Anglican High School
2020 Secondary 4 Preliminary Examination Mathematics Paper 2

Answer **All** the questions

1 **(a)** **(i)** Factorise $2px - 2p + 3qx - 3q$ completely.

Answer …….. [2]

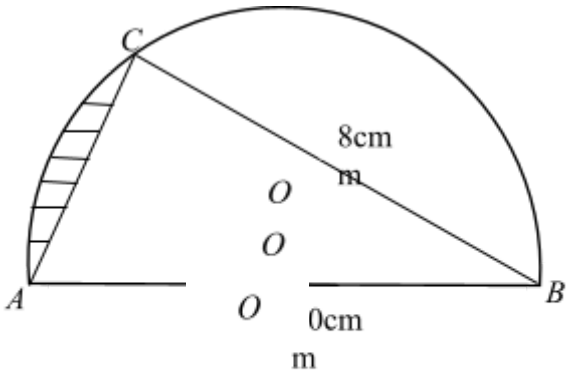
(ii) Given that p and q are positive constants, find the value of x for which $2px - 2p + 3qx - 3q = 0$.

Answer $x =$…….. [1]

(b) Simplify $\left(-3p^2q^{-1}\right)^2\left(p^{-2}q^2\right)^3$, expressing your final answer in positive indices.

Answer …….. [2]

2 In the diagram, $BC = 8$ cm and $AB = 10$ cm is the diameter of the semi-circle.



Find

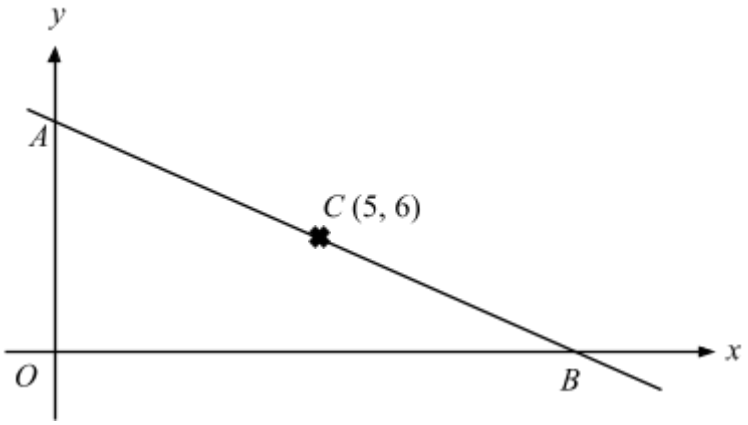
(a) angle COA in radians,

Answer ……radians [2]

(b) area of the shaded region.

Answer ……..cm^2 [3]

3 In the diagram, not drawn to scale, point A lies on the y-axis and point B lies on the x-axis. The coordinates of C is (5, 6).



(a) Given that C lies on the line AB and that $5OA = 3OB$, show that the y-intercept of the line AB is 9.

Answer

[3]

(b) Given that point D lies on the y-axis, state the coordinates of D such that triangle ACD is an isosceles triangle.

Answer (…… , ……) [1]

(c) Given further that $OCEA$ is a parallelogram, state the coordinates of the point E.

Answer (…… , ……)
[1]

(d) Find the area of the parallelogram, $OCEA$.

Answer ……..……..units^2 [2]

4 Mrs Tan bought x kg of rice for \$65 in December 2019. In February 2020, the price of rice increased and she received 6 kg less for the same amount of money spent.

(a) Write down an expression for the price of rice per kilogram, in terms of x,

(i) in December 2019,

Answer \$…….. [1]

(ii) in February 2020.

Answer \$…….. [1]

(b) If the increase in price is \$2 per kilogram of rice, form an equation in x and show that it reduces to $x^2 - 6x - 195 = 0$.

Answer

[3]

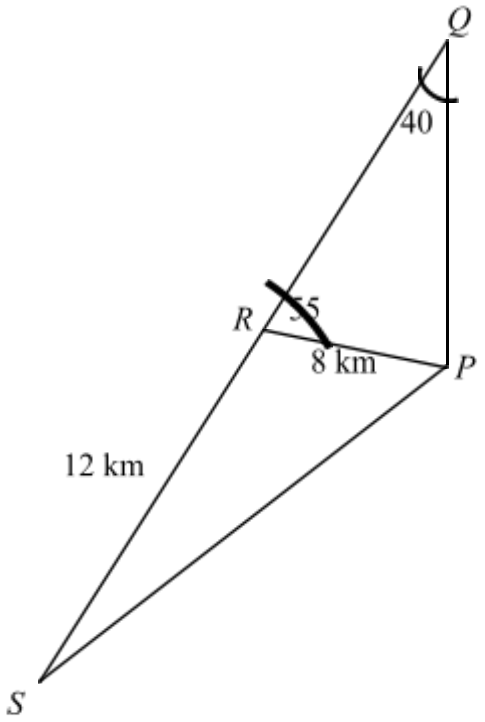
(c) Solve the equation, giving your answers correct to 2 decimal places.

Answer x =……or …….. [2]

(d) Hence, find the price of rice per kilogram in February 2020, leaving your answer to the nearest cent.

Answer …….. [1]

5 In the diagram, *PQRS* are four points on level ground, and *Q* is due north of *P*.



Given that angle $PQR = 40°$, angle $PRQ = 55°$, $PR = 8$ km, $RS = 12$ km and that *QRS* is a straight line,

(a) show that the distance *PS* is 17.836 km.

Answer

[3]

(b) Using the result in **(a)**, find,

(i) the bearing of P from S,

°

Answer …….. [3]

(ii) the shortest distance from R to PS.

Answer ……..km [2]

(c) A vertical tower stands at point R. Evan, walking along PS and stops at a point where the greatest angle of elevation of the top of the tower, T, is 3°. Find the height of the tower.

Answer ……..km [2]

6 **(a)** The table below shows the amount of flour, sugar and number of eggs needed for each type of pastry sold in a cafe.

	Flour (g)	Sugar (g)	Number of eggs
Cookie	90	50	1.5
Cake	220	200	8
Pancake	60	80	2

The above information is represented by a matrix $\mathbf{D} = \begin{pmatrix} 90 & 50 & 1.5 \\ 220 & 200 & 8 \\ 60 & 80 & 2 \end{pmatrix}$.

Each kg of flour and sugar costs \$1.50 and \$1.80 respectively. A dozen eggs costs \$2.70. Complete the cost in \$ for every gram of flour and sugar as well as the cost in \$ for each egg in the table below :

Flour(\$/g)	Sugar(\$/g)	1 Egg(\$)

[2]

(b) Write down a 3×1 matrix **E** such that its elements represent the unit cost of each ingredient needed for the various pastries.

Answer …….. [1]

(c) Calculate **DE** and state what the elements of **DE** represent.

Answer ……

……

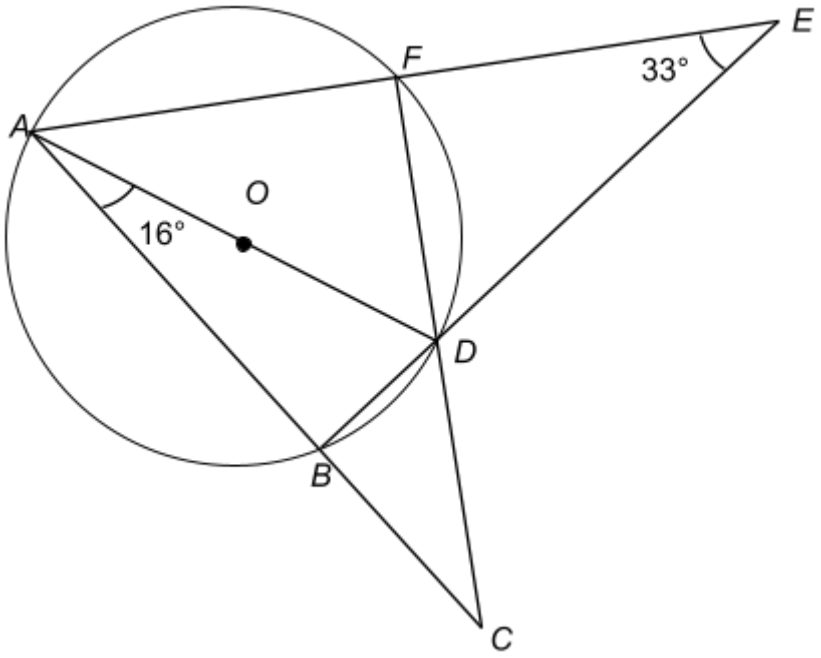
…… [2]

(d) The cafe prepared 70 cookies, 30 cakes and 120 pancakes on a particular day. Using matrix multiplication, calculate the total cost of the basic ingredients for all the pastries prepared on this day.

Answer $…….. [2]

7 **(a)** In the diagram, AD is the diameter of the circle $ABDF$ with centre O. Given BD and AF produced meet at E, AB and FD produced meet at C,

$\angle FED = 33^o$ and $\angle DAB = 16^o$.



Calculate, stating your reasons clearly,

(i) $\angle ADB$,

°

Answer …….. [2]

(ii) $\angle AFB$,

°

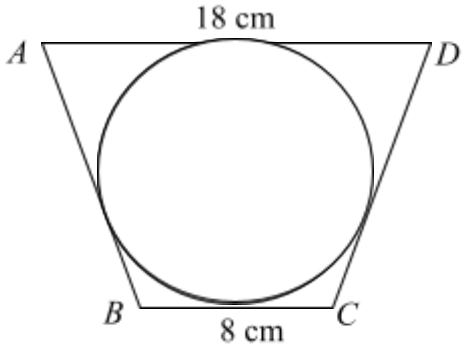
Answer …….. [1]

(iii) $\angle DAF$.

°

Answer …….. [2]

(b) Given $ABCD$ is a trapezium with AD = 18 cm, BC = 8 cm and $AB = DC$. A circle is inscribed in the trapezium as shown.



(i) Show that the length of AB is 13 cm, stating your reason(s) clearly.

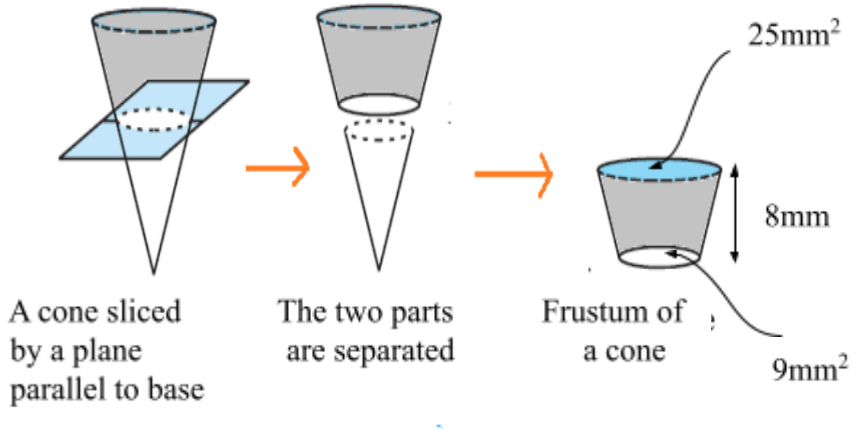
Answer

[2]

(ii) Calculate the radius of the circle.

Answer ……..cm [2]

8 **(a)** A solid cone is cut into a smaller cone and a frustum as shown in the diagram.



The height of the frustum is 8mm. The area of the two circular bases are 9 mm^2 and 25 mm^2 respectively.

(i) Show that the height of the smaller cone is 12 mm.

Answer

[2]

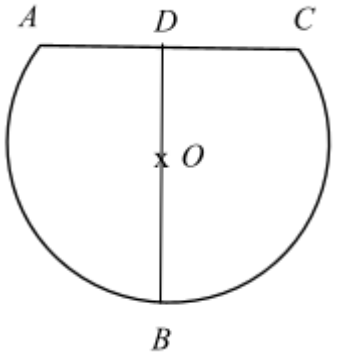
(ii) Find the volume of the frustum.

Answer ……mm^3 [3]

(iii) Find the curved surface area of the smaller cone.

Answer ……..mm^2 [3]

(b) In the figure, *ADCB* is a major segment of a circle, centre *O* and radius 7 cm. *BD* = 12 cm. *AC* is perpendicular to the line *BOD*. Find the perimeter of the major segment.



Answer ……..cm [4]

9 Mr Tan is looking to purchase a new car to drive to and from work on weekdays and for leisure on weekends. He estimates the total distance travelled is about 1500 km per month. The average cost of petrol is $2.20 per litre. To buy a new car, he must pay a down payment of 30% of the selling price of the car before he can take a car loan for the remaining amount from a bank. He has to pay back the loan by monthly instalment. A 5-year car loan simple interest rate offered by most banks is 2.28% per annum. Mr Tan shortlisted 3 cars with all the relevant cost as shown in the table.

	Car A	Car B	Car C
Selling Price	$87 999	$108 999	$107 888
Fuel Consumption(km per litre)	17.2	14.9	17.8
Car Insurance per year	$ 1 200	$ 1500	$ 1 500
Engine Capacity(in cubic cm)	1 598	1 499	1 197
Monthly Car Maintenance	$200	$200	$200
Monthly Car Park charges	$150	$150	$150

He also finds out that the annual road tax of the car is determined by the engine capacity of the car is as follows:

$$\text{Annual Road Tax} = [\$500 + 0.75(\text{Engine Capacity minus } 1000)] \times 0.782$$

(a) After reviewing the information, Mr Tan decides to buy Car A.

(i) calculate the down payment.

Answer $…….. [1]

(ii) calculate the total **monthly** expenditure including monthly instalment payment and all the other monthly costs.

Answer $…….. [7]

(b) An alternative to buying a car is to rent an electric car that is easily accessible from his house and workplace. Subscription per month is $15 and it costs 33 cents per minute of use. Mr Tan estimates that the average daily travel time to and from work is about 1 hour and 20 minutes. On weekends, he needs to use the car for about 5 hours for leisure activities. Calculate his monthly expenditure to rent a car.

Answer $…….. [2]

(c) Do you think Mr Tan should buy or rent a car? Justify your answer.

Answer……

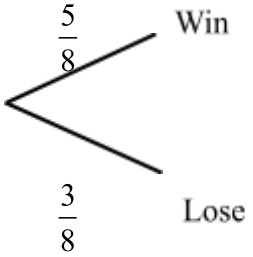
……..……..……..[1]

10 **(a)** Team Alpha and Delta are competing in the badminton finals. Each game will only result in a win or a loss. The competition ends when a team wins 2 games out of 3.

The probability of Alpha team winning in a game is $\frac{5}{8}$.

(i) Draw a tree diagram to show all the possible outcomes for Alpha team.

Answer



[2]

(ii) Calculate the probability, expressing your answers in fraction, that Alpha team wins the competition.

Answer …….. [2]

10 (b) The frequency table shows the weight in kg of 80 persons who join an exercise club.

Weight	$30 < x \leq 40$	$40 < x \leq 50$	$50 < x \leq 60$	$60 < x \leq 70$	$70 < x \leq 80$
Frequency	8	17	34	18	3

(i) Find the mean weight.

Answer ……kg [1]

(ii) Find the standard deviation.

Answer ……..kg [1]

(iii) After 6 months of exercising, the mean and standard deviation of these 80 persons are 50kg and 12.2 kg respectively. Give 2 comments on the effect of exercise.

1 ……

……

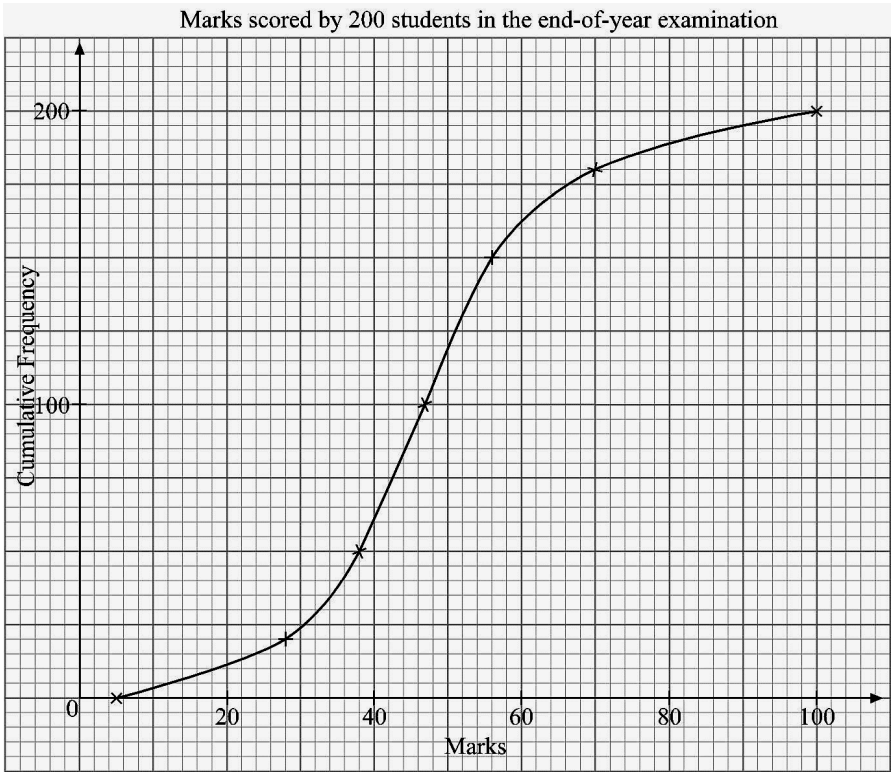
……

2 ……

……

……... [2]

10 (c) The cumulative frequency graph below shows the end-of-year Mathematics examination marks of a group of 200 students.



From the graph, find the

(i) number of students who scored 28 marks or less,

Answer …….. [1]

(ii) number of students who scored more than 76 marks,

Answer …….. [1]

(iii) the upper quartile,

Answer …….. [1]

(iv) minimum mark obtained by the top 2.5% of the cohort,

Answer …….. [1]

(v) probability that one student scored 28 marks or less and the other student scored more than 76 marks when two students are chosen at random.

Answer ……..[2]

11 The variables x and y are connected by the equation

$$y = \frac{5x}{4} + \frac{1}{x^2}.$$

The table below shows some values of x and the corresponding values of y, correct to 1 decimal place.

x	0.5	1	1.5	2	3	4	5
y	4.6	2.3	2.3	2.8	3.9	5.1	p

(a) Calculate the value of p.

Answer p = ……..… [1]

(b) Using a scale of 2 cm to represent 1 unit on each axis, draw a horizontal x-axis for $0 \le x \le 5$ and a vertical y-axis for $0 \le y \le 8$. On your axes, plot the points given in the table and join them with a smooth curve. [3]

(c) From the graph, find the values of x for which $\frac{5x}{4} + \frac{1}{x^2} - 3 = 0$.

Answer x = ……. or……..…. [2]

(d) By drawing a tangent, find the gradient of the curve at the point $(1,\ 2.3)$.

Answer ……..…. [2]

(e) **(i)** On the same axes, draw the graph of $y = \frac{1}{2}x + 2$. [2]

(ii) Write down the x-coordinates of the points where the two graphs intersect.

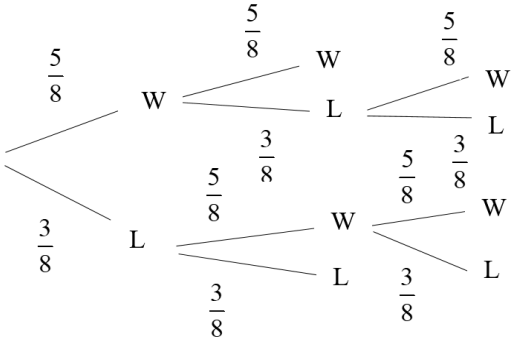
Answer x = ……..……or…… [1]

(iii) Find the equation in the form $3x^3 + ax^2 + bx + c = 0$, which is satisfied by the values of x found in part **(e)(ii).**

Answer …….. [1]

END OF PAPER

Answer key for paper 2

1	**(a)(i)** $(2p+3q)(x-1)$ (ii) $x=1$ (b) $\frac{9q^4}{p^2}$	**2**	**(a)** 1.29 radians **(b)** $4.09cm^2$
3	(b) A(0, 9). D(0, 3) (c) *E* (5, 15) (d) 45 unit^2	**4**	**(a) (i)** \$ $\frac{65}{x}$ (ii) \$ $\frac{65}{x-6}$ (c) $x = 17.28$ or $x = -11.28$ (d) \$5.76 (to nearest cent)
5	(b)(i) 061.6° (ii) 4.41 km (c) 0.231 km.		
7	(a) (i) 74^o (ii) $74°$ (iii) $41°$ (b) (i) 13 (Tangent from ext. point) (ii) 6 cm	**6**	**(a)** 0.0015, 0.0018, 0.225(egg) (b) **E** = $\begin{pmatrix}0.0015\\0.0018\\0.225\end{pmatrix}$ (c) $\begin{pmatrix}0.5625\\2.49\\0.684\end{pmatrix}$ (c) The elements in **DE** represents the cost price (in dollars) needed for the making of one cookie, one cake and one pancake respectively (d) \$196.16
8	(a) (ii) $130\frac{2}{3}mm^2$ *or* $131mm^2(3sf)$ (iii) $64.4mm^2$ (b) $42.9cm(3sf)$		
9	(a)(i) \$26,399.70 (ii) \$1847.36 (b) \$924 (Accept answers from **\$939 to \$1064.40** as some students may assume up to 23 working days and 4 weekends or 21 working days and 5 weekends) (c) Acceptable responses such as 1. Renting is better as there is no need to pay down payment and monthly instalment. 2. Although it is more expensive to buy a car, it is very convenient. 3. Any other reasonable justification.		
10 (a)	(a)(i)  	**11**	(a) $p \approx 6.3$

(ii) $\frac{175}{256}$

(b) (i) 53.875 kg or $\frac{431}{8} kg \text{ or } 53\frac{7}{8} kg$

(ii) 9.87 kg

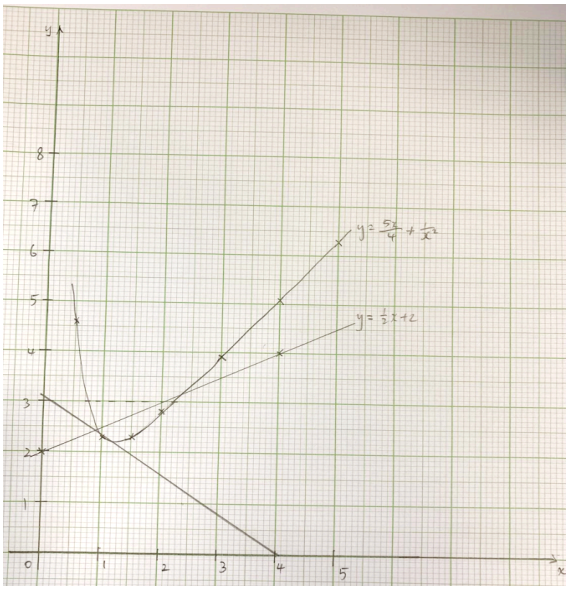
(iii) (1) The mean weight is lower. Jogging has helped to reduce weight.

(2) Standard deviation is higher implies that weight loss after exercising is more spread out. (accept "not consistent")

(c) (i) 20 students (ii) 15 students

(iii) 56 marks (iv) 90 marks (v) $\frac{3}{199}$

(b)



(c) From the graph, when $y = 3$,

$x = 0.7 \text{ or } x = 2.2$ (Accept ± 0.2 0)

(d) From the tangent at (1, 2.3),

the gradient is – 0.75 (± 0.2)

(e) (i) Draw graph of $y = \frac{1}{2}x + 2$

(ii) The *x* coordinates are 0.9 and 2.3

(iii) $3x^3 - 8x^2 + 4 = 0$