1}{1.2} dv$$

$$\int \frac{1}{1-1.2y} dy = -\frac{1}{1.2} ln\,|v| + C_2$$

$$\int \frac{1}{1-1.2y} dy = -\frac{1}{1.2} ln\,|1 - 1.2y| + C_2 \; (3)$$

Substituting equations (2) and (3) into equation (1), we get

$$\frac{1}{1.2} ln|2 + 1.2x| + C_1 = -\frac{1}{1.2} ln\,|1 - 1.2y| + C_2$$

$$\frac{1}{1.2} ln|2 + 1.2x| + \frac{1}{1.2} ln\,|1 - 1.2y| = C$$

2.3 Unsteady flow of opposite impinging streams (Part III)

2.3.1 Problem Statement

In Part III, we will be using the same mixing system referenced in Part II (Figure 2) in which we have been given a mixing system where the flows of the fluid inside a stream flow in opposite directions until they both impinge onto one another in a certain plane.

The only difference is that now the driving pressure gradients are not constants and that they both change over time. This will cause the vertical component (y-axis) to oscillate with a frequency of 1 Hz, which corresponds to the angular velocity $\omega = 2\pi$rad/s.

<u>Assumptions:</u>

(a) The driving pressure gradient of both the strains present is changing with respect to time.

(b) The flow is steady
(c) The density, ρ, is constant as there is only one fluid present, the flow is incompressible
(d) 2-D flow (x and y component present)

The two equations for the flow are given such that:

$$u = 2.0 + 1.2x \qquad v = 1.0 + 1.5\,cos(\omega t) - 1.2y$$

The domain of the fluid in interest and time is

$$x \in (0, 11)\text{m and } y \in (-5, 6)\text{m}$$
$$t \in (0, 2)\text{sec}$$

2.3.1.1 Analytical expression for the time-dependent location of the stagnation point in this flow field.

As previously mentioned, the stagnation point is where the velocity is zero and since both equations, u and v are changing in respect to time, we can set them both to zero and we will get this:

$$0 = u = \frac{dx}{dt} \qquad 0 = v = \frac{dy}{dt}$$

With these concepts, we can then find the integration of each side with the respective component that they are changing in respect of whether it be time or position. The following will be the Eulerian equations we have integrated:

x-component:

$$u = \frac{dx}{dt}$$

$$2.0 + 1.2x = \frac{dx}{dt}$$

$$\int_{x_o}^{x} \frac{dx}{2+1.2x} = \int_{t_o}^{t} dt$$

$$\frac{1}{1.2}[ln(2 + 1.2x) - ln(2 + 1.2x_o)] = t - t_0$$

$$\frac{1}{1.2}\left[ln\,\frac{(2+1.2x)}{(2+1.2x_0}\right] = t - t_0$$

$$e^{\frac{1}{1.2}\left[ln\,\frac{(2+1.2x)}{(2+1.2x_0}\right]} = e^{t-t_0}$$

$$\frac{2+1.2x}{2+1.2x_0} = e^{(t-t_0)^{1.2}}$$

$$2 + 1.2x = \left[e^{(t-t_0)^{1.2}}\right]\left[2 + 1.2x_0\right]$$

$$x(t) = \frac{\left[\left[e^{1.2(t-t_0)}\right]\left[2+1.2x_0\right]\right]-2}{1.2}$$

<u>y-component:</u>

$$v = \frac{dy}{dt}$$

$$1.0 + 1.5\,cos(\omega t) - 1.2y = \frac{dy}{dt}$$

$$\frac{dy}{dt} + 1.2y = 1.5\,cos(2\pi t) + 1$$

Using the following integrating factor,

$$e^{\int 1.2dt} = e^{1.2t} \text{ (Integration factor)}$$

Multiplying the integrating factor on both sides, we obtain the following:

$$e^{1.2t}\left[\frac{dy}{dt}\right] + e^{1.2t}[1.2y] = e^{1.2t}\left[1.5\,cos(2\pi t) + e^{1.2t}[1]\right]$$

$$\frac{d}{dt}(ye^{1.2t}) = e^{1.2t} + e^{1.2t}[1.5\,cos(2\pi t)]$$

$$\int_{y_0}^{y} d(ye^{1.2t}) = \int_{t_0}^{t}\left[e^{1.2t} + e^{1.2t}[1.5\,cos(2\pi t)]\right]dt$$

$$\int_{y_0}^{y} d(ye^{1.2t}) = \int_{t_0}^{t} e^{1.2t}dt + \int_{t_0}^{t} e^{1.2t}[1.5\,cos(2\pi t)]dt$$

$\int_{t_0}^{t} e^{1.2t}\,[1.5cos(2\pi t)]$ *can be solved the following way:*

$$\Rightarrow \int e^{1.2t}[1.5\,cos(2\pi t)]dt = \frac{e^{1.2t}[1.5\,sin(2\pi t)]}{2\pi} - \int \frac{[1.5\,sin(2\pi t)]\,e^{1.2t}(1.2)}{2\pi}$$

$$= \frac{e^{1.2t}[1.5\,sin(2\pi t)]}{2\pi} - \frac{-[1.5\,cos(2\pi t)]e^{1.2t}(1.2)}{4\pi^2} + \int \frac{-1.5cos(2\pi t)]e^{1.2t}(1.2)^2}{4\pi^2}$$

$$= \frac{e^{1.2t}[1.5sin(2\pi t)]}{2\pi} + \frac{1.5cos(2\pi t)e^{1.2t}(1.2)}{4\pi^2}$$

$$=\frac{\frac{e^{1.2t}[1.5sin(2\pi t)]}{2\pi}+\frac{[1.5cos(2\pi t)e^{1.2t}(1.2)}{4\pi^2}}{1+\frac{(1.2)^2}{4\pi^2}}$$

Plugging back the integral into the equation, we obtain the following:

$$\int_{y_0}^{y} d(ye^{1.2t}) = \int_{t_0}^{t} e^{1.2t}dt + \int_{t_0}^{t} e^{1.2t}[1.5cos(2\pi t)]dt$$

$$\left[ye^{1.2t}\right]_{y_0}^{y} = \left[\frac{e^{1.2t}}{1.2}\right]_{t_o}^{t} + \left[\frac{\frac{e^{1.2t}[1.5sin(2\pi t)]}{2\pi}+\frac{[1.5cos(2\pi t)e^{1.2t}(1.2)}{4\pi^2}}{1+\frac{(1.2)^2}{4\pi^2}}\right]_{t_o}^{t}$$

$$y - y_0 = \frac{\frac{1.5sin(2\pi t)}{2\pi}+\frac{1.5cos(2\pi t)(1.2)}{4\pi^2}}{1+\frac{(1.2)^2}{4\pi^2}} - \frac{\frac{1.5sin(2\pi t_o)}{2\pi}+\frac{1.5cos(2\pi t_o)(1.2)}{4\pi^2}}{1+\frac{(1.2)^2}{4\pi^2}}$$

$$y = y_0 + [0.23\, sin(6.28t) + 0.04\, cos(6.28t)] - [0.23\, sin(6.28t_0) + 0.04\, cos(6.28t_0)]$$

$$y = y_0 + 0.23[sin(6.28t) - sin(6.28t_0)] + 0.04[cos(6.28t) - cos(6.28t_0)]$$

Next, in order to find the level set function of both the x and y components of the streamline, we shall make $t_0 = 0$. We will use the equations that we have formulated for F in terms of x and y by getting rid of the t component in the said equations.

For the x component, we just have to replace t with the function of x in the pathline y(t):

$$\frac{1}{1.2}\, ln\frac{(2+1.2x)}{(2+1.2x_0)} = t - t_0$$

When $t_0 = 0$,

$$\frac{1}{1.2}\, ln\frac{(2+1.2x)}{(2+1.2x_0)} = t$$

$$t = \frac{5}{6}\, ln\frac{(2+1.2x)}{(2+1.2x_0)}$$

$$t = \ln((2+1.2x)/(2+1.2x_0))^{5/6}$$

Now that we have found what t is, we can substitute the value of t into the y component y(t) and also making $t_0 = 0$.

When $t_0 = 0$,

$$y = y_0 + 0.23\,sin(6.28t) - 0.23sin(6.28 * 0) + 0.04cos(6.28t) - 0.04cos(6.28* 0))$$

Substituting $t = ln((2+1.2x)/(2+1.2x_0))^{5/6}$,

$$y = y_0 + \left[0.23\,sin\left[6.28\left(ln\frac{2+1.2x}{2+1.2x_0}^{5/6}\right)\right]\right] + \left[0.04\,cosn\left[6.28\left(ln\frac{2+1.2x}{2+1.2x_0}^{5/6}\right)\right]\right] - 0.04$$

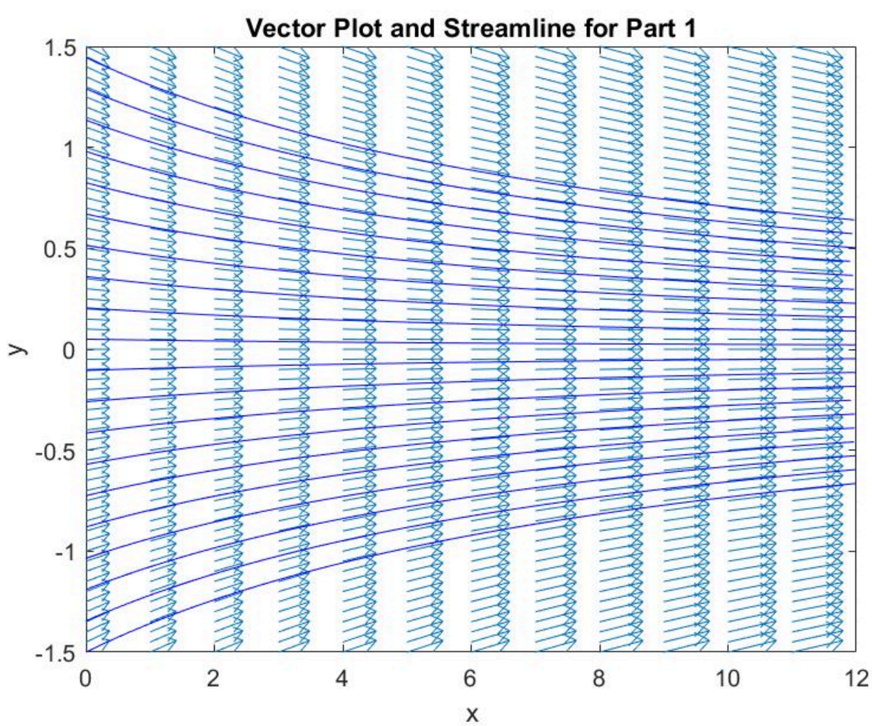
Finally,

$$F(x,y) = y_0 - y + \left[0.23\,sin\left[6.28\left(ln\frac{2+1.2x}{2+1.2x_0}^{5/6}\right)\right]\right] + \left[0.04\,cos\left[6.28(ln\frac{2+1.2x}{2+1.2x_0}^{5/6})\right]\right] - 0.04$$

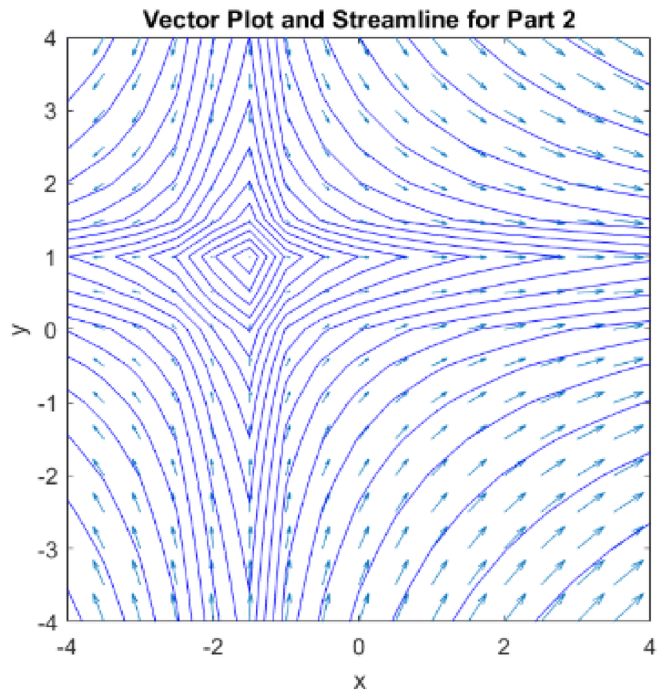
3. RESULTS

(Please note that x-axis for each graph represents the x-position and y-axis for each graph represents the y-position of the flow.)

3.1 Part I

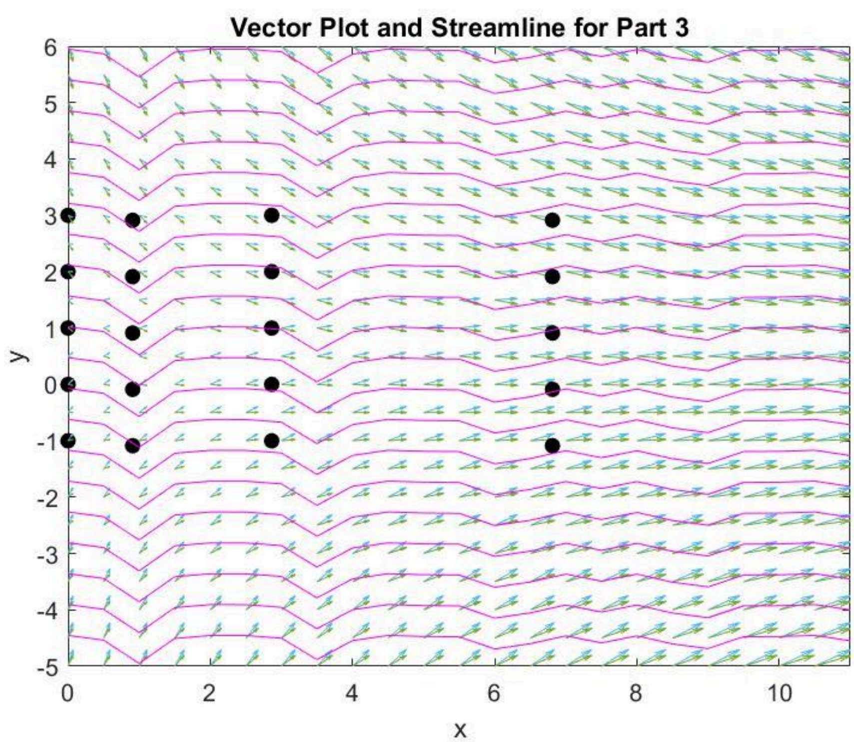


3.2 Part II



3.3 Part III

Vector Plot, Streamlines, and Pathlines for 5 particles with 4 time particles for Part 3



4. DISCUSSION

4.1 Interpretation of the results in light of the example applications identified in the introduction

As Part I and Part II are similar problems with different velocity fields from steady-state flows, the vector field of Part I needs to represent the flow inside a converging blood vessel. This is because the velocity in the positive x-direction must increase with increasing x-value increases as they are linearly related with a positive slope. The magnitude of velocity in the y-direction must decrease with decreasing distance from the y-value of 0 as they are linearly related with a negative slope.

The vector field of Part II is supposed to be similar to the vector field of Part I in terms of how the direction of the velocity vector changes based on the change in both x and y-direction. In this case, the x-value and velocity in the x-direction are directly proportional and linearly related with a positive slope just like in Part I. The y-value and velocity in the y-direction are directly proportional and linearly related with a negative slope just like in Part I. Even though this kind of flow is caused by opposite impinging jets rather than a converging blood vessel, the streamlines of Part I and Part II are to follow the displayed velocities of every represented position of the vector field in a similar way with different magnitudes of position and velocity.

The vector field of Part III was supposed to be represented in a way that is similar to how the vector field of Part II was based on how the velocities of the fluid are to change in position. The convective accelerations between all positions of the fluid for Part II and Part III are to be exactly the same if not for what causes local acceleration in the y-direction. As the velocities in the y-direction are to change with respect to time in a way that represents simple harmonic motion in Part III, the represented streamlines in Part III are to be represented in a sinusoidal manner due to time being inside a cosine function of the y-velocity. The pathlines represented in this vector field display the trajectory of how 5 particles move with respect to time in a duration of 2 seconds.

4.2 Comparison between part II and III

What makes the flow in Part ll steady is the driving pressure gradients of both streams to be constant. What makes the flow in Part lll unsteady is the driving pressure gradients of both streams to change over time. Even though velocities change with respect to the position in both parts, the velocities for each particular position of the fluid remain constant for Part II while the velocities for each particular position of the fluid change over time for Part III. For Part II, the stagnation point is constant. For Part III, the stagnation point changes over time.

Regarding how the vector plots are displayed, Part II only shows streamlines of the vector field. Streamlines of Part II show a group of curves that are tangent to the displayed velocities of every represented position of the vector field with x-values ranging from -4 to 4 and y-values ranging from - 4 to 4. Part III shows pathlines of 5 different particles' trajectories as well as streamlines. Part II did not need pathlines as the streamlines are to remain constant. Part

III needed pathlines as the velocity and position of the particles needed to change over time and that the streamlines needed to look different over time.

4.3 Comparison between streamlines and pathlines in part III

The streamlines in Part III show a group of curves that are tangent to the displayed velocities of every represented position of the vector field with x-values ranging from 0 to 11 and y-values ranging from - 5 to 6. As the difference between each adjacent represented position of the vector field is 0.25 in both x and y directions,

The pathlines of Part III show trajectories that certain particles follow as time changes. The position of each particle based on certain time durations would show on the vector field to make up the pathline. The graphs showed trajectories that particles from certain initial positions are to follow. As the initial x-positions of all 5 particles equal zero, the initial y-positions of all 5 particles are all the integers that range from - 1 to 3. All 5 of the different trajectories displayed on the vector field show the position of the particles when the time is equal to 0 seconds, 0.5 seconds, 1 second, 1.5 seconds, and 2 seconds.

4.4 Limitations of the models and analysis included in this project

Even though we were able to figure out the velocities of certain points in both x and y-direction in all 3 parts, we were only able to use only certain x and y positions instead of every single x and y position from negative infinity to positive infinity. Despite making a video in Part III that helps determine how the streamlines and positions of certain particles change with respect to time, the video was only able to track certain particles from certain positions rather than every single particle of the entire fluid. However, we were not able to make the other video and the video we made was not what we wanted to see. For the first video, we are expecting the vector plot to change over time as the direction of the arrows represented in the graph is to change with respect to time. However, everything in the graph did not change and a blurry square took up the graph's space instead. For the second video, we are expecting the vector plot, streamlines, pathlines, and position of all 5 particles to change over time as the direction of the arrows represented in the graph are to change with respect to time. However, the second video turned out not very different from the first video.

5. CONCLUSION

In this term paper, we studied different flow patterns of blood using basic analytical modeling for three different cases: steady flow in a converging blood vessel, steady flow of opposite impinging streams, and unsteady flow of opposite impinging streams. These analytical models were then used to visualize using MATLAB tools. The two-dimensional models discussed in this term paper helped us better understand some of the basic concepts of fluid mechanics discussed in this class, and learn about their applications in the field of biomedical engineering.

6. REFERENCES

Atherosclerosis | NHLBI, NIH. (n.d.). NIH. Retrieved May 15, 2021, from https://www.nhlbi.nih.gov/health-topics/atherosclerosis#:%7E:text=Over%20time%2C%20plaque%20hardens%20and,%2C%20stroke%2C%20or%20even%20death

Brimmo, A. T., & Qasaimeh, M. A. (2017). Stagnation point flows in analytical chemistry and life sciences. *RSC Advances*, *7*(81), 51206–51232. https://doi.org/10.1039/c7ra11155j

Institute of Medicine, Board on the Health of Select Populations, & Criteria, C. O. S. S. C. D. (2010). *Cardiovascular Disability: Updating the Social Security Listings*. National Academies Press.

Jayamohan, H., Sant, H. J., & Gale, B. K. (2012). Applications of Microfluidics for Molecular Diagnostics. *Microfluidic Diagnostics*, 305–334. https://doi.org/10.1007/978-1-62703-134-9_20

7. STATEMENTS OF ACADEMIC INTEGRITY

Statement of Academic Integrity By signing below, we affirm that the work on this term paper (including report, figures, scripts and simulations) is solely our own and that we have not received or obtained assistance from any other team.

Also, we affirm that we have not shared our work with other teams. We also affirm that, wherever we have included information, data or opinions from reputable sources (e.g. journal articles, books, conference proceedings), the sources were appropriately cited in the text and referenced in the bibliography.

We affirm that we have reviewed and understand the definitions of cheating and plagiarism, as included in the San Jose State University's Academic Integrity Policy, posted in Canvas and also available via the website of the Office of Student Conduct and Ethical Development at http://sa.sjsu.edu/student_conduct or the university catalog at http://info.sjsu.edu/static/catalog/integrity.html.

We affirm that we understand that cheating, plagiarism or academic dishonesty can result in disciplinary action – including academic and administrative sanctions – according to the San Jose State University's Academic Integrity Policy. We also understand that faculty members are required to report all infractions to the office of Student Conduct and Ethical Development. The Student Conduct and Ethical Development website is available at http://www.sa.sjsu.edu/judicial_affairs/index.html.

Team member #1

Printed Name: Smriti Bhalerao Date: 5/14/2021

Signature: DocuSigned by: Smriti Bhalerao EB02191CAEEE48A...

Team member #2

Printed Name: kayalvili Vivekanandan Date: 5/14/2021

Signature: DocuSigned by: kayalvili Vivekanandan BCB58DDF2513498...

Team member #3

Printed Name: Rahul Srinivas Date: 5/14/2021

Signature: DocuSigned by: Rahul Srinivas F0B5CACC1E1C422...

8. ACKNOWLEDGMENTS

Kayalvili Vivekanandan	Rahul Srinivas	Smriti Bhalerao
- Section 2 (Models and Analysis) - Section 3 (Results) - Code and Graph for Part I - Section 5 (Conclusion) - Section 6 (References)	- Section 2 (Models and Analysis) - Section 4 (Discussion) - Code and Graph for Part II - Section 6 (References)	- Section 1 (Introduction) - Section 2 (Models and Analysis) - Section 4 (Discussion) - Code and Graph for Part II - Section 6 (References)

9. APPENDIX (MATLAB Code)

9.1 Code for Part I

```
% define constants
L = 12; % length of blood vessel (given)
R0 = 1.5; % radius of the inlet (given)
R1 = 1; % radius of the outlet (given)
a = 1.25; % constant a (calculated)
b = 6.5625; % constant b (calculated)
U0 = 63; % velocity of inlet (calculated)
% creates a meshgrid of this x-interval [0,12] and this y-interval [-1.5,1.5]
[x,y] = meshgrid(0:1:12, -1.5:0.05:1.5); % x = length, y = radius
% given equations of the flow
u = U0*((a*x)/L + 1);
v = -b*y;
% plot a vector plot and streamlines
quiver(x,y,u,v); % vector plot
[startx, starty] = meshgrid(0,-1.5:0.155:1.5); % meshgrid helps set the number of streamlines,
%need to keep changing interval size until desired 20 streams are found
streamline(x,y,u,v,startx,starty); % displays all 20 streamlines
% plot the graph
xlabel('x') % labels the x-axis of the graph x-position of the flow
ylabel('y') % labels the y-axis of the graph y-position of the flow
title('Vector Plot and Streamline for Part 1') % title of the graph
axis([0 12 -1.5 1.5]) % sets the range of x-axis from 0 to 12 and the y-axis from -1.5 to 1.5
```

9.2 Code for Part II

```
[x,y]=meshgrid(-4:0.5:4, -4:0.5:4) ; % creates a meshgrid of this x-interval [-4,4] and this
%y-interval [-4,4]
u = 2 + 1.2*x ; % function of velocity in x-direction
v = 1 - 1.2*y ; % function of velocity in y-direction
quiver(x,y,u,v) ; % displays the vector plot
daspect([1 1 1]) ;
hold on
xlabel('x') % labels the x-axis of the graph x-position of the flow
ylabel('y') % labels the y-axis of the graph y-position of the flow
title('Vector Plot and Streamline for Part 2') % gives the graph a title
F = (1/1.2).*log(abs(2 + 1.2.*x)) + (1/1.2).*log(abs(1 - 1.2.*y)); % defining function that should
%be contoured
contour(x,y,F,20, 'LineColor','b') % contouring appropriately to make 20 streamlines
axis([-4 4 -4 4]) % sets the range of x-axis from -4 to 4 and the y-axis from -4 to 4.
```

9.3 Code for Part III

```
tstag = [0:0.1:2]; % shows values of time for function of y-position for stagnation point
ystag = (1 - 1.5*cos(2*pi.*tstag))./1.2; % gives a function of y-position for stagnation point with
%respect to time
[x,y] = meshgrid(0:0.5:11, -5:0.5:6); % creates a meshgrid of this x-interval [0,11] and this
%y-interval [-5,6]
x0 = 0; % x-position is 0 for all particles
t0 = 0; % time is 0 for all particles
yi0 = -1; % y-position is -1 for 1st particle
yi1 = 0; % y-position is 0 for 2nd particle
yi2 = 1; % y-position is 1 for 3rd particle
yi3 = 2; % y-position is 2 for 4th particle
yi4 = 3; % y-position is 3 for 5th particle
for t = [0:0.5:1.5] % talking about particles at these time intervals: time = 0s, time = 0.5s, time =
%1s, and time = 1.5s
    xt = ((2 + 1.2.*x0)*exp((t-t0).^1.2) - 2)/1.2 % function of x-position with respect to time for
%all 5 particles
   a = (1.5/(2*pi))*sin(2*pi.*t);
   a0 = (1.5/(2*pi))*sin(2*pi.*t0);
   b = (1.5*1.2/(4*pi*pi))*cos(2*pi.*t);
   b0 = (1.5*1.2/(4*pi*pi))*cos(2*pi.*t0);
   c = a + b
   c0 = a0 + b0
   d = (1 + (1.2*1.2/(4*pi*pi))).^-1;
   u = 2 + 1.2.*x; % function of velocity in x-direction
```

```
   v = 1 + 1.5.*cos(2.*pi.*t) - 1.2*y; % function of velocity in y-direction
   XC = (1/1.2).*(log((2 + 1.2.*x)/(2+1.2.*x0)));
   AC = (1.5/(2*pi))*sin(2*pi.*XC);
   BC = (1.5*1.2/(4*pi*pi))*cos(2*pi.*XC);
   C1 = 0.23*log(abs(6.28*(sin(2+1.2*x)/(2+1.2*x0)).^(5/6)))
   C2 = 0.04*log(abs(6.28*(cos(2+1.2*x)/(2+1.2*x0)).^(5/6)))
   quiver(x,y,u,v);  % displays the vector plot
   hold on
     plot(xt, yi0 + d.*(c - c0), 'ok', 'markerface', 'k') % plot of 1st particle's pathline based on
%parametric equations
    F0 = yi0 + C1 + C2 - y - 0.04  ; % defining function that should be contoured based on the 1st
%particle
     contour(x,y,F0,20, 'LineColor','b') % contouring appropriately to make 20 streamlines based
%the 1st particle
     plot(xt, yi1 + d.*(c - c0), 'ok', 'markerface', 'k') % plot of 2nd particle's pathline based on
%parametric equations
    F1 = yi1 + C1 + C2 - y - 0.04 ;    % defining function that should be contoured based on the
%2nd particle
     contour(x,y,F1,20, 'LineColor','r') % contouring appropriately to make 20 streamlines based
%the 2nd particle
     plot(xt, yi2 + d.*(c - c0), 'ok', 'markerface', 'k') % plot of 3rd particle's pathline based on
%parametric equations
    F2 = yi2 + C1 + C2 - y - 0.04 ; %  defining function that should be contoured based on the 3rd
%particle
     contour(x,y,F2,20, 'LineColor','g') % contouring appropriately to make 20 streamlines based
%the 3rd particle
     plot(xt, yi3 + d.*(c - c0), 'ok', 'markerface', 'k') % plot of 4th particle's pathline based on
%parametric equations
    F3 = yi3 + C1 + C2 - y - 0.04 ; % defining function that should be contoured based on the 4th
%particle
     contour(x,y,F3,20, 'LineColor','y') % contouring appropriately to make 20 streamlines based
%the 4th particle
     plot(xt, yi4 + d.*(c - c0), 'ok', 'markerface', 'k') % plot of 5th particle's pathline based on
%parametric equations
    F4 = yi4 + C1 + C2 - y - 0.04 ; %  defining function that should be contoured based on the 5th
%particle
    contour(x,y,F4,20, 'LineColor','m')  % contouring appropriately to make 20 streamlines based
%the 5th particle
   axis([0 11 -5 6]) % sets the range of x-axis from 0 to 11 and the y-axis from -5 to 6.
end
```

```
VIDEO = VideoWriter('1stpart3IIIfinal.avi'); % names video file
open(VIDEO); % opens video
for n = 1:0.01:2
    t = n - 1; % for certain time frames
    v = 1 + 1.5*cos(2*pi.*t) - 1.2*y; % velocity in the y direction
    surf(x,F0,v); % creates appropriate surface
    axis(([0 11 -5 6 -1 1]));  % sets the range of x-axis from 0 to 11 and the y-axis from -5 to 6.
    CORRECTFRAME = getframe;
    writeVideo(VIDEO,CORRECTFRAME);
    n = n + 0.01;
end
close(VIDEO);  % closes video
VIDEO = VideoWriter('2ndpart3IIIfinal.avi');  % names video file
open(VIDEO); % opens video
for n = 1:0.01:2
    t = n - 1 % for certain time frames
    v = 1 + 1.5*cos(2*pi.*t) - 1.2*y; % velocity in the y direction
    surf(x,y,v);% creates appropriate surface
    axis(([0 11 -5 6 -1 1])); % sets the range of x-axis from 0 to 11 and the y-axis from -5 to 6.
    CORRECTFRAME = getframe;
    writeVideo(VIDEO,CORRECTFRAME);
    n = n + 0.01;
end
close(VIDEO); % closes video
```