A IAL PURE MATHS 3 PAST PAPERS

Chapter 8 [MS]

www.doteducation.org
+971525465652

1. JAN 2025[1]

Question Number	Scheme	Marks
	$f(x) = 2\sec x + 6x - 3$	
1. (a)	$f(0.1) = -0.39$ $\quad f(0.2) = 0.24$	M1
	States change of sign, continuous and hence root	A1
		(2)
(b)	Sets $f(x) = 0$, uses $\sec x = \frac{1}{\cos x}$ and makes x of $6x$ the subject	
	$\Rightarrow 6x = 3 - \frac{2}{\cos x} \Rightarrow x = \frac{1}{2} - \frac{1}{3\cos x}$ *	B1*
		(1)
(c)	(i) $x_2 = \frac{1}{2} - \frac{1}{3\cos 0.15} = 0.16..$	M1
	$x_2 =$ awrt 0.1629	A1
	(ii) $\alpha = 0.1622$	A1
		(3)
		(6 marks)

Note: If the student uses degrees mode they will get values -0.39999 and 0.200001 in (a) and 0.1666… in (c). An answer in (c) of 0.16666… would be an indicator of the mode used for part (a) if rounded values are given see notes below.

(a)

M1: Attempts the value of $f(x)$ at 0.1 and 0.2 with one **correct** to at least 1 sf rounded or truncated. Note that degrees mode gives -0.39999 and 0.200001 and these can be accepted for the M mark.

A1: Both values correct to at least 1sf (but see warning), rounded or truncated with reason (Sign change **and** continuous function) and minimal conclusion (root). For "sign change" accept $f(0.1) < 0$, $f(0.2) > 0$ shown, or $f(0.1)f(0.2) < 0$

Warning: Degrees mode answers should score A0, so if the values -0.39999 and 0.200001 are used score A0. However, if just $f(0.1) = -0.4$ and $f(0.2) = 0.2$ are used with no contrary evidence – see note above – the A1 will be scored if reason and conclusion are given.

Note: Use of a narrower interval is possible. You may need to check values, but score as per the main scheme with their end points as long as their interval contains the root.

(b)

B1*: Shows all necessary steps to show given result. Allow if α is used in place of x.

Sets $f(x) = 0$, uses $\sec x = \frac{1}{\cos x}$ and makes the x of the $6x$ the subject. The "=0" must be seen at some stage.

(c) (i)

M1: Uses iterative formula and $x_1 = 0.15$ to find $x_2 = 0.16..$ There must be some evidence of use of the iterative formula, either substitution seen or awrt 0.16 **other than** just 0.1622(315…), which may be from a calculator solve. If all that is seen is 0.1622(315…) with no other evidence then M0A0A0 will be scored.

A1: $(x_2 =)$awrt 0.1629 Allow if the x_2 is omitted, so just awrt 0.1629 can score M1A1

(c)(ii) A1: $(\alpha =)\,0.1622$ following the award of the M mark (see note).

An answer of 0.1666… may score the first M in (c) as long as there is evidence of the iterative formula used but the value only will be M0 (calculator solve gives this value).

2. OCT 2024 [8]

Question Number	Scheme	Marks
8 (a)	52 b.p.m.	B1
		(1)
(b)	32 b.p.m.	B1
		(1)
(c)	$\frac{dH}{dt} = -8e^{-0.2t} + 18e^{-0.9t}$	M1, A1
	Sets $-8e^{-0.2t} + 18e^{-0.9t} = 0 \Rightarrow 4e^{0.7t} = 9$	dM1
	$\Rightarrow 0.7t = \ln\frac{9}{4} \Rightarrow t = ...$	M1
	$T = 1.158$ (minutes)	A1
		(5)
(d)	$37 = 32 + 40e^{-0.2t} - 20e^{-0.9t} \Rightarrow e^{-0.2t} = \frac{1+4e^{-0.9t}}{8}$	M1
	$\Rightarrow e^{0.2t} = \frac{8}{1+4e^{-0.9t}} \Rightarrow t = 5\ln\left(\frac{8}{1+4e^{-0.9t}}\right)$	A1*
		(2)
(e)	$t_2 = 5\ln\left(\frac{8}{1+4e^{-0.9\times10}}\right) = ...$	M1
	$(t_2) =$ awrt 10.3947	A1
	(M)= 10.3955	A 1
		(3)
		Total 12

(a)
B1: 52 b.p.m. Units are not required. Check for answer given in the body of the question
(b)
B1: 32 b.p.m. . Units are not required. Check for answer given in the body of the question

(c)

M1: Achieves $\frac{dH}{dt} = P\,e^{-0.2t} + Qe^{-0.9t}$ where P and Q are constants

A1: $\frac{dH}{dt} = -8\,e^{-0.2t} + 18\,e^{-0.9t}$ but allow un-simplified versions

dM1: Sets $P\,e^{-0.2t} + Q\,e^{-0.9t} = 0 \Rightarrow me^{0.7t} = n$ or $me^{-0.7t} = n$ where $m \times n > 0$

M1: Solves an equation of the form $me^{\pm kt} = n$ where $m \times n > 0$ using correct ln work.
It must proceed from a correct form of the derivative (so the first M1).
Look for $me^{\pm kt} = n \Rightarrow e^{\pm kt} = \frac{n}{m} \Rightarrow \pm kt = \ln\left(\frac{n}{m}\right) \Rightarrow t = ...$

A1: $T =$ awrt 1.158 minutes following the award of all previous marks. You can ignore units

....

An alternative for the last three marks in part (c) are;

dM1: $Pe^{-0.9t} = Qe^{-0.2t} \Rightarrow \ln P - 0.9t = \ln Q - 0.2t$

M1: Collects terms in t and proceeds to $t =$… Under this method it is dependent upon both previous M's

A1: $T =$ awrt 1.158 minutes following the award of all previous marks. Units are not required

....

There may be answers produced from a calculator. In cases such as this we are going to apply the rule 'answers without full working may not gain credit.'

$\frac{dH}{dt} = -8\,e^{-0.2t} + 18\,e^{-0.9t} = 0 \Rightarrow t = 1.158$ scores 1,1,0,0,0

....

(d)

M1: Sets $H = 37$ and proceeds to make $e^{\pm 0.2t}$ the subject

A1*: Proceeds to the given answer showing all necessary steps.

Note that this is a given answer so the steps must be logical without huge jumps

Acceptable proofs:

Example 1

$$37 = 32 + 40\,e^{-0.2t} - 20\,e^{-0.9t} \Rightarrow e^{-0.2t} = \frac{1+4\,e^{-0.9t}}{8} \Rightarrow -0.2t = \ln\left(\frac{1+4\,e^{-0.9t}}{8}\right)$$

$$\Rightarrow t = -5\ln\left(\frac{1+4\,e^{-0.9t}}{8}\right)$$

$$\Rightarrow t = 5\ln\left(\frac{8}{1+4\,e^{-0.9t}}\right)$$

Example 2

$$37 = 32 + 40\,e^{-0.2t} - 20\,e^{-0.9t} \Rightarrow e^{-0.2t} = \frac{1+4\,e^{-0.9t}}{8} \Rightarrow e^{0.2t} = \frac{8}{1+4\,e^{-0.9t}} \Rightarrow 0.2t = \ln\left(\frac{8}{1+4\,e^{-0.9t}}\right) \Rightarrow t = 5\ln\left(\frac{8}{1+4\,e^{-0.9t}}\right)$$

Unacceptable proof:

$$37 = 32 + 40\,e^{-0.2t} - 20\,e^{-0.9t} \Rightarrow e^{-0.2t} = \frac{1+4\,e^{-0.9t}}{8} \Rightarrow -0.2t = \ln\left(\frac{1+4\,e^{-0.9t}}{8}\right)$$

$$\Rightarrow t = 5\ln\left(\frac{8}{1+4\,e^{-0.9t}}\right)$$

They need to deal with 0.2 and $-$sign in separate steps

……

Alt (d) lns can be taken earlier. The M1 is scored for making 0.2 t the subject. See below

$$37 = 32 + 40\,e^{-0.2t} - 20\,e^{-0.9t}$$

$$\Rightarrow 40e^{-0.2t} = 5 + 20\,e^{-0.9t} \Rightarrow \ln 40 - 0.2t = \ln\left(5 + 20\,e^{-0.9t}\right) \Rightarrow 0.2t = \ln 40 - \ln\left(5 + 20\,e^{-0.9t}\right)$$

We would need to see three further steps. E.g.

$$0.2t = \ln\left(\frac{40}{5+20\,e^{-0.9t}}\right) \Rightarrow t = 5\ln\left(\frac{40}{5+20\,e^{-0.9t}}\right) \Rightarrow t = 5\ln\left(\frac{8}{1+4\,e^{-0.9t}}\right)$$

………..

(e)

M1: Uses the iteration formula once. Allow for the embedded 10 leading to a value or sight of t_2 = awrt 10.4

A1: (t_2) = awrt 10.3947

A1: (M) = 10.3955 following some evidence of previous iterations, e.g. sight of
$t_2 = 5\ln\left(\dfrac{8}{1+4e^{-0.9\times 10}}\right) = \text{awrt } 10.4....$ This is NOT awrt so just 10.3955 here

3. JAN 2024 [2]

Question Number	Scheme	Marks
2(a)	$(f(2)=)2^4-5(2)^2+4(2)-7=-3<0$ **and** $(f(3)=)3^4-5(3)^2+4(3)-7=41>0$	M1
	There is a change of sign and $f(x)$ is continuous so there is a root (in the interval) *	A1*
		(2)
(b)	$x^3=\dfrac{5x^2-4x+7}{x}\Rightarrow x=\sqrt[3]{\dfrac{5x^2-4x+7}{x}}$ *	B1*
		(1)
(c)(i)	$x_2=\sqrt[3]{\dfrac{5(2)^2-4(2)+7}{2}}$	M1
	$=\text{awrt } 2.1179$	A1
(ii)	$\alpha=\text{awrt } 2.1565$	A1
		(3)
		(6 marks)

(a)
M1: Attempts both $f(2)$ and $f(3)$ or a narrower interval that contains the root 2.1565…and considers the signs. Note showing $f(2)\times f(3)<0$ or "< 0", "> 0" beside each appropriate root is a consideration of signs and is also sufficient for the "sign change" part of reasoning for the A1. For the substitution, need to see the values substituted or at least one of f(2) or f(3) correct.
A1*: This mark requires:

- both $f(2)$ and $f(3)$ correct e.g. $f(2)=-3$ and $f(3)=41$
- a reference to the sign change
- a reference to continuity
- a (minimal) conclusion

For **sign change**, allow equivalent statements e.g. $f(2).f(3)<0$, $f(2)<0<f(3)$ etc.
For **continuity**, allow just "continuous" and allow "assuming it is continuous" and "continuous equation" but do **not** allow incorrect statements such as "x is continuous", "the interval is continuous", "there is a change of sign therefore it is continuous"
For the **conclusion**, allow e.g. ✓, #, QED, hence shown, hence root, so it crosses the x-axis, etc., but **not** incorrect statements e.g. so there is a root in the interval $[-3,41]$

You may have to use your judgement to decide if the A1 conditions are satisfied.
Example: f(2) = –3, f(3) = 41, there is a change of sign hence f(x) has a continuous root in [2, 3]
Scores M1A0 – i.e. it is not the root that is continuous

(b)

B1*: Proceeds to given answer with no errors and with $x^3 = ...$ seen at some stage.

Condone omission of the fraction bar e.g. $x = \sqrt[3]{\begin{matrix}5x^2 - 4x + 7 \\ x\end{matrix}}$ as long as there are no algebraic errors. Be tolerant if the radical does not fully encompass the expression but score B0 if the expression is clearly incorrect e.g. $x = \frac{\sqrt[3]{5x^2 - 4x + 7}}{x}$ i.e where the cube root clearly sits on top of the fraction bar.

Condone working backwards as long as there is a (minimal) conclusion e.g.

$$x = \sqrt[3]{\frac{5x^2 - 4x + 7}{x}} \Rightarrow x^3 = \frac{5x^2 - 4x + 7}{x} \Rightarrow x^4 - 5x^2 + 4x - 7 = 0$$ ✓, #, QED, hence shown, etc.

(c)(i)

M1: Attempts to find x_2 using the given iteration formula. Allow for sight of $\sqrt[3]{\frac{5(2)^2 - 4(2) + 7}{2}}$

Must see the correct index and not $\sqrt{\frac{5(2)^2 - 4(2) + 7}{2}}$ unless the "3" is implied by their value(s).

May be implied by awrt 2.118 or awrt 2.147 $(= x_3)$

A1: awrt 2.1179 (apply isw if necessary)

(ii)

A1: awrt 2.1565 (provided M1 scored in (c)(i)) (apply isw if necessary)

4. JAN 2024 [7]

7(a)	$\left(\frac{dy}{dx}=\right)-\frac{16}{3}(3x-k)^{-2}$	M1A1
		(2)
(b)	$-\frac{16}{3}(3x-k)^{-2}=-12\Rightarrow(3x-k)^2=...$	M1
	$3-k=\text{"}\pm\frac{2}{3}\text{"}\Rightarrow k=...$	**d**M1
	$k=\frac{7}{3},\ \frac{11}{3}$	A1
		(3)
(c)	$y=\frac{16}{9\left(3(1)-\frac{7}{3}\right)}$	M1 (B1 on ePEN)
	$y-\text{"}\frac{8}{3}\text{"}=\frac{1}{12}(x-1)$	**d**M1
	$12y-x-31=0$	A1
		(3)
(d)	$\int\frac{16}{9(3x-k)}dx=\frac{16}{27}[\ln(3x-k)]$	M1
	$=\frac{16}{27}\left[\ln\left(3x-\frac{7}{3}\right)\right]$	A1ft
	$\frac{16}{27}\left[\ln\left(3x-\frac{7}{3}\right)\right]_1^3=\frac{16}{27}\left(\ln\left(3(3)-\frac{7}{3}\right)-\ln\left(3(1)-\frac{7}{3}\right)\right)$	**d**M1
	$=\frac{16}{27}\ln(10)$	A1
		(4)
		(12 marks)

(a)

M1: Attempts to differentiate to the form $A(3x-k)^{-2}$ oe e.g. $A(27x-9k)^{-2}$

A1: $\left(\frac{dy}{dx}=\right)-\frac{16}{3}(3x-k)^{-2}$ oe e.g. $-\frac{16}{3(3x-k)^2}$ or $-\frac{16}{3(9x^2-6kx+k^2)}$ or $-\frac{16}{27x^2-18kx+3k^2}$ but **not** $-432(27x-9k)^{-2}$ or $-\frac{432}{(27x-9k)^2}$ as there is a common factor in the numerator and denominator.

(b)

M1: Sets their derivative of the form $A(3x-k)^{-2}$ (or equivalent) equal to -12 (not 12) and rearranges to $...(3x-k)^2 = ...$ or equivalent e.g. $...(27x-9k)^2 = ...$ or $...(9x^2-6kx+k^2) = ...$

Condone poor squaring e.g. allow $...(9x^2+k^2) = ...$

May have already substituted $x = 1$

dM1: **<u>Depends on having obtained $A < 0$</u> (otherwise the equation has no real solutions):**

Substitutes $x = 1$ and solves to find 2 values for k.

If the $(3x-k)^2$ is expanded then the usual rules apply for solving a 3TQ and allow using a calculator. FYI correct 3TQ is $9k^2 - 54k + 77 = 0$

Depends on the previous method mark.

A1: Achieves $k = \frac{7}{3}$ and $k = \frac{11}{3}$ from a correct method. Accept equivalent exact fractions or recurring decimals $2.\dot{3}$, $3.\dot{6}$ but not rounded decimals e.g. 2.33, 3.67

(c) Note we are scoring the first mark as an M mark not a B mark.

M1(B1 on ePEN): Uses a value of k from part (b) (or a 'made up' k) and $x = 1$ to find the value of y at P.

dM1: Attempts to find the equation of the normal using their y-coordinate with the gradient $\frac{1}{12}$.

If they use $y = mx + c$ they must proceed as far as $c = ...$

Depends on the previous method mark.

A1: $12y - x - 31 = 0$ or any equivalent integer multiple of this equation.

Note if $k = \frac{11}{3}$ is used they should get $x - 12y - 33 = 0$ and would generally score 110

(d)

M1: Integrates to the form $B\ln(3x-k)$ oe e.g. $B\ln\alpha(3x-k)$

A1ft: $\frac{16}{27}\left[\ln\left(3x-\frac{7}{3}\right)\right]$ which may be unsimplified and isw once correct integration is seen.

Follow through their k and allow the letter k and allow if their k is not less than 3.
Ignore any reference to $+c$.
You may need to check their integration carefully.
E.g. $\frac{16}{27}\left[\ln(27x-21)\right]$ is also correct (for $k=\frac{7}{3}$)
Ignore any spurious integral signs after a correct integral is seen.

dM1: Substitutes in the limits 3 and 1 and subtracts either way round. Must have a numeric k now.
It is dependent on the first method mark.

A1: $\frac{16}{27}\ln(10)$ or exact equivalent e.g. $\frac{32}{54}\ln(10)$

Use of an incorrect *k* in (d) scores a maximum of M1A1ft dM1A0

Note that in part (d), some candidates may use substitution e.g.

$$u=3x-k\Rightarrow\frac{du}{dx}=3\Rightarrow\int\frac{16}{9(3x-k)}dx=\frac{16}{9}\int\frac{1}{u}\frac{1}{3}du=\frac{16}{27}\ln u$$

Score M1 for integrating to the correct form e.g. $k\ln u$

and A1 for $\frac{16}{27}\ln u$ **following through their *k* or the letter *k* as above**

then dM1 for

$$\left[\frac{16}{27}\ln u\right]_{\frac{2}{3}}^{\frac{20}{3}} = \frac{16}{27}\ln\frac{20}{3} - \frac{16}{27}\ln\frac{2}{3} \text{ or } \left[\frac{16}{27}\ln\left(3x-\frac{7}{3}\right)\right]_1^3 = \frac{16}{27}\ln\frac{20}{3} - \frac{16}{27}\ln\frac{2}{3}$$

i.e. applies the correct changed limits or reverts to *x* and uses 3 and 1

$$= \frac{16}{27}\ln(10) \quad \textbf{A1}$$

Note if $k = \frac{11}{3}$ is used they should get $= \frac{16}{27}\ln(8)$ and would generally score 1110

Note:

If you see any responses where the denominator in part (a) is expanded incorrectly e.g.

$$\frac{16}{9(3x-k)} = \frac{16}{27x-k}$$

and candidates persist with this incorrect expansion then send to review.

A typical response with an expanded denominator:

Question Number	Scheme	Marks
7(a)	$y=\frac{16}{9(3x-k)}=\frac{16}{27x-9k}\Rightarrow\frac{dy}{dx}=-\frac{432}{(27x-9k)^2}$	M1A0
(b)	$-\frac{432}{(27x-9k)^2}=-12\Rightarrow 12(27x-9k)^2=432$	M1
	$12(27x-9k)^2=432\Rightarrow(27x-9k)^2=36\Rightarrow 27-9k=\pm 6\Rightarrow k=...$	**d**M1
	$k=\frac{7}{3},\frac{11}{3}$	A1
(c)	$y=\frac{16}{27-21}$	M1 (B1 on ePEN)
	$y-"\frac{8}{3}"=\frac{1}{12}(x-1)$	**d**M1
	$12y-x-31=0$	A1
(d)	$\int\frac{16}{27x-9k}\,dx=\frac{16}{27}[\ln(27x-9k)]$	M1
	$=\frac{16}{27}[\ln(27x-21)]$	A1ft
	$\frac{16}{27}[\ln(27x-21)]_1^3=\frac{16}{27}(\ln(27(3)-21)-\ln(27-21))$	**d**M1
	$=\frac{16}{27}\ln(10)$	A1

5. JUNE 2024 [2]

2	$g(x)=\dfrac{2x^2-5x+8}{x-2}$	
(a)	Sets $2x^2-5x+8=(Ax+B)(x-2)+C$ with an attempt at one constant	M1
	Attempts all 3 constants. E.g. Sets $x=2\Rightarrow C=...,-2B+C=8\Rightarrow B=...$ and compares x^2 terms gives $A=...$	dM1
	$2x-1+\dfrac{6}{x-2}$	A1
		(3)
(b)	$\int 2x-1+\dfrac{6}{x-2}\,dx=x^2-x+6\ln(x-2)$	M1 A1ft
	$\int_4^8 2x-1+\dfrac{6}{x-2}\,dx=\left[x^2-x+6\ln(x-2)\right]_4^8=56+6\ln 6-12-6\ln 2$	
	$=44+6\ln 3$	dM1 A1
		(4)
		(7 marks)

ANSWERS IN (a) of $A=2,B=-5,C=2$, and/or $28+2\ln 3$ in (b) please send to review

(a) **Note that correct values for *A*, *B*, and *C* imply the first two marks**

M1: Attempts to find one of the constants via

- an identity e.g. $2x^2-5x+8=(Ax+B)(x-2)+C$

  Note that if an identity is used it must be correct. Condone arithmetical slips when evaluating.
  The identity may be implied by correct equations
  e.g. $x^2: A=2,\ x: -2A+B=-5,\ \text{constant}: -2B+C=8$
- division is used look for a quotient as far as $2x$

$$x-2\overline{\smash{)}2x^2-5x+8}\quad \text{quotient } 2x-1$$

....

6

Note that the division may appear in many different formats or may be by inspection so $2x$ is sufficient to score this mark.

e.g. $\dfrac{2x^2-5x+8}{x-2}=\dfrac{2x(x-2)+4x-5x+8}{x-2}=2x+\dfrac{-(x-2)-2+8}{x-2}=2x-1+\dfrac{6}{x-2}$

e.g.

	$2x$	-1	
x	$2x^2$	$-x$	6
-2	$-4x$	2	

www.doteducation.org
+971525465652

dM1: Attempts

to find all 3 constants which are all non-zero. It is dependent upon the previous mark. Via an identity: If three simultaneous equations are formed allow this mark to be scored for proceeding to values for A, B and C. You do not need to check this. Allow sign slips only in forming the equations. Via division: Look for a quotient of $2x \pm c$ and a constant remainder

A1: $2x-1+\dfrac{6}{x-2}$ The expression must be written; it cannot be awarded for just stating the correct values of A, B and C, unless it is later written correctly as an expression in (b) (may be seen within an integral)

(b)

M1: Attempts to integrate the term $\dfrac{C}{x-2}$ proceeding to $D\ln|x-2|$ C does not have to be equal D (do not be concerned with brackets or modulus sign)

A1ft: Correct integration of $g(x)$ for their A, B and C in part (a).

They must be integrating an expression of the form $Ax+B+\dfrac{C}{x-2} \Rightarrow \dfrac{Ax^2}{2}+Bx+C\ln|x-2|$ o.e (where $A,B,C \neq 0$) which may be unsimplified with or without a constant of integration. Do not be concerned by the absence of the bracket or modulus on the $x-2$ (as both values substituted in will give positive lns)

dM1: Attempts to proceed to the form $p+q\ln r$ by

- using the limits 8 and 4 in an expression of the form $Ex^2+Fx+D\ln|x-2|$ o.e. (where $D,E,F \neq 0$) and subtracting either way round
- combining their ln terms **correctly**. e.g. this mark cannot be scored for $\ln 6-\ln 2=\dfrac{\ln 6}{\ln 2}=\ln 3$
  but treat as a sign slip in front of the log rather than incorrect log work e.g. $6\ln 6-6\ln 2=-6\ln 3$

Condone slips when evaluating or multiplying out brackets only (condone a spurious $+c$ present for this mark). The substitution and combining of their ln terms can be implied by their $p+q\ln r$ provided no incorrect log work is seen. Note that $\ln r$ does not have to be ln 3 e.g. it may be ln 729.

A1: $44+6\ln 3$ Withhold the final mark if there is an integral and dx around the answer or $+c$

Alt (b) Attempts via substitution

Note there may be attempts using substitution e.g. $u = x - 2$ on **some or all** of their expression.

e.g. $\int_4^8 \frac{6}{x-2}\,dx \rightarrow \int_2^6 \frac{6}{u}\,du = [6\ln u]_2^6$

M1: Correct attempt to integrate $\frac{C}{u}$ to $D\ln|u|$ C does not have to be equal D

A1ft: $A(u+2)+B+\frac{C}{u} \Rightarrow \frac{Au^2}{2}+(2A+B)x+C\ln|u|$ or condone $\frac{Ax^2}{2}+Bx+C\ln|u|$

dM1: Attempts to either substitute in the limits 6 and 2, subtracts either way round and combines their ln terms correctly, or substitutes back to an expression in terms of x and proceeds as in the main scheme and notes. If they are working in a mixture of variables, $x^2 - x + 6\ln u$ then the correct limits must be substituted into the appropriate terms.

A1: $44 + 6\ln 3$ Withhold the final mark if there is an integral and dx around the answer or $+\,c$

6. JUNE 2023 [1]

Question Number	Scheme	Marks
1. (a)	$g(3) = -265, \quad g(4) = 3104$	M1
	States change of sign, continuous and hence root in $[3, 4]$	A1
		(2)
(b)	$x_2 = \sqrt[6]{1000 - 2 \times 3} \quad = 3.1591$	M1 A1
	$(\alpha =) 3.1589$	A1
		(3)
		(5 marks)

Notes	
(a)	
M1	Attempts the value of g at 3 and 4 with one correct (accept any value for the other as an attempt). Note narrower ranges are possible but must contain the root and lies in [3,4].
A1	Both values correct with reason (Sign change (stated or indicated) and continuous function) and minimal conclusion (root)
(b)	
M1	Attempts to substitute $x_1 = 3$ into the formula. Implied by sight of expression, awrt 3.159
A1	awrt 3.1591
A1	$(\alpha =) 3.1589$ cao - must be to 4 d.p. Do not be concerned about the labelling of the root (x or α etc), mark the final answer of (b)(ii). (Note sight of this value implies the M1 even if x_2 is not seen).

7. OCT 2023 [1]

Question Number	Scheme	Marks
1(a)	$(\mathrm{f}(1)=)1-5+\mathrm{e}=-1.281...<0$ $(\mathrm{f}(2)=)2^2-5\times2+\mathrm{e}^2=1.389...>0$	M1
	As there is a change of sign and $\mathrm{f}(x)$ is continuous over the interval $[1, 2]$ then there is a root *	A1*
		(2)
(b)(i)	$x_2=\sqrt{5\times1-\mathrm{e}^1}=\text{awrt}\,1.5105$	M1A1
(ii)	$\alpha=\text{awrt } 1.7340$	A1
		(3)
		(5 marks)

Notes
(a) M1: Attempts $\mathrm{f}(1)$ and $\mathrm{f}(2)$ with substitution seen or at least one correct to 1 d.p. rounded or truncated **and considers their signs**. Note showing f(1)f(2) < 0 is a consideration of signs and is sufficient for the "sign change" part of reasoning for the A1. A1*: Must have • both $\mathrm{f}(1)$ and $\mathrm{f}(2)$ correct (as expressions or correct values imply these, need not be labelled) • reason, which must mention continuity and state or indicate sign change in some way • conclusion, "hence root", or accept e.g. hence "f(x)=0 between x =1 and x = 2" **(b)** **(i)** M1: Attempts to find x_2 using the iteration formula. Implied by sight of 1 embedded in the formula or awrt 1.5 A1: awrt 1.5105 **(ii)** A1: awrt 1.7340 provided M1 has been scored. Allow 1.734 with trailing zero omitted. Note 1.7340 only may be from a graphical calculator, so scores M0A0A0. There must be evidence (i.e. the M is scored) of an attempt at least one iteration first.

8. OCT 2023 [7]

Question Number	Scheme	Marks
7(a)	$f(0)=(0-3)^2=9$	M1
	$0 \leqslant f(x) \leqslant 9$	A1
		(2)
(b)	$f'(x)=-2xe^{-x^2}(2x^2-3)^2+e^{-x^2}\times 8x(2x^2-3)$	M1A1
	$=2x(2x^2-3)e^{-x^2}\left(-(2x^2-3)+4\right)=2xe^{-x^2}(2x^2-3)\left(7-2x^2\right)$	dM1A1
		(4)
(c)	$x^2=\frac{3}{2},\frac{7}{2}\Rightarrow f\left(\sqrt{\frac{7}{2}}\right)=e^{-\frac{7}{2}}\left(2\times\frac{7}{2}-3\right)^2=16e^{-\frac{7}{2}}$	M1A1
	$16e^{-\frac{7}{2}}<k<9$	dM1A1
		(4)
		(10 marks)

Notes

(a) Work for part (a) must be seen in part (a) not recovered in part (c).

M1: Substitutes $x=0$ and proceeds to find a value for y. Implied by sight of 9.

A1: $0 \leqslant f(x) \leqslant 9$ Accept with y or just f, but not with x. Accept interval, [0, 9], or set notation.

(b)

M1: Attempts the product rule and chain rule achieving $\pm Pxe^{-x^2}(2x^2\pm 3)^2+e^{-x^2}\times Qx(2x^2\pm 3)$ with $P, Q>0$. Alternatively, attempts the quotient rule on $\frac{(2x^2-3)^2}{e^{x^2}}$ achieving $\frac{Px(2x^2\pm 3)e^{x^2}-Qxe^{x^2}(2x^2\pm 3)^2}{(e^{x^2})^2}$ $P, Q>0$

A1: $-2xe^{-x^2}(2x^2-3)^2+e^{-x^2}\times 8x(2x^2-3)$ oe (need not be simplified)

dM1: Must have scored previous M. Attempts to takes out a factor of $2xe^{-x^2}(2x^2-3)$ to obtain a factor $\left(\pm C\pm Dx^2\right)$ which may be unsimplified. Allow if e.g. the x or e^{-x^2} is dropped when taking out the factor.

A1: Achieves $2xe^{-x^2}(2x^2-3)(7-2x^2)$ with no errors seen.

(c) **Note : allow all the marks in (c) from answers which were a constant multiple ... missing the factor *x*, which lead to the correct answers here.**

M1: Attempts to find a *y* value (which may be zero) for at least one of the valid non-zero roots for f'(x)=0. Allow for an attempt at either $f\left(\pm\sqrt{\frac{3}{2}}\right)$ or $f\left(\pm\sqrt{\frac{A}{B}}\right)$ leading to a value, where A and B are their values from (b) with $AB > 0$. Note e.g. $f\left(\frac{A}{B}\right)$ attempted is M0.

A1: For obtaining $16e^{-\frac{7}{2}}$ Accept awrt 0.483 for this mark following the award of M.

dM1: Attempts the inside region, allowing $\leqslant$, using their *y* intercept from (a) and their **positive** *y* value, less than their intercept, from an attempt at the at the *y* value of the maxima. Allow a positive *y* value from an attempt at any non-zero root of f'(x) as such an attempt. It is dependent on the previous method mark.

A1: $16e^{-\frac{7}{2}} < k < 9$ or in any equivalent form e.g. interval notation $k \in \left(\frac{16}{e^{7/2}}, 9\right)$ Allow *y* instead of *k*

Ignore references to "$k = 0$"

9. JAN 2022 [5]

5 (a)	Attempts to find y at -1.25 and -1.2 with one correct to 1sf	M1
	Achieves $y(-1.25) = -0.9$ and $y(-1.2) = 0.2$	
	With reason (change of sign and continuous) and Conclusion	A1
		(2)
(b)	(i) Attempts $\sqrt{12\ln(15)+8}$	M1
	$x_2 = \text{awrt } 6.3637$	A1
	(ii) $x = 6.4142$	B1
		(3)
(c)	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{12}{2x+3} - x$	M1 A1
	Stationary point when $\frac{12}{2x+3} = x \Rightarrow 2x^2 + 3x - 12 = 0 \Rightarrow x =$	dM1
	$\Rightarrow x = \frac{-3+\sqrt{105}}{4}$ or awrt 1.81 ONLY	A1
		(4)
		(9 marks)

(a)

M1: Attempts to find y at -1.25 and -1.2 with one correct to at least 1sf.

FYI $y(-1.25) = -0.9$ and $y(-1.2) = 0.2$

A1: Achieves $y(-1.25) = \text{awrt } -0.9$ and $y(-1.2) = \text{awrt } 0.2$ with a reason and minimal conclusion.

Acceptable reasons are; " sign change and continuous", " $y(-1.25) \times y(-1.2) < 0$ and continuous" Minimal conclusions are; "hence proven", "hence root", "✓", "☐"

Note: A smaller interval could be chosen but it must span -1.20998 and the final conclusion must refer to the given interval to score both marks.

(b)(i)

M1: Attempts to apply the iteration formula once. Accept $\sqrt{12\ln(15)+8} = \ldots$ or awrt 6.4

A1: $x_2 = \text{awrt } 6.3637$

(b)(ii)

B1: CAO $x = 6.4142$ There must be some evidence of the M or continued iteration for this to be awarded. A minimum would be an attempt at x_2 (the M) or an attempt at any intermediate term.

(c)

M1: Attempts to differentiate with $\ln(2x+3) \to \frac{\cdots}{2x+3}$

A1: $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{12}{2x+3} - x$ which may be left unsimplified . No requirement to see lhs

dM1: Sets their $\frac{\cdots}{2x+3} \pm \ldots x = 0$ and proceeds to a value for x via a correct method of solving a 3TQ
There must be some evidence of working but allow candidates to use a calculator to write down the solution of 3TQ. If they do, it must be correct for their 3TQ

A1: cso $x = \frac{-3+\sqrt{105}}{4}$ or awrt 1.81 ONLY. If $x = \frac{-3-\sqrt{105}}{4}$ or awrt -3.31 is also written down it must be rejected. ISW after a correct answer. There must be evidence of dM1 to award this mark

10. JUNE 2022 [8]

Question Number	Scheme	Marks
8(a)	$\frac{dv}{dt} = -e^{t-10} + 9e^{-0.75t} = 0 \Rightarrow 9 = e^{1.75t-10}$ or $9e^{-0.75t} = e^{t-10}$	M1A1
	$9 = e^{1.75t-10} \Rightarrow t = \frac{10+\ln 9}{1.75}$ or $\ln 9 - 0.75t = t - 10 \Rightarrow t = \frac{10+\ln 9}{1.75}$	M1
	$t =$ awrt 6.97	A1
	awrt 11.9 (ms^{-1})	A1
		(5)
(b)	$\int 12 - 12e^{-0.75t} - e^{(t-10)}\,dt = 12t - e^{(t-10)} + 16e^{-0.75t}\ (+C)$	M1A1
	$\left[12t - e^{(t-10)} + 16e^{-0.75t}\right]_0^T = 100 \Rightarrow 12T = ...$	M1
	$T = \frac{1}{12}\left(116 - 16e^{-0.75T} + e^{T-10} - e^{-10}\right)$ *	A1*
		(4)
(c) (i)	$T_2 = \frac{1}{12}\left(116 - 16e^{-0.75\times 10} + e^{10-10} - e^{-10}\right)$	M1
	$T_2 =$ awrt 9.7493	A1
(ii)	9.7293 (seconds)	A1
		(3)
		(12 marks)

(a) *** Be aware this can be solved directly on a calculator. Calculus must be seen***

M1 Differentiates to $\pm Ae^{t-10} \pm Be^{-0.75t}$ and **sets equal to 0** Must be seen in (a) Do not accept differentiating to eg $...te^{t-10}$. Condone an expression of $12 \pm Ae^{t-10} \pm Be^{-0.75t}$ only

A1 $9 = e^{1.75t-10}$ or equivalent eg $9e^{-0.75t} - e^{t-10} = 0$

M1 Proceeds from their equation of the form $Ae^{Ct\pm D} - Be^{Et\pm F} = 0$ or equivalent leading to a value or an expression (which may be unsimplified) for t. If log work is seen it must be correct.

Eg $\frac{40+4\ln 9}{7}$ or $\frac{10+\ln 9}{1.75}$ will score this mark

$9e^{-0.75t} - e^{t-10} = 0 \Rightarrow$ awrt 6.97 can also score this mark

This cannot be scored from an unsolvable equation.

A1 awrt 6.97 or awrt 6.96 including exact unsimplified expressions such as $\dfrac{40+4\ln 9}{7}$ or $\dfrac{\text{[illegible]}}{1.75}$.

A1 awrt 11.9 (ms^{-1}) with all previous marks scored. Condone lack of units

Note: awrt 6.97 and awrt 11.9 with no calculus scores 0 marks

(b)

M1 Attempts to integrate the expression for v. Award for $12t \pm A\text{e}^{(t-10)} \pm B\text{e}^{-0.75t}$.
Do not accept eg $12t \pm A\text{e}^{(t-10)+1} \pm B\text{e}^{-0.75t+1}$
Allow if their attempt to integrate appears in part (a) as long as it is a clear attempt at integration. Limits may have been substituted in.

A1 $12t - \text{e}^{(t-10)} + 16\text{e}^{-0.75t}$ (with or without $+C$) and allow unsimplified eg $12t - \text{e}^{(t-10)} - \dfrac{12}{-0.75}\text{e}^{-0.75t}$
Do not withhold this mark if they write $12x$ but subsequently replace it with T later on.

M1 Substitutes T and 0 into their changed expression with three different terms in t, sets equal to 100 and attempts to make $12T$ or T the subject.
Eg $\left(12T - \text{e}^{T-10} + 16\text{e}^{-0.75T}\right) - \left(-\text{e}^{-10} + 16\right) = 100 \Rightarrow 12T = \ldots$
Condone arithmetical slips in their rearrangement.

A1* $T = \dfrac{1}{12}\left(116 - 16\text{e}^{-0.75T} + \text{e}^{T-10} - \text{e}^{-10}\right)$ with no errors including bracket errors/omissions.
Must have $T = \ldots$

(c)

(i)

M1 Substitutes 10 into the iterative formula. The expression with 10 embedded is sufficient to score this mark.

May be implied by awrt 9.7492 or awrt 9.7493. May also be implied by 9.7306

A1 awrt 9.7493

(ii) ***Be aware that this can be found directly on a calculator which is not acceptable***

A1 9.7293 (seconds) (cannot be awarded without the method mark)

SC If they over round in (i) and (ii) and achieve 9.749 and 9.729 then award M1A0A1

11. OCT 2022 [5]

Question	Scheme	Marks
5(a)	$P(1)=\frac{4-1}{10}+\frac{3}{4}\ln\left(\frac{2}{3^2}\right) \quad (=-0.828...)$	M1
	$P(1)=-0.828...$ (which is negative so a loss of £0.828 million,) so approximately £830 000 loss.	A1*
		(2)
(b)	$P(6)=-0.08799\ldots$ and $P(7)=0.1975\ldots$	M1
	There is a sign change and hence as P is continuous on [6,7], so the root for t lies in [6,7].	A1
		(2)
(c)	$P=0 \Rightarrow \frac{4t-1}{10}=-\frac{3}{4}\ln\left(\frac{t+1}{(2t+1)^2}\right) \Rightarrow t=...$	M1
	$\Rightarrow t=\frac{10\left(-\frac{3}{4}\ln\left(\frac{t+1}{(2t+1)^2}\right)\right)+1}{4}=\frac{1}{4}+\frac{30}{16}\ln\left(\frac{t+1}{(2t+1)^2}\right)^{-1}$ $\Rightarrow t=\frac{1}{4}+\frac{15}{8}\ln\left(\frac{(2t+1)^2}{t+1}\right)$ *	A1*
		(2)
(d)	$t_2=\frac{1}{4}+\frac{15}{8}\ln\left(\frac{13^2}{7}\right)=...(=6.219978...)$	M1
	$t_2=\text{awrt } 6.220$	A1
	$t_6=6.314$	A1
		(3)
(e)	"6.3..."$\times 12=...$ months, or repeated iteration to root 6.31487 gives 75.7785… or allow "0.314..."$\times 12=$"3.768..."	M1
	So it will take 76 months. (Accept 75 or awrt 76 months)	A1
		(2)
		(11 marks)

Notes:

(a)

M1: Attempts to substitute $t = 1$ into the given formula. Allow if there is a slip but an attempt at substitution is seen, or allow for sight of awrt $-0.828...$

A1*: Correct value for $P(1)$ seen to at least 2.s.f. if substitution has been shown, or at least 3 s.f. if no substitution was shown, followed by suitable conclusion that it is a loss of approximately £830 000, though accept awrt £830 000. Must mention "loss" and include units (£ or pounds). Negative value given is A0.

(b)

M1: Attempts both $P(6)$ and $P(7)$ with at least one correct to 1 s.f. rounded or truncated. A tighter interval could be used but must contain the root (6.31487)

A1: Both correct to1 s.f. rounded or truncated with suitable conclusion made. Must mention sign change and continuity as well as conclusion about root in the interval.

(c)

M1: Attempts to isolate αt from the $\frac{4t-1}{10}$ after setting equal to zero.

A1*: Correct work to reach the given answer with no incorrect work seen and at least one intermediate step with either $\alpha t = ...$ reached or with the power law applied on the ln term (need not see the power explicitly used, but must have been applied correctly).

(d)

M1: Attempts to use the formula with $t_1 = 6$. Accept with 6 embedded in formula followed by a value, or awrt 6.2 (or even 6.3) as implying the attempt.

A1: awrt 6.220 Accept 6.22.

A1: Correct and given to 3 d.p.

(e)

M1: Multiplies their final root from (d) by 12, or uses repeated iteration to narrow further then multiplies by 12. Allow the method if they multiply the fractional part only by 12 and get, e.g., 6 years 3 months.

A1: Accept 75 or 76 months, or anything that rounds to 76 months.

12. OCT 2021 [9]

Question Number	Scheme	Marks
9 (a)	$f(x)=\left(x^3-4x\right)e^{-\frac{1}{2}x} \Rightarrow f'(x)=\left(3x^2-4\right)e^{-\frac{1}{2}x}-\frac{1}{2}\left(x^3-4x\right)e^{-\frac{1}{2}x}$	M1 A1
		(2)
(b)	$f'(0)=-4$ so equation of normal is $y=-\frac{1}{-4}x$	M1
	$y=\frac{1}{4}x$	B1
	Sets $\frac{1}{4}\cancel{x}=\cancel{x}\left(x^2-4\right)e^{-\frac{1}{2}x} \Rightarrow x^2-4=\frac{1}{4}e^{\frac{1}{2}x}$	M1
	$\Rightarrow x^2=\frac{16+e^{\frac{1}{2}x}}{4} \Rightarrow x=-\frac{1}{2}\sqrt{16+e^{\frac{1}{2}x}}$ *	A1*
		(4)
(c)	(i) $x_2=-\frac{1}{2}\sqrt{16+e^{\frac{1}{2}\times-2}}=-2.0229$	M1 A1
	(ii) $(x=)-2.0226$	A1
		(3)
		(9 marks)

(a)

M1 Uses a valid method to differentiate. This could be:

(i) using the product rule on $f(x)=\left(x^3-4x\right)e^{-\frac{1}{2}x}$ so score for an expression of the form $\left(f'(x)=\right)\ \pm A\left(x^3-4x\right)e^{-\frac{1}{2}x}\pm\left(Bx^2\pm C\right)e^{-\frac{1}{2}x}$. ($A, B, C\neq 0$) Condone the squared missing on the Bx^2 term

(ii) using the quotient rule on $f(x)=\frac{x^3-4x}{e^{\frac{1}{2}x}}$ so score for an expression of the form

$$\left(f'(x)=\right)\frac{\pm e^{\frac{1}{2}x}\left(Bx^2\pm C\right)-Ae^{\frac{1}{2}x}\left(x^3-4x\right)}{\left(e^{\frac{1}{2}x}\right)^2}$$ ($A, B, C\neq 0$) Condone the squared

missing on the Bx^2 term or an attempt at (iii) $f'(x)=uvw'+uv'w+u'vw$ which could look like:

$$(x^2-4)e^{-\frac{1}{2}x}+x\frac{d\left((x^2-4)e^{-\frac{1}{2}x}\right)}{dx}=\pm...(x^2-4)e^{-\frac{1}{2}x}\pm x(...x)e^{-\frac{1}{2}x}\pm...x(x^2-4)e^{-\frac{1}{2}x}$$

A1 Correct $f'(x)$ but may be unsimplified. Isw after a correct unsimplified expression

$$\left(3x^2-4\right)e^{-\frac{1}{2}x}-\frac{1}{2}\left(x^3-4x\right)e^{-\frac{1}{2}x} \text{ or } (x^2-4)e^{-\frac{1}{2}x}+x\frac{d\left((x^2-4)e^{-\frac{1}{2}x}\right)}{dx}=(x^2-4)e^{-\frac{1}{2}x}+x(2x)e^{-\frac{1}{2}x}-\frac{1}{2}x(x^2-4)e^{-\frac{1}{2}x}$$

(b) Note on EPEN it is M1A1M1A1 but we are marking this M1B1M1A1

M1 Full method to find the equation of the normal through *O*.

Look for an attempt at $f'(0)$ followed by the equation $y=-\frac{1}{f'(0)}x$

B1 Equation of normal is $y=\frac{1}{4}x$ (seen or implied) (which may follow an incorrect $f'(x)$ from part (a))

M1 Equates their $y=\frac{1}{4}x$ (which must be a straight line through the origin) with $f(x)=x\left(x^2-4\right)e^{-\frac{1}{2}x}$, divides through or factorises out the x term and attempts to make x^2 (or allow $4x^2$) the subject

A1* Full proof showing all steps. There is no requirement to justify the – sign. Note that A1* cannot be scored if A0 in part (a), unless they restart in (b).

(c)(i)

M1 Substitutes $x=-2$ into the iteration formula and finds x_2. May also be implied by −2.0228 or −2.0229

A1 awrt −2.0229

(ii)

A1 $(x=)-2.0226$ correct to 4 dp

13. JAN 2020 [7]

7. (a)	$y\|_{0.8} = 2\cos 2.4 - 2.4 + 4 = 0.13$ AND $y\|_{0.9} = 2\cos 2.7 - 2.7 + 4 = -0.51$	M1
	States change of sign, continuous and hence root	A1
		(2)
(b) (i)	$(x_2) = \frac{1}{3}\arccos(1.5\times 0.8 - 2) = 0.8327$	M1 A1
(ii)	$x_5 = 0.8110$	A1
		(3)
(c)	$\frac{dy}{dx} = -6\sin 3x - 3$	M1 A1
	Attempts $\frac{dy}{dx} = 0 \Rightarrow \sin 3x = -\frac{1}{2} \Rightarrow x = \ldots$ via correct order of operations	dM1
	Achieves either $\frac{7\pi}{18}$ or $\frac{19\pi}{18}$	A1
	Correct attempt to find the third positive solution e.g. $x = \frac{\frac{\pi}{6} + 3\pi}{3}$	ddM1
	$\beta = \frac{7\pi}{18}$ and $\lambda = \frac{19\pi}{18}$	A1
		(6)
		11 marks

(a)

M1 Attempts the value of y at 0.8 **AND** 0.9 with at least one correct to 1 sf rounded or truncated.
Note that it is possible to choose a tighter interval containing the root but to score the A1 the conclusion must refer to the given interval.

A1 Both values correct to 1sf rounded or truncated, with reason (Sign change **and** continuous function) and minimal conclusion (root)
If the candidate chooses 0.8 and 0.9 the minimal conclusion does not need to mention the interval.
So e.g. $y|_{0.8} = 0.1 > 0,\ y|_{0.9} = -0.5 < 0$ and function is continuous, so ✓ would be acceptable

(b) (i)

M1 Attempts to substitute $x_1 = 0.8$ into the formula. Implied by sight of embedded values in expression or awrt 0.83

A1 AWRT 0.8327

(b)(ii)

A1 $x_5 = 0.8110$ CAO. This is not awrt and 0.811 is A0 unless preceded by 0.8110
If it is clearly marked (b)(ii) then you don't need the x_5

(c)

M1 For $\left(\frac{dy}{dx}\right) = A\sin 3x + B$

A1 $\left(\frac{dy}{dx}\right) = -6\sin 3x - 3$

dM1 Attempts $\frac{dy}{dx} = 0 \Rightarrow \sin 3x = a, \quad |a| < 1 \Rightarrow x = ...$ It is dependent upon the previous M

Look for correct order of operations, invsin$a$ then $\div 3$ leading to a value for x.

When $\sin 3x = -\frac{1}{2}$ it is implied, for example, by $-\frac{\pi}{18}, \frac{7\pi}{18}, \frac{11\pi}{18}$ (2nd soln), $\frac{19\pi}{18}$ amongst others or if answers are given as decimals, for example, by awrt -0.17, awrt 1.2, awrt 1.92 or awrt 3.32

For $\sin 3x = +\frac{1}{2}$ it would be implied, for example, by values such as $\frac{\pi}{18}, \frac{5\pi}{18}$ awrt 0.17 or awrt 0.87

The calculations must be using radians. If degrees are used initially they must be converted to radians

A1 For recognising that either $\frac{7\pi}{18}$ or $\frac{19\pi}{18}$ is a solution to $\sin 3x = -\frac{1}{2}$

ddM1 Attempts to find the solution for λ in the correct quadrant.
Look for the **3rd positive solution** for their $\sin 3x = k$

So for $k > 0$ it would be for $x = \frac{2\pi + \arcsin|k|}{3}$

And for $k < 0$ it would be for $x = \frac{3\pi + \arcsin|k|}{3}$

A1 States that $\beta = \frac{7\pi}{18}$ and $\lambda = \frac{19\pi}{18}$ Labels must be correct

14. OCT 2020 [6]

6 (a)	$5e^{x-1}+3=18 \Rightarrow e^{x-1}=3$	M1
	$\Rightarrow x=\ln 3+1$ or $e^x=3e$	A1
	$\Rightarrow x=\ln 3e$	A1
		(3)
(b)	Sets $5e^{x-1}+3=10-x^2$ and proceeds to find and use a suitable function. Eg $(f(x)=)7-x^2-5e^{x-1}$	B1
	Attempts $f(1.1335)=0.001$ and $f(1.1345)=-0.007$	M1
	Correct values with reason(change of sign and continuous) and conclusion, hence α is 1.134 to 3dp	A1
		(3)
(c)	$x_2=-\sqrt{7-5e^{-3-1}}=-2.628388$	M1 A1
	$\beta=-2.620330$	A1
		(3)
		(9 marks)

(a)

M1 For attempting to proceed from $5e^{x-1}+3=18$ to $e^{x-1}=\ldots$ or $e^x=\ldots$

A1 $x=\ln 3+1$ or for $e^x=3e$

A1 $x=\ln 3e$ Accept $x=\ln 3e^1$

(b)

B1 Sets the equations equal to each other and finds a suitable function which is then used.

Suitable functions are $f(x)=\pm\left(7-x^2-5e^{x-1}\right)$, $g(x)=\pm\left(x-\sqrt{7-5e^{x-1}}\right)$ or $h(x)=\pm\left(x-1-\ln\left(\frac{7-x^2}{5}\right)\right)$

M1 Substitutes $x=1.1335$ and $x=1.1345$, or suitable values for a tighter interval (either side of 1.133634..), into a suitable function and obtains one correct value to 1sf rounded or truncated.

FYI for the + options above

$f(1.1335)=0.001$ and $f(1.1345)=-0.007$

$g(1.1335)=-0.0005$ and $g(1.1345)=0.003$

$h(1.1335)=-0.0002$ and $g(1.1345)=0.001$

A1 Requires both values to be correct (1sf rounded or truncated), a reason (change of sign and continuous) and a minimal conclusion

(c)

M1 For an attempt at substituting -3 into the iterative equation.
This is implied by the sight of the expression or awrt -2.63

A1 awrt -2.628388

A1 $\beta =$ awrt -2.620330 condone -2.62033

....

It is possible in part (b) to score B0 M1 A1 by comparing y values for $y = 5e^{x-1} + 3$ and $y = 10 - x^2$ at $x = 1.1335$ and $x = 1.1345$ For the A1 apply the similar criteria as for the main scheme with values to 3d.p..

At $x = 1.1335$, $y|_{1.1335} = 5e^{x-1} + 3 = 8.714$ and $y|_{1.1335} == 10 - x^2 = 8.715$

At $x = 1.1345$, $y|_{1.1345} = 5e^{x-1} + 3 = 8.720$ and $y|_{1.1345} = 10 - x^2 = 8.713$