Advanced Math
Date____
Period____

Name________________________

5.1-5.5 Review

1 and 2: A – Find a recursive-form definition that matches the table.
B - Then find a closed-form definition that matches the table.

1.

Input	Output	
0	10	+4
1	14	+4
2	18	+4
3	22	+4
4	26	

2.

Input	Output	
0	2	(6)
1	12	(6)
2	72	(6)
3	432	(6)
4	2592	

A. Recursive

$$f(n) = \begin{cases} 10 & \text{if } n=0 \\ f(n-1)+4 & \text{if } n>0 \end{cases}$$

Previous output + 4

B. Closed

$g(n) = 10 + 4n$ ← multiplication is shortcut for repeated addition

A. Recursive

$$f(n) = \begin{cases} 2 & \text{if } n=0 \\ 6f(n-1) & \text{if } n>0 \end{cases}$$

6 · previous output

B. Closed

$g(n) = 2 \cdot 6^n$ ← Exponents are shortcuts for repeated multiplication

3-5: Show each pair of function definitions are identical for nonnegative integer inputs with induction.

$f(x) = 7x + 2$

3. $g(x) = \begin{cases} 2 & \text{if } x = 0 \\ g(x-1)+7 & \text{if } x > 0 \end{cases}$

Base Case

x	$f(x)=7x+2$
0	$f(0) = 7(0)+2$ $= 2$
1	$f(1) = 7(1)+2$ $= 9$
2	$f(2) = 7(2)+2$ $= 16$
3	$f(3) = 7(3)+2$ $= 23$

x	$g(x) = g(x-1)+7$
0	$g(0) = 2$ ← given in top line of g
1	$g(1) = g(1-1)+7$ $= g(0)+7$ $= 2+7 = 9$
2	$g(2) = g(2-1)+7$ $= g(1)+7$ $= 9+7 = 16$
3	$g(3) = g(3-1)+7$ $= g(2)+7$ $= 16+7 = 23$

Goal $f(k+1) = 7(k+1)+2$
$= 7k+7+2$
$= 7k+9$

Inductive step

From the base case, we know $f(k) = g(k)$ up to some input. We will show $f(k+1) = g(k+1)$

$g(k+1)$	Given
$= g(k+1-1)+7$	Def of g
$= g(k)+7$	simplify
$= f(k)+7$	Substitute from base case
$= 7k+2+7$	Def of f
$= 7k+9$	simplify
$= f(k+1)$	Def of f (see goal)

∴ $f(x) = g(x)$ for all nonnegative integer inputs.

3-5: Show each pair of function definitions are identical for nonnegative integer inputs with induction.

4.
$f(x) = 7 \cdot 4^x$

$g(x) = \begin{cases} 7 & \text{if } x = 0 \\ 4 \cdot g(x-1) & \text{if } x > 0 \end{cases}$

Base Case

x	$f(x) = 7 \cdot 4^x$
0	$f(0) = 7 \cdot 4^0 = 7$
1	$f(1) = 7 \cdot 4^1 = 28$
2	$f(2) = 7 \cdot 4^2 = 112$
3	$f(3) = 7 \cdot 4^3 = 448$

x	$g(x) = 4g(x-1)$
0	$g(0)$ 7
1	$g(1) = 4g(1-1) = 4g(0) = 4(7) = 28$
2	$g(2) = 4g(2-1) = 4g(1) = 4(28) = 112$
3	$g(3) = 4g(3-1) = 4g(2) = 4(112) = 448$

Inductive Step

From the base case, we know $f(k) = g(k)$ up to some input k.
We will show $f(k+1) = g(k+1)$.

$g(k+1)$	Given
$= 4g(k+1-1)$	Def of g
$= 4g(k)$	Simplify
$= 4f(k)$	Substitute from base case
$= 4 \cdot 7 \cdot 4^k$	Def of f
$= 7 \cdot 4^{k+1}$	Simplify
$= f(k+1)$	Def of f $f(k+1) = 7 \cdot 4^{k+1}$

5.
$f(x) = x^2$

$g(x) = \begin{cases} 0 & \text{if } x = 0 \\ g(x-1) + 2x - 1 & \text{if } x > 0 \end{cases}$

Base case

x	$f(x) = x^2$
0	$f(0) = 0^2$ 0
1	$f(1) = 1^2$ 1
2	$f(2) = 2^2$ 4
3	$f(3) = 3^2$ 9

x	$g(x) = g(x-1) + 2x - 1$
0	$g(0) = 0$
1	$g(1) = g(1-1) + 2(1) - 1$ $g(0) + 2(1) - 1$ $0 + 2 - 1 = 1$
2	$g(2) = g(2-1) + 2(2) - 1$ $= g(1) + 2(2) - 1$ $= 1 + 4 - 1 = 4$
3	$g(3) = g(3-1) + 2(3) - 1$ $= g(2) + 2(3) - 1$ $4 + 6 - 1 = 9$

Goal:

$f(k+1) = (k+1)^2$
$= (k+1)(k+1)$
$= k^2 + 1k + 1k + 1$
$= k^2 + 2k + 1$

Inductive Step

From the base case we know $f(k) = g(k)$ up to some input k.
We will show $f(k+1) = g(k+1)$

$g(k+1)$	Given
$= g(k+1-1) + 2(k+1) - 1$	Def of g
$= g(k) + 2(k+1) - 1$	Simplify
$= f(k) + 2(k+1) - 1$	substitute from base case
$= k^2 + 2(k+1) - 1$	def of f $f(k) = k^2$
$= k^2 + 2k + 2 - 1$	Simplify
$= k^2 + 2k + 1$	Simplify
$= f(k+1)$	Def of f (see goal)

$\therefore$ $f(n) = g(n)$ for all nonnegative integer inputs

6. Given $f(x) = \begin{cases} 4 & \text{if } x = 0 \\ 7 & \text{if } x = 1 \\ 2f(x-2) + f(x-1) & \text{if } x > 1 \end{cases}$, find f(2) and f(3).

$f(2) = 2f(2-2) + f(2-1)$
$f(2) = 2f(0) + f(1)$
$f(2) = 2(4) + 7 = 15$

$f(3) = 2f(3-2) + f(3-1)$
$f(3) = 2f(1) + f(2)$
$f(3) = 2(7) + 15 = 29$

7. Justin owes ~~[illegible]~~ \$8000 on a credit card that charges 3% interest each month (36% APR). He plans to make payments of \$400 each month. (Justin is already past his grace period so assume interest is charged on his remaining balance and then the payment is applied.)

A. Complete the table to find his balance (to nearest penny) after each of the given months.

Month	Balance
0	\$8,000
1	~~[illegible]~~ $8000 + .03(8000) - 400$ $= \$7840$
2	~~[illegible]~~ 7840 $+ .03(7840) - 400$ $= \$7675.20$
3	$7675.2 + .03(7675.2) - 400$ $\approx \$7505.46$
4	$7505.46 + .03(7505.46) - 400$ $\approx \$7330.62$

B. Create a recursive form definition that will find the balance, B after n months.

$$B(n) = \begin{cases} 8000 & \text{if } n = 0 \\ B(n-1) + .03\,B(n-1) - 400 & \text{if } n > 0 \end{cases}$$

OR

$$B(n) = \begin{cases} 8000 & \text{if } n = 0 \\ 1.03\,B(n-1) - 400 & \text{if } n > 0 \end{cases}$$