

Pair off everyone in the classroom; if there's a student left over, they become the helper up front. In each pair, one person is given a 4-sided die, the other is given a 6-sided die. We're going to play a game, where each person rolls their die, and the person with the higher number wins

Question: who will win?

If they answer "probably/likely the 6-sided die", ask why they gave their answer; if they simply say "the 6-sided die", ask whether the 6-sided die will always win, and then ask why once they understand that the result isn't certain.

Question: now that we know the 6-sided die is more likely to win, how much more likely?

Look for suggestions; if none are forthcoming, first let's all roll a few times and record how many wins go to the 4-sided die and how many go to the 6-sided die. These observations allow us to estimate the probability that the 6-sided die will win on any given roll; we do this by taking the number of 6-sided wins and dividing by the total number of rolls. Notice that this is a fraction!

However, this is a simple enough case that we can *exactly determine* the probability the 6-sided die wins. Look for suggestions on how to do this; if none are forthcoming, let's enumerate all possible outcomes. One possible outcome is that [roll the dice and write this up on the board]. What are the other possible outcomes? Write down all the suggestions in an organized way until all 24 possible outcomes are enumerated.

Questions:

1. **How many possible outcomes did we find?** [24]
2. **In how many of these outcomes does the 6-sided die win?** [14]
3. **What then is the probability the 6-sided die wins?** [14/24]
4. **What then is the probability the 4-sided die wins?**

Note that they may accidentally include ties in their initial determination of the probability; there are 4 ties, and so the probability is 6/24.

5. **Why are there 4 ties? Do you see a relationship between the number of ties and the number of sides?**

The number of sides on the *smaller* die is the number of ties.

Question: what about a 6-sided die versus an 8-sided die?

Again, we can enumerate all the possible outcomes, but it will be a lot.

Question: how many possible outcomes do you think there will be?

If no suggestions are forthcoming, ask if they notice a relationship between the number of sides on the dice and the number of outcomes in the 4-sided versus 6-sided situation. If they don't see it, point out it's the product of the number of sides. Hopefully they should now get $6 \times 8 = 48$.

48 outcomes is a lot to write out. Is there a simpler way to do it?

One way is to write a program to brute force the answer (show with the computer projector).

Another way is to notice a pattern in how many wins go to the smaller die.