1.

Show that $\ln(x^3 - 4x) - \ln(x^2 - 2x) \equiv \ln(x + 2)$. [3]

2.

(i) Solve the inequality $|3x - 5| < |x + 3|$. [4]

(ii) Hence find the greatest integer n satisfying the inequality $\left|3^{0.1n+1} - 5\right| < \left|3^{0.1n} + 3\right|$. [2]

3.

Find the equation of the normal to the curve

$$x^2 \ln y + 2x + 5y = 11$$

at the point $(3, 1)$. [7]

4.

(a) Find $\int \tan^2 3x \, dx$. [3]

(b) Find the exact value of $\int_0^1 \frac{e^{3x} + 4}{e^x} \, dx$. Show all necessary working. [4]

5.

The polynomial $p(x)$ is defined by

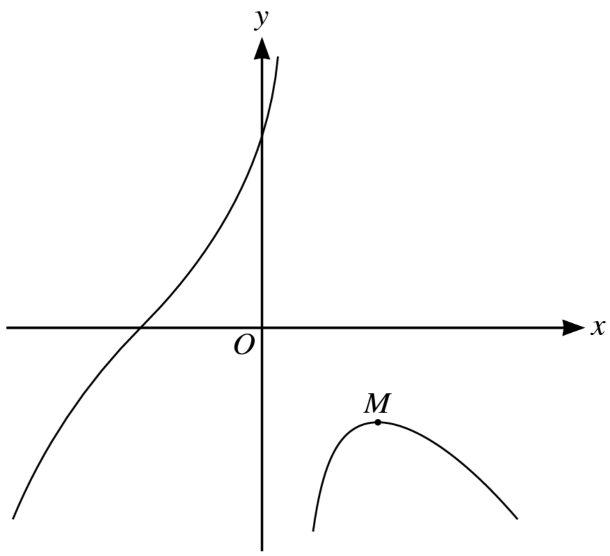
$$p(x) = 5x^3 + ax^2 + bx - 16,$$

where a and b are constants. It is given that $(x - 2)$ is a factor of $p(x)$ and that the remainder is 27 when $p(x)$ is divided by $(x + 1)$.

(i) Find the values of a and b. [5]

(ii) Hence factorise $p(x)$ completely. [3]

6.



The diagram shows the curve with equation $y = \dfrac{8 + x^3}{2 - 5x}$. The maximum point is denoted by M.

(i) Find an expression for $\dfrac{dy}{dx}$ and determine the gradient of the curve at the point where the curve crosses the x-axis. [4]

(ii) Show that the x-coordinate of the point M satisfies the equation $x = \sqrt{(0.6x + 4x^{-1})}$. [2]

(iii) Use an iterative formula, based on the equation in part **(ii)**, to find the x-coordinate of M correct to 3 significant figures. Give the result of each iteration to 5 significant figures. [3]

7.

(i) Show that $2\operatorname{cosec} 2\theta \cot\theta \equiv \operatorname{cosec}^2\theta$. [3]

(ii) Hence show that $\operatorname{cosec}^2 15° \tan 15° = 4$. [2]

(iii) Solve the equation $2\operatorname{cosec}\phi\cot\frac{1}{2}\phi + \operatorname{cosec}\frac{1}{2}\phi = 12$ for $-360° < \phi < 360°$. Show all necessary working. [5]