MHT CET 14 OCTOBER 2020 Morning Session (GROUP NO. 28)

****

Q- 26 : The measure of the acute angle between the lines given by the equation $3x^2 - 4\sqrt{3}xy + 3y^2 = 0$ is

(a) 60^0 (b) 70^0 **(c) 30^0** (d) 45^0

Solution : XII – Pair of Straight Lines

Given equation is $3x^2 - 4\sqrt{3}xy + 3y^2 = 0$

Comparing with $ax^2 + 2hxy + by^2 = 0$ then a = 3 , h = $-2\sqrt{3}$, b = 3

Let θ be the acute angle between them , then $tan\theta = \left|\frac{2\sqrt{h^2-ab}}{a+b}\right| = \left|\frac{2\sqrt{(-2\sqrt{3})^2-(3)(3)}}{3+3}\right|$

$tan\theta = \left|\frac{2\sqrt{12-9}}{6}\right| = \left|\frac{2\sqrt{3}}{6}\right| = \frac{1}{\sqrt{3}} \Rightarrow \theta = \left(\frac{1}{\sqrt{3}}\right) \quad \therefore \theta = 30^0$

Answer : Option C : 30^0

Q- 27 : If $\int x^x(1 + logx)dx = k.x^x + c$ then k = ……

(a) $log_e(e^2)$ (b) $log_e\left(\frac{1}{e^2}\right)$ **(c)** $log_e e$ (d) $log_e\left(\frac{1}{e}\right)$

Solution : XII – Integration

Put $x^x = t$

$\therefore x^x(1 + logx)dx = dt$

$\therefore \int dt = k.x^x + c$

$\therefore t + c = k.x^x + c$

$\therefore x^x + c = k.\ x^x + c \qquad \therefore k = 1 = log_e e$

Answer : Option : C : $log_e e$

Q- 28 : $cos\left(\frac{3\pi}{4} + x\right) - sin\left(\frac{\pi}{4} - x\right) = ...$

(a) $\sqrt{2}\,cosx$ (b) $-\sqrt{2}\,cosx$ (c) $-\sqrt{2}\,sinx$ (d) $\sqrt{2}\,sin\,x$

Solution : XI – Trigonometry – II

$$cos\left(\frac{3\pi}{4} + x\right) - sin\left(\frac{\pi}{4} - x\right)$$

$$= cos\left(\frac{\pi}{2} + \frac{\pi}{4} + x\right) - sin\left(\frac{\pi}{4} - x\right)$$

$$= - sin\left(\frac{\pi}{4} + x\right) - sin\left(\frac{\pi}{4} - x\right)$$

$$= - \left[sin\left(\frac{\pi}{4}\right).cosx + cos\left(\frac{\pi}{4}\right).sinx\right] - \left[sin\left(\frac{\pi}{4}\right).cosx - cos\left(\frac{\pi}{4}\right).sinx\right]$$

$$= - \frac{1}{\sqrt{2}}.cosx - \frac{1}{\sqrt{2}}.sinx - \frac{1}{\sqrt{2}}.cosx + \frac{1}{\sqrt{2}}.sinx$$

$$= - \frac{1}{\sqrt{2}} cosx - \frac{1}{\sqrt{2}} cosx = \left(\frac{-2}{\sqrt{2}}\right)cosx = -\sqrt{2} cosx$$

Answer : Option (b) : $-\sqrt{2}\, cosx$

Q -29 : If the radius of a circle increases at the rate of 7 cm /sec , then the rate of increase of its area after 10 minutes is

(a) 1,88,400 cm^2/sec (b) 1,68,400 cm^2/sec (c) 1,64,800 cm^2/sec **(d) 1,84,800 cm^2/sec**

Solution : XII : Applications of Derivative

Let r be the radius of the circle.

$\therefore \frac{dr}{dt} = r$ cm / sec ……..(1)

Area of the circle = πr^2

$\therefore \frac{dA}{dt} = 2\pi r\left(\frac{dr}{dt}\right)$

$\therefore \frac{dA}{dt} = 2\pi r(r)$ ………From (1)

Area after 10 minutes is required.

In 1 second radius is increasing by r cm

10 minutes ie. In 600 second

1 second → r

600 second → x

$\therefore x = \frac{600 \times r}{1} = 600 \times 7 = 4200$ cm

$\therefore \frac{dA}{dt} = 2 \times \frac{22}{7} \times 4200 \times 7 = 8400 \times 22 \frac{cm^2}{sec}$

$\therefore \frac{dA}{dt} = 1,84,800$ cm^2 / sec.

Answer : Option : (d) $1,84,800$ **cm² / sec**

Q-30 : If the vectors ($2\hat{i} - q\hat{j} + 3\hat{k}$) and $\left(4\hat{i} - 5\hat{j} + 6\hat{k}\right)$ are collinear, then the value of q is

(a) $-\frac{2}{5}$ (b) $\frac{2}{5}$ (c) $-\frac{5}{2}$ **(d)** $\frac{5}{2}$

Solution : XI : Vectors

If two vectors are collinear then $\bar{a} = t\,\bar{b}$.

$\therefore\left(2\hat{i} - q\hat{j} + 3\hat{k}\right) = t\left(4\hat{i} - 5\hat{j} + 6\hat{k}\right)$

2 = 4t ; - q = - 5t ; 3 = 6t

t = $\frac{1}{2}$ and q = 5t $\Rightarrow 5\left(\frac{1}{2}\right) \Rightarrow q = \frac{5}{2}$

Answer : (d) : $\frac{5}{2}$

Q-31 : If the planes $2x - 5y + z = 8$ & $2\lambda x - 15y + \lambda z + 6 = 0$ are parallel to each other, then value of λ is

(a) -3 **(b) 3** (c) 2 (d) $\frac{1}{3}$

Solution : XII : Line and Plane

If the planes are parallel to each other then the normal are also parallel to each other.

Therefore coefficients of normal are in proportion .

$\therefore \frac{2}{2\lambda} = \frac{5}{15} = \frac{1}{\lambda}$

$\therefore \frac{1}{\lambda} = \frac{1}{3}$ $\therefore \lambda = 3$

Answer : Option (b) : $\lambda = 3$

Q-32 : y = $c^2 + \frac{c}{x}$ is the solution of the differential equation

(a) $x^4\left(\frac{dy}{dx}\right)^2 + x\left(\frac{dy}{dx}\right) - y = 0$ (b) $x^4\left(\frac{dy}{dx}\right)^2 + x\left(\frac{dy}{dx}\right) + y = 0$

(c) $x^4\left(\frac{dy}{dx}\right)^2 - x\left(\frac{dy}{dx}\right) - y = 0$ (d) $x^4\left(\frac{dy}{dx}\right)^2 - x\left(\frac{dy}{dx}\right) + y = 0$

Solution : XII : Differential Equation

Let y = $c^2 + \frac{c}{x}$ ……..(1)

Diff. w.r.t.x.

$\frac{dy}{dx} = 0 + \frac{-c}{x^2}$

$\therefore\ -c = x^2\left(\frac{dy}{dx}\right)$

$\therefore c = -x^2\left(\frac{dy}{dx}\right)$

Equation (1) becomes ,

$y = \left[-x^2\left(\frac{dy}{dx}\right)\right]^2 + \frac{-x^2\left(\frac{dy}{dx}\right)}{x}$

$y = x^4\left(\frac{dy}{dx}\right)^2 - x\left(\frac{dy}{dx}\right)$

$\therefore x^4\left(\frac{dy}{dx}\right)^2 - x\left(\frac{dy}{dx}\right) - y = 0$

Answer : Option (c) : $x^4\left(\frac{dy}{dx}\right)^2 - x\left(\frac{dy}{dx}\right) - y = 0$

Q-33 : The cumulative distribution function of a continuous random variable X is given by ,

F(X= x) = $\frac{\sqrt{x}}{2}$, then P [x > 1] is

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{1}{\sqrt{2}}$ (d) $\frac{1}{3}$

Solution : XII : Probability Distribution

P[X > 1] = 1 – P [X ≤ 1]

We know that , P [X ≤ 1] = F (1)

$\therefore P\,[X > 1] = 1 - F(X = 1)$

$\therefore P[X > 1] = 1 - \frac{1}{2}$

$\therefore P[X > 1] = \frac{1}{2}$

Answer : Option : (a) : $\frac{1}{2}$

Q-34 : If $cosec\theta + cot\theta = 5$ then $sin\theta =$

(a) $\frac{5}{26}$ **(b)** $\frac{5}{13}$ (c) $\frac{1}{5}$ (d) $\frac{1}{13}$

Solution : XI : Trigonometry – I

$cosec\theta + cot\theta = 5$

$cosec\theta = 5 - cot\theta$

$cosec^2\theta = (5 - cot\theta)^2$

$1 + cot^2\theta = 25 - 10cot\theta + cot^2\theta$

$10cot\theta = 24$

$cot\theta = \frac{24}{10}$ $\quad \therefore cot\theta = \frac{12}{5}$

$cosec\theta = \sqrt{1 + cot^2\theta}$

$cosec\theta = \sqrt{1 + \frac{144}{25}}$ $\quad \therefore cosec\theta = \sqrt{\frac{169}{25}}$

$\therefore cosec\theta = \frac{13}{5}$ $\quad \therefore sin\theta = \frac{5}{13}$

Answer : Option : (b) : $\frac{5}{13}$

Q-35 : If $y = tan^{-1}\left[\sqrt{\frac{1+\cos cos\left(\frac{x}{2}\right)}{1-cos\left(\frac{x}{2}\right)}}\right]$ then $\frac{dy}{dx} =$

(a) $\frac{-1}{4}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) $\frac{-1}{3}$

Solution : XII : Differential Equation

Let $y = \left[\sqrt{\frac{1+cos\left(\frac{x}{2}\right)}{1-cos\left(\frac{x}{2}\right)}}\right] = \left[\sqrt{\frac{2cos^2\frac{x}{4}}{2sin^2\frac{x}{4}}}\right] = \left[\sqrt{cot^2\left(\frac{x}{4}\right)}\right] = \left[cot\left(\frac{x}{4}\right)\right]$

$y = \left[tan\left(\frac{\pi}{2} - \frac{x}{4}\right)\right]$

$y = \frac{\pi}{2} - \frac{x}{4} \quad \therefore \frac{dy}{dx} = \frac{-1}{4}$

Answer : Option : (a) $\frac{-1}{4}$

Q-36 : $\int_{-\pi}^{\pi} \frac{2x}{1+cos^2x}\, dx =$

(a) $-\pi$ (b) π **(c) 0** (d) 1

Solution : XII : Definite Integral

$f(x) = \frac{2x}{1+cos^2x}$; $f(-x) = \frac{2(-x)}{1+cos^2(-x)} = \frac{-2x}{1+cos^2x}$

$f(x) =- f(x)$

$\therefore f$ is odd

Answer : Option : (c) 0

Q-37 : If the population grows at the rate of 8% per year, then the time taken for the population to be doubled is (Given : log2 = 0.6912)

(a) 4.3 years (b) 10.27 years **(c) 8.64 years** (d) 6.8 years

Solution : XII : Applications of Differential Equation

$$\frac{dP}{dt} = \frac{8}{100} P$$

$$\frac{dP}{P} = \frac{8}{100} dt$$ ....(V. S . form)

$$\int \frac{dP}{P} = \frac{8}{100} \int dt$$

$$logP = \frac{2}{25} t + c$$

When t = 0 ; P = x

$$logP = \frac{2}{25} t + c$$

$$logx = \frac{2}{25} (0) + c$$

$$logx = c$$ ....(1)

When P = $2x$ t = ?

$$logP = \frac{2}{25} t + c$$

$$log2x = \frac{2}{25} t + logx$$ ......from (1)

$$log2x - logx = \frac{2}{25} t$$

$$log\left(\frac{2x}{x}\right) = \frac{2}{25} t$$

$$t = \frac{25}{2} \log log\ 2 = \frac{25}{2} (0.6912) = 12.5(0.6912) = 8.64$$

Answer : Option (c) 8.64 years

Q-38 : If $\frac{1-tan\,\theta}{1+\tan tan\,\theta} = \frac{1}{\sqrt{3}}$, where $\theta \in \left(0\,,\ \frac{\pi}{2}\right)$ then θ =

(a) $\frac{\pi}{4}$ **(b)** $\frac{\pi}{12}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{3}$

Solution : XI : Trigonometry –I

Let $\frac{1-tan\theta}{1+tan\theta} = \frac{1}{\sqrt{3}}$

$$\sqrt{3} - \sqrt{3}\,tan\theta = 1 + tan\theta$$

$$\sqrt{3} - 1 = tan\theta + \sqrt{3}\,tan\theta$$

$$\sqrt{3} - 1 = \left(\sqrt{3} + 1\right)tan\theta$$

$$tan\theta = \frac{\sqrt{3}-1}{\sqrt{3}+1}$$

$$tan\theta = \frac{\sqrt{3}\left(1-\frac{1}{\sqrt{3}}\right)}{\sqrt{3}\left(1+\frac{1}{\sqrt{3}}\right)}$$

$$tan\theta = \frac{\left(1-\frac{1}{\sqrt{3}}\right)}{\left(1+\frac{1}{\sqrt{3}}\right)}$$

$$tan\theta = \frac{1-tan\frac{\pi}{6}}{1+\left(1\times tan\frac{\pi}{6}\right)}$$

$$tan\theta = \frac{tan\frac{\pi}{4}-tan\frac{\pi}{6}}{1+\left(tan\frac{\pi}{4}\times tan\frac{\pi}{6}\right)}$$

$$tan\theta = \tan tan\left(\frac{\pi}{4}-\frac{\pi}{6}\right)$$

$$tan\theta = tan\left(\frac{6\pi-4\pi}{24}\right)$$

$$tan\theta = tan\left(\frac{2\pi}{24}\right)$$

$$tan\theta = tan\left(\frac{\pi}{12}\right)$$

$$\theta = \frac{\pi}{12}$$

Answer : Option (b) $\frac{\pi}{12}$