AP PRECALCULUS

UNIT 1.1 — CHANGE IN TANDEM

TEACHER GUIDED NOTES · RED ANSWERS MATCH STUDENT BLANKS

Name: _____ Date: _____

Learning Goal: I can explain how two quantities change together, identify input and output variables, and describe functions using words, tables, equations, and graphs.

PART I — GUIDED NOTES

Section 1: What Is a Function?

A function is a rule that connects an **input** to an **output**. Every input must have exactly **one** output. Two different inputs may have the **same** output, but one input cannot have **two** different outputs.

Term	Student-friendly meaning
Domain	**All allowed input values**
Range	**All output values produced by a function**
Independent variable	**The input, usually x**
Dependent variable	**The output, usually y**
Function rule	**The rule that connects each input to its output**

Input = **Domain** = Independent variable (x). Output = **Range** = Dependent variable (y).

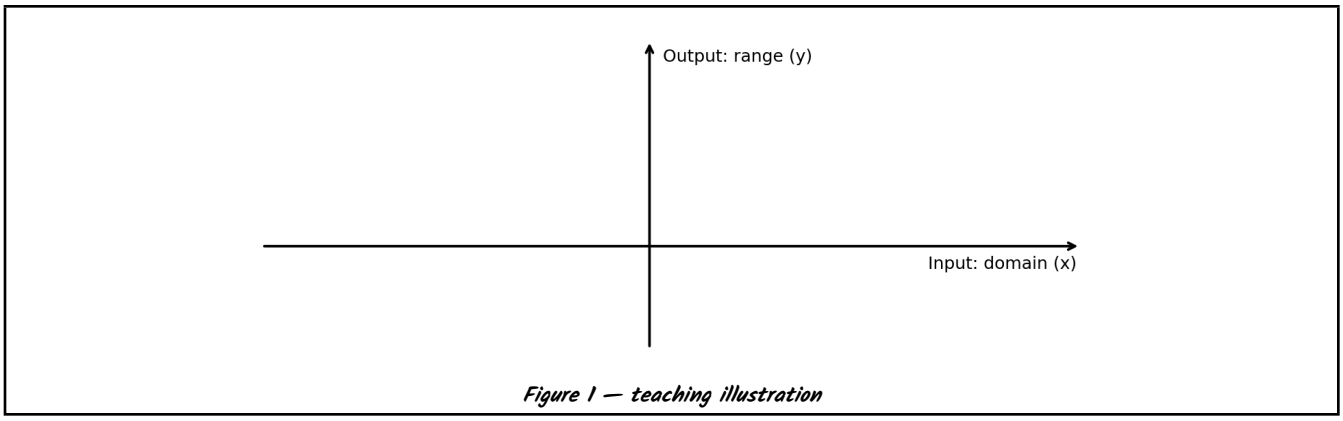


Figure 1 — teaching illustration

Example: Throwing a Football

Suppose t is the number of seconds since a football was thrown and h is its height. The height depends on **time**. Time (t) is the **independent** variable; height (h) is the **dependent** variable. Write **h(t)**, read "h of t." This is called function **notation**.

Example 1: Water in a Swimming Pool

The function w(x) uses x for the **length** of a pool and w for the amount of **water** in the pool (gallons). (a) Independent variable: **length of the pool** (b) Dependent variable: **amount of water**.

KEY IDEA | A function assigns exactly one output to each input.

CHECK FOR UNDERSTANDING

1. What is the difference between domain and range? **The domain contains all input values; the range contains the output values.**

2. For d(t), distance traveled after t hours, identify the independent and dependent variables. Independent: time (t); dependent: distance d(t).

3. Can a function assign two outputs to the same input? Explain. No. Every input must have exactly one output.

Section 2: Change in Tandem and VANG

"Change in tandem" means two quantities change together according to a relationship. A function rule describes how the output changes when the input changes.

Representation	Meaning	Example
Verbal (V)	Words	As hours worked rise, total pay rises.
Analytical (A)	Equation	P(h) = 15h
Numerical (N)	Table	h: 1, 2, 3, 4; P: 15, 30, 45, 60
Graphical (G)	Graph	Plot (1,15), (2,30), (3,45), (4,60)

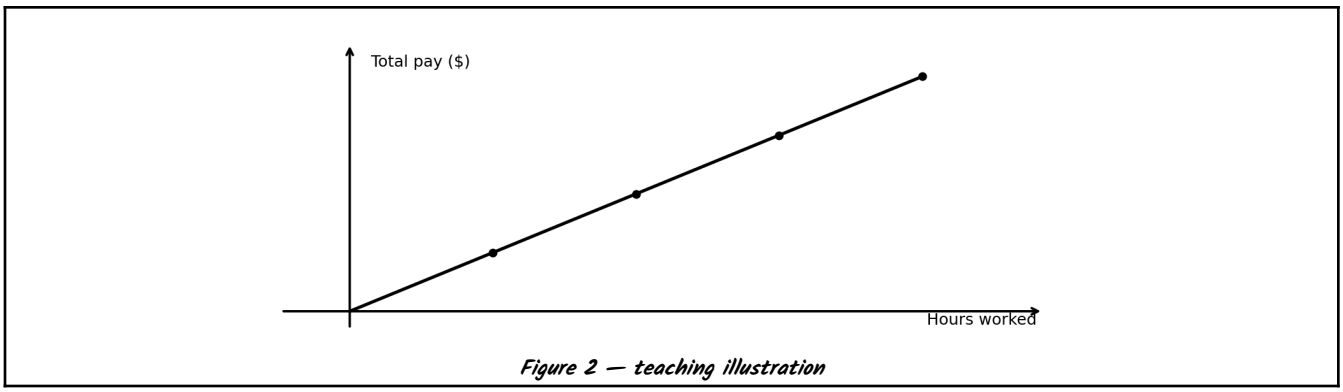


Figure 2 — teaching illustration

KEY IDEA | VANG = V: Verbal A: Analytical N: Numerical G: Graphical.

CHECK FOR UNDERSTANDING

1. What does VANG stand for? Verbal, Analytical, Numerical, Graphical.

2. Which representation uses a table? Numerical (N).

3. Write a pay function for $20 per hour. P(h) = 20h.

Section 3: Increasing Functions

A function is INCREASING on an interval when its output goes up as its input goes up. An interval is a connected set of input values.

Verbal: As x increases, f(x) increases. Analytical: If a < b, then f(a) ≤ f(b) for any a and b in the interval.

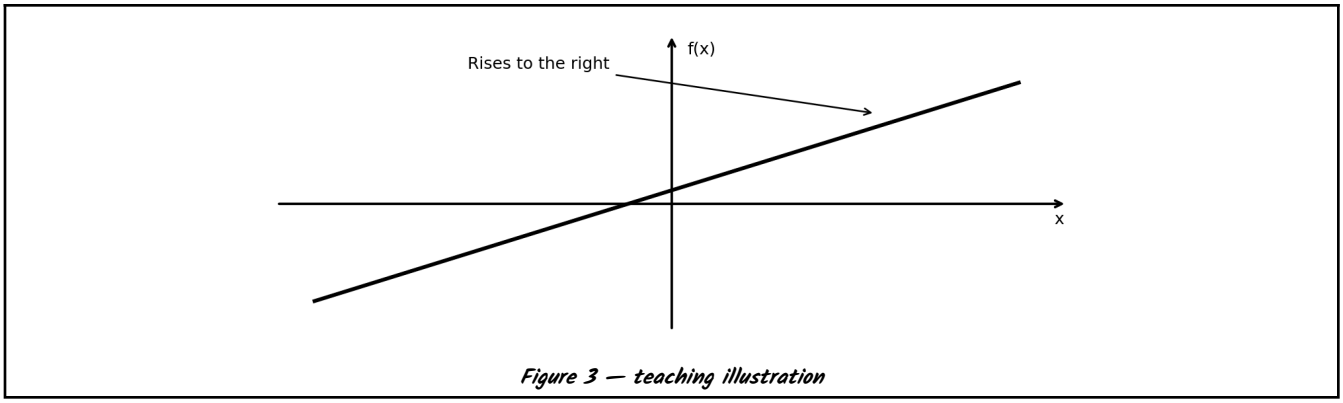


Figure 3 — teaching illustration

Steps: 1. Read inputs from smallest to largest. 2. Compare the outputs. 3. Check whether each output increases. 4. State the behavior on the interval.

x	1	2	3	4
f(x)	3	6	9	12

As inputs rise, outputs rise. Therefore, the function is increasing.

KEY IDEA | For increasing functions, larger inputs have larger outputs.

CHECK FOR UNDERSTANDING

1. What happens to f(x) as x rises? It increases.

2. Are outputs 2, 5, 8, 11 increasing? Yes.

3. If $a < b$, what inequality compares f(a) and f(b)? $f(a) < f(b)$.

Section 4: Decreasing Functions

A function is DECREASING on an interval when its output falls as its input rises. Verbal: As x increases, f(x) decreases. Analytical: If $a < b$, then $f(a) \geq f(b)$.

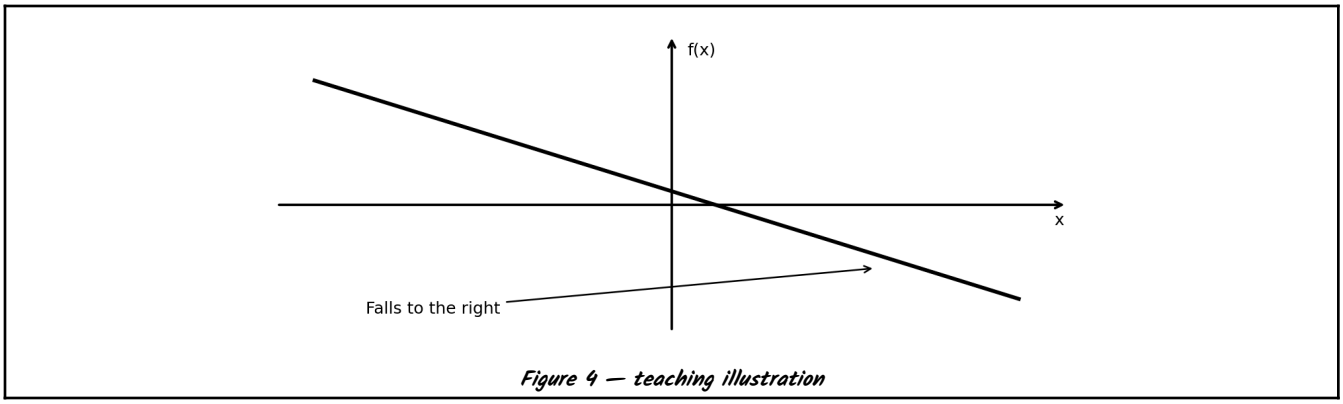


Figure 4 — teaching illustration

Steps: 1. Order inputs from least to greatest. 2. Read the matching outputs. 3. Check if outputs decrease. 4. State the behavior on the interval.

x	1	2	3	4
f(x)	20	15	10	5

As inputs rise, outputs fall. Therefore, the function is decreasing.

Increasing	Decreasing
Outputs rise	Outputs fall
Graph rises to the right	Graph falls to the right
$a < b \rightarrow f(a) \leq f(b)$	$a < b \rightarrow f(a) \geq f(b)$

KEY IDEA | For decreasing functions, larger inputs have smaller outputs.

CHECK FOR UNDERSTANDING

1. What direction does a decreasing graph move? Down as we move to the right.

2. Are $f(2) = 10$ and $f(5) = 4$ consistent with a decreasing function? Yes, because $10 > 4$ while $2 < 5$.

3. Write the analytical inequality for decreasing functions. If $a < b$, then $f(a) > f(b)$.

Section 5: Reading Function Tables

Original Example 2: f is known to be increasing or decreasing, but not both. Decide which and justify.

x	4	6	7	10	20

f(x)	1	1.01	1.04	1.06	1.29

Step 1: Inputs 4, 6, 7, 10, 20 increase. Step 2: Outputs 1, 1.01, 1.04, 1.06, 1.29 increase. Step 3: Therefore, f is increasing on the interval.

AP Exam Tip: When asked to JUSTIFY, explain the pattern in both the inputs and the outputs.

KEY IDEA | Read tables in increasing input order. Use the output trend to justify.

CHECK FOR UNDERSTANDING

1. Is a function decreasing when its outputs are 30, 25, 20, 15 as x rises? Yes, the outputs fall as x rises.

2. Why read the inputs in increasing order? To compare output values in left-to-right input order.

3. Write a one-sentence justification for this example. As x increases from 4 to 20, f(x) increases from 1 to 1.29, so f is increasing.

Section 6: Basic Elements of a Function's Graph

A. Zeros

A ZERO is an x-value that makes f(x) = 0. On the graph, find where the curve crosses or touches the x-axis. Write the x value, not the entire ordered pair.

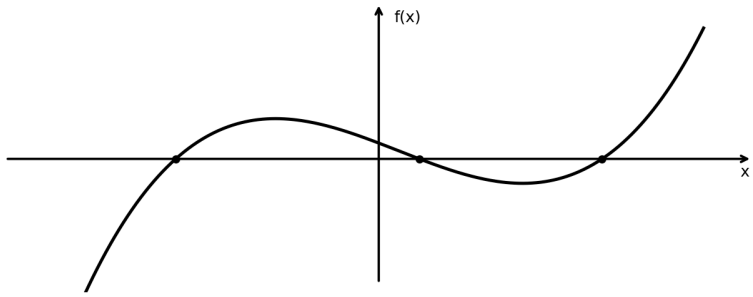


Figure 5 — teaching illustration

B. The y-Intercept

The y-intercept is where the graph meets the y-axis, so x = 0. The output is f(0), and the point is (0, f(0)).

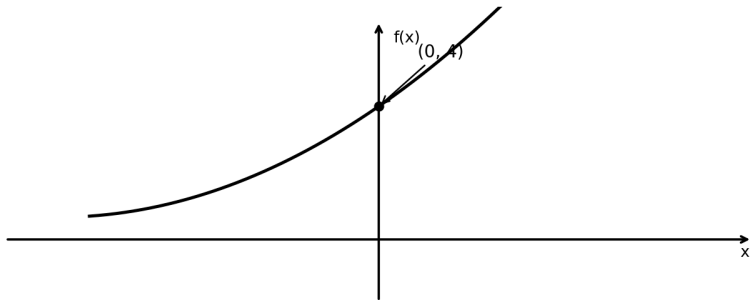


Figure 6 — teaching illustration

C. Concavity

CONCAVITY describes how a curve bends. Concave UP bends like a smile and its slopes increase from left to right. Concave DOWN bends like an upside-down cup and its slopes decrease. Slope describes steepness and direction. A straight line has no upward or downward bending.

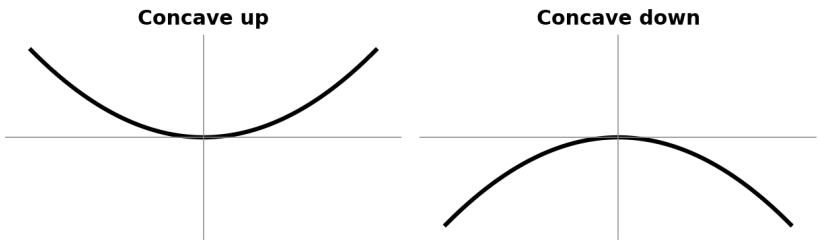


Figure 7 — teaching illustration

D. Point of Inflection

A POINT OF INFLECTION is a point where the graph changes concavity: from up to down, or the reverse. A turning point is not automatically an inflection point.

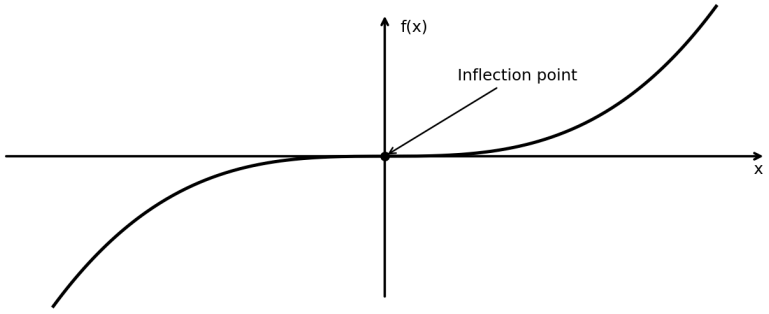


Figure 8 — teaching illustration

E. Increasing and Decreasing Intervals

Read the graph from left to right. Identify where the curve rises (increasing) and falls (decreasing). Record the x-intervals separated by turning points.

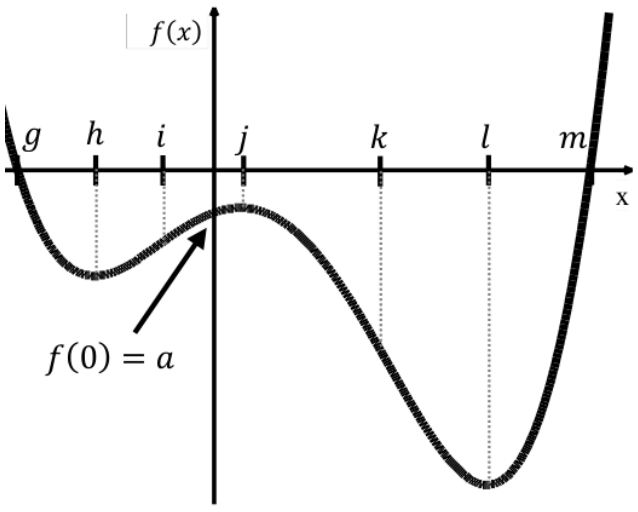


Figure 9 — reproduced from the original APPC packet

For Figure 9, answer: (a) concave-up intervals; (b) concave-down intervals; (c) zeros; (d) y-intercept; (e) increasing intervals; (f) decreasing intervals.

Feature	What to look for
Zeros	Where the graph meets the x-axis
y-intercept	Where x = 0
Increasing	Curve rises from left to right
Decreasing	Curve falls from left to right
Concave up	Slopes increase
Concave down	Slopes decrease
Inflection	Concavity changes

KEY IDEA | Find axis crossings, rising/falling trends, and curve bending to describe a graph.

CHECK FOR UNDERSTANDING

1. How is a zero different from a y-intercept? A zero is an x-value where f(x)=0; a y-intercept is the point (0, f(0)).

2. What does concave up mean? The curve bends like a smile; its slopes increase.

3. Can a graph decrease and be concave up? Explain. Yes. A decreasing curve can flatten as x increases (decreasing and concave up).

PART 2 — UNIT 1.1 PRACTICE

Name: _____ Date: _____

Section 7: Independent and Dependent Variables

Problem 1

$b(s)$ is the number of books in a library; s is the number of students in the school.

Dependent variable: number of books in the library, b Independent variable: number of students, s

Problem 2

$p(t)$ is the number of Pokémon cards; t is the number of years since kindergarten.

Dependent variable: number of Pokémon cards, p Independent variable: years since kindergarten, t

Problem 3

$c(t)$ is the number of cups of coffee consumed; t is the number of teachers at school.

Dependent variable: number of cups of coffee, c Independent variable: number of teachers, t

Problem 4

The graph compares the area of a circle with its radius.

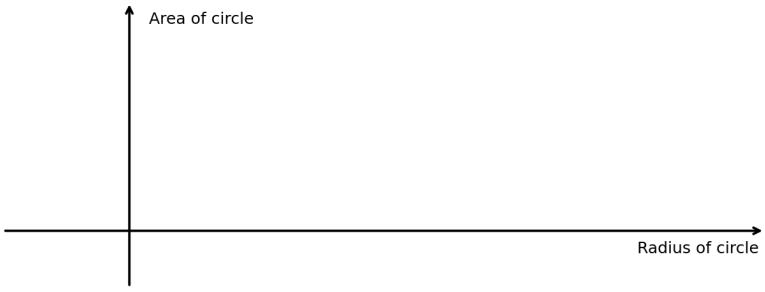


Figure 10 — teaching illustration

Dependent: area of circle Independent: radius of circle

Problem 5

The graph compares the volume of a box with its side length.

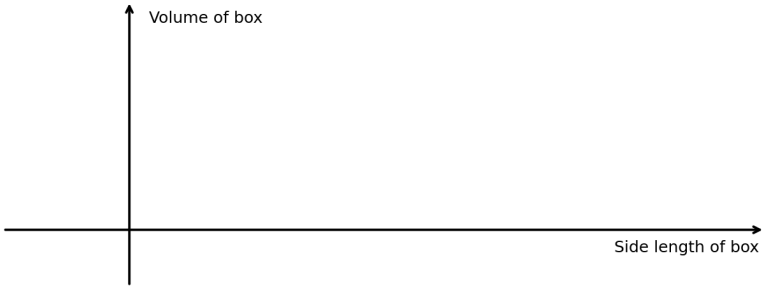


Figure 11 — teaching illustration

Dependent: volume of box Independent: side length of box

KEY IDEA | The input is independent; the output is dependent.

CHECK FOR UNDERSTANDING

1. Which variable is inside the parentheses? The independent/input variable.

2. In A(r), what does r mean? Radius (input).

3. In V(s), what does V mean? Volume (output).

Section 8: Increasing or Decreasing?

For Questions 6–11, f is either increasing or decreasing, but not both, on the stated interval. State which and justify.

Problem 6

x	1	2	3	4	5
f(x)	95	90	75	50	10

Increasing or decreasing? Decreasing Why? As x increases from 1 to 5, f(x) falls from 95 to 10.

Problem 7

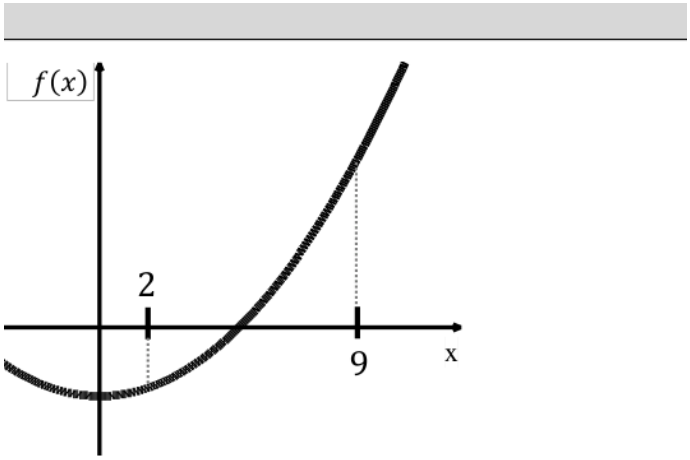


Figure 12 — reproduced from the original APPC packet

Increasing or decreasing from x = 2 to x = 9? Justify: Increasing; f(x) rises as x moves from 2 to 9.

Problem 8

x	12	16	18	25	31
f(x)	8	7.99	7.8	7.75	7.7

Increasing or decreasing? Decreasing Why? Outputs decrease from 8 to 7.7 as inputs increase from 12 to 31.

Problem 9

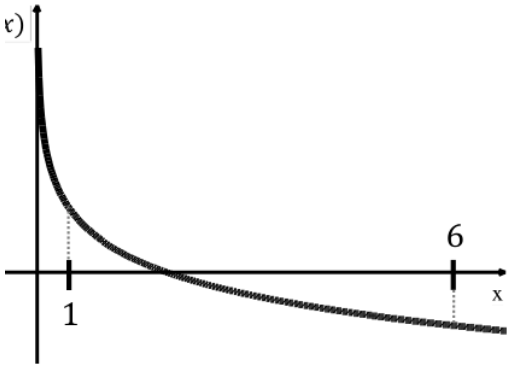


Figure 13 — reproduced from the original APPC packet

Increasing or decreasing from x = 1 to x = 6? Justify: Decreasing; the graph falls between x=1 and x=6.

Problem 10

x	101	201	301	401	501
f(x)	10	11	12	13	14

Increasing or decreasing? Increasing **Why?** Outputs rise from 10 to 14 as x increases from 101 to 501.

Problem 11

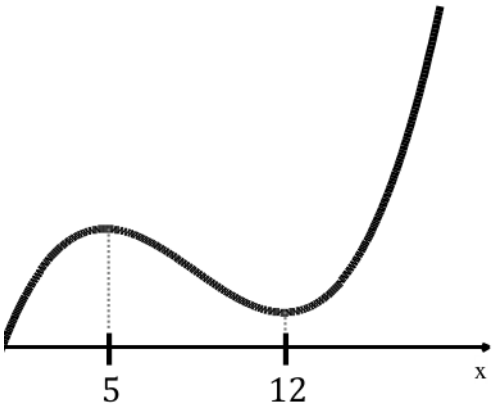


Figure 14 — reproduced from the original APPC packet

Increasing or decreasing from x = 5 to x = 12? Justify: Decreasing; f(x) falls as x moves from 5 to 12.

KEY IDEA | In a table, follow outputs as inputs rise. On a graph, read left to right.

CHECK FOR UNDERSTANDING

1. What does a decreasing graph do? Falls from left to right.

2. If outputs rise 4, 7, 12, what is the behavior? Increasing.

3. Why must a justification mention both inputs and outputs? The relationship between rising inputs and changing outputs is the justification.

Section 9: Analyzing Function Graphs

For Questions 12–15, find concavity, increasing/decreasing intervals, zeros, and y-intercepts from the pictured curves.

Problem 12

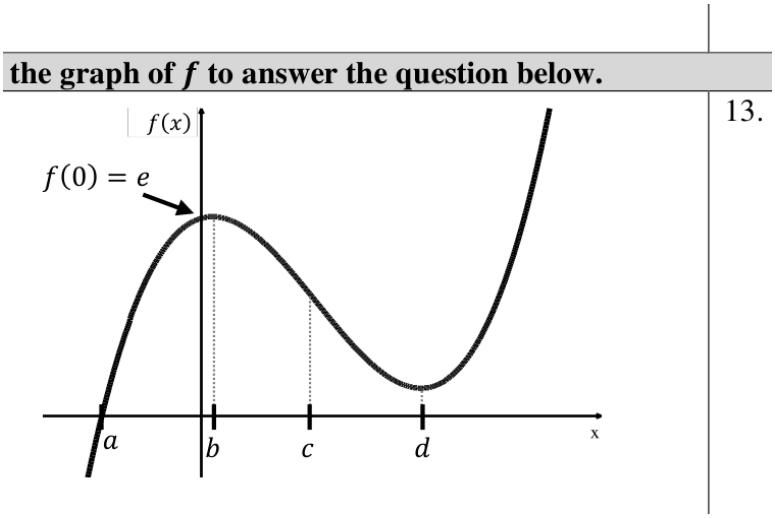


Figure 15 — reproduced from the original APPC packet

a. On what interval(s) is the graph concave up? (c, ∞)

b. On what interval(s) is the graph concave down? $(-\infty, c)$

c. On what interval(s) is the graph increasing? $(-\infty, b) \cup (d, \infty)$

d. On what interval(s) is the graph decreasing? (b, d)

e. Find the zeros of the function. a

f. Find the y-intercept of the function. $(0, e)$

Problem 13

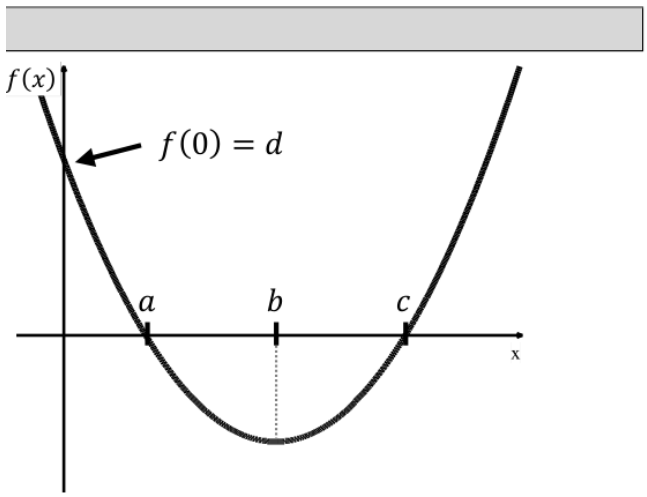


Figure 16 — reproduced from the original APPC packet

a. On what interval(s) is the graph concave up? $(-\infty, \infty)$

b. On what interval(s) is the graph concave down? None

c. On what interval(s) is the graph increasing? (b, ∞)

d. On what interval(s) is the graph decreasing? $(-\infty, b)$

e. Find the zeros of the function. a and c

f. Find the y-intercept of the function. $(0, d)$

Problem 14

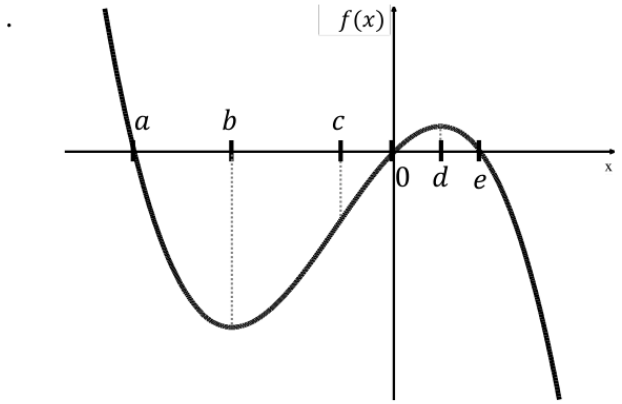


Figure 17 — reproduced from the original APPC packet

a. On what interval(s) is the graph concave up? $(-\infty, c)$

b. On what interval(s) is the graph concave down? (c, ∞)

c. On what interval(s) is the graph increasing? (b, d)

d. On what interval(s) is the graph decreasing? $(-\infty, b) \cup (d, \infty)$

e. Find the zeros of the function. a, 0, e

f. Find the y-intercept of the function. $(0, 0)$

Problem 15

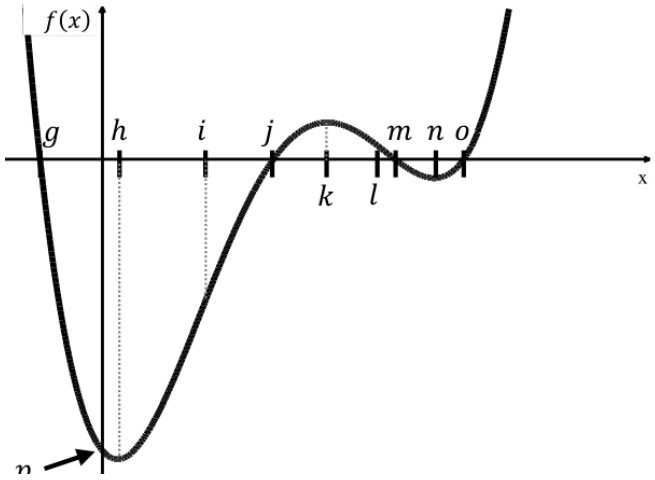


Figure 18 — reproduced from the original APPC packet

a. On what interval(s) is the graph concave up? Approximately $(-\infty, i) \cup (m, \infty)$

b. On what interval(s) is the graph concave down? Approximately (i, m)

c. On what interval(s) is the graph increasing? $(h, k) \cup (n, \infty)$

d. On what interval(s) is the graph decreasing? $(-\infty, h) \cup (k, n)$

e. Find the zeros of the function. g, j, l, o

f. Find the y-intercept of the function. (0, f(0)), below the x-axis

KEY IDEA | Locate intercepts; identify increasing / decreasing intervals; check where concavity changes.

CHECK FOR UNDERSTANDING

1. What is f(x) at a zero? 0.

2. How do you locate f(0) on a graph? Read the point where the curve meets the y-axis.

3. Can a decreasing graph be concave up? Yes, if the slope increases while staying negative.

PART 3 — UNIT 1.1 TEST PREPARATION

Section 10: Multiple Choice

Question 16 — Decreasing Function

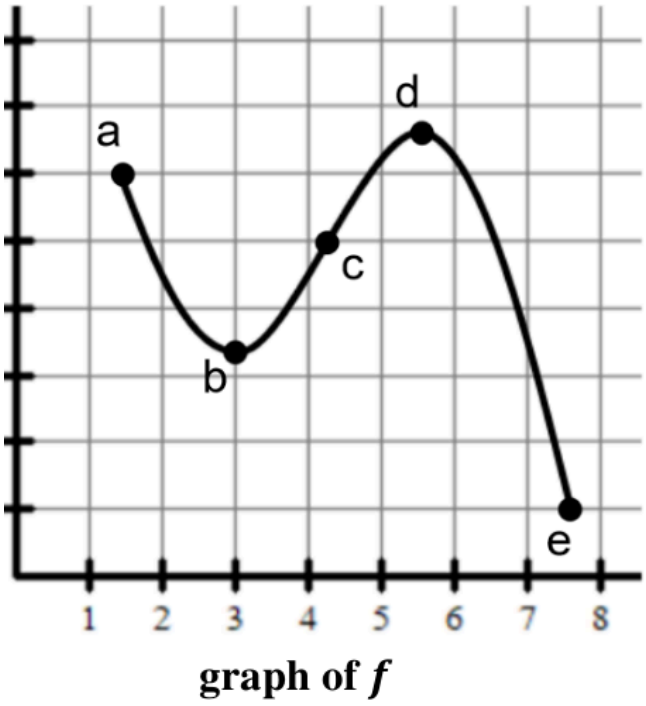
The function f is always decreasing. Which inequality gives ALL possible values of a?

x	f(x)
−5	12
−2	4
0	a
3	−2
7	−15

A. $-5 < a < 7$ B. $-2 < a < 0$ C. $0 < a < 3$ D. $-2 < a < 4$ E. $-15 < a < 12$

Helpful strategy: (1) As inputs rise, outputs fall. (2) Find the outputs directly before and after a. (3) Write the bounds: $-2 < a < 4$. (4) Select the matching option.

Questions 17–18 — Use the Same Graph



graph of *f*

Figure 19 — reproduced from the original APPC packet

Question 17 — Point of Inflection

Which point is an inflection point? A. Point a B. Point b C. Point c D. Point d E. Point e

Helpful strategy: Find where the curve changes from concave up to concave down, or the reverse.

Question 18 — Decreasing and Concave Down

On which interval is f BOTH decreasing and concave down?

A. a to b B. b to c C. c to d D. d to e E. None of the above

Helpful strategy: Confirm BOTH conditions: the graph falls and it is concave down over the SAME interval.

KEY IDEA | In a two-condition question, both conditions must be true over the interval.

CHECK FOR UNDERSTANDING

1. If $a < b$ and f is decreasing, compare $f(a)$ and $f(b)$. $f(a) > f(b)$.
2. What change defines an inflection point? A change in concavity.
3. How would you check "increasing and concave down"? Check the graph rises and its slopes are decreasing throughout that interval.

FINAL UNIT 1.1 REVIEW

Concept	Remember
Function	Exactly one output per input
Domain / Range	Inputs / Outputs
Independent / Dependent	Input / Output
Function notation	$f(x)$ gives the output for input x
Change in tandem	Quantities change together
VANG	Verbal, Analytical, Numerical, Graphical
Increasing / Decreasing	Outputs rise / fall as inputs rise
Zero / y-intercept	$f(x) = 0$ / $x = 0$
Concave up / down	Slopes increase / decrease
Inflection point	Concavity changes

Five steps: 1. Find zeros. 2. Find the y-intercept. 3. Identify increasing intervals. 4. Identify decreasing intervals. 5. Check concavity and inflection points.

SELF-ASSESSMENT: ☐ Domain and range ☐ Input and output ☐ Function notation ☐ VANG ☐ Increasing and decreasing ☐ Zeros and intercepts ☐ Concavity ☐ Inflection points ☐ Justifications