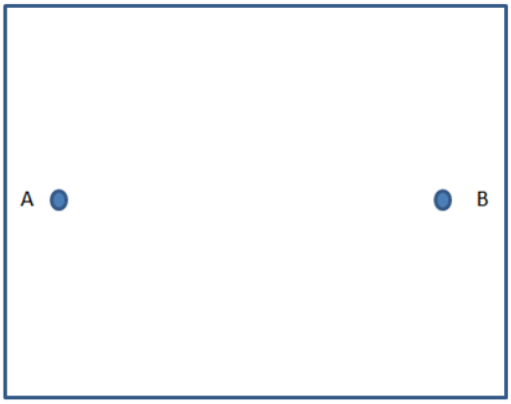
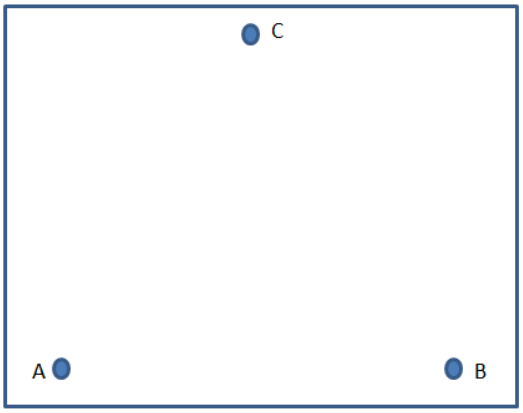
Voronoi Diagrams

Divide each rectangle into regions so that each region represents the collection of all the points for which the site (A, B or C) is the closest of all sites.





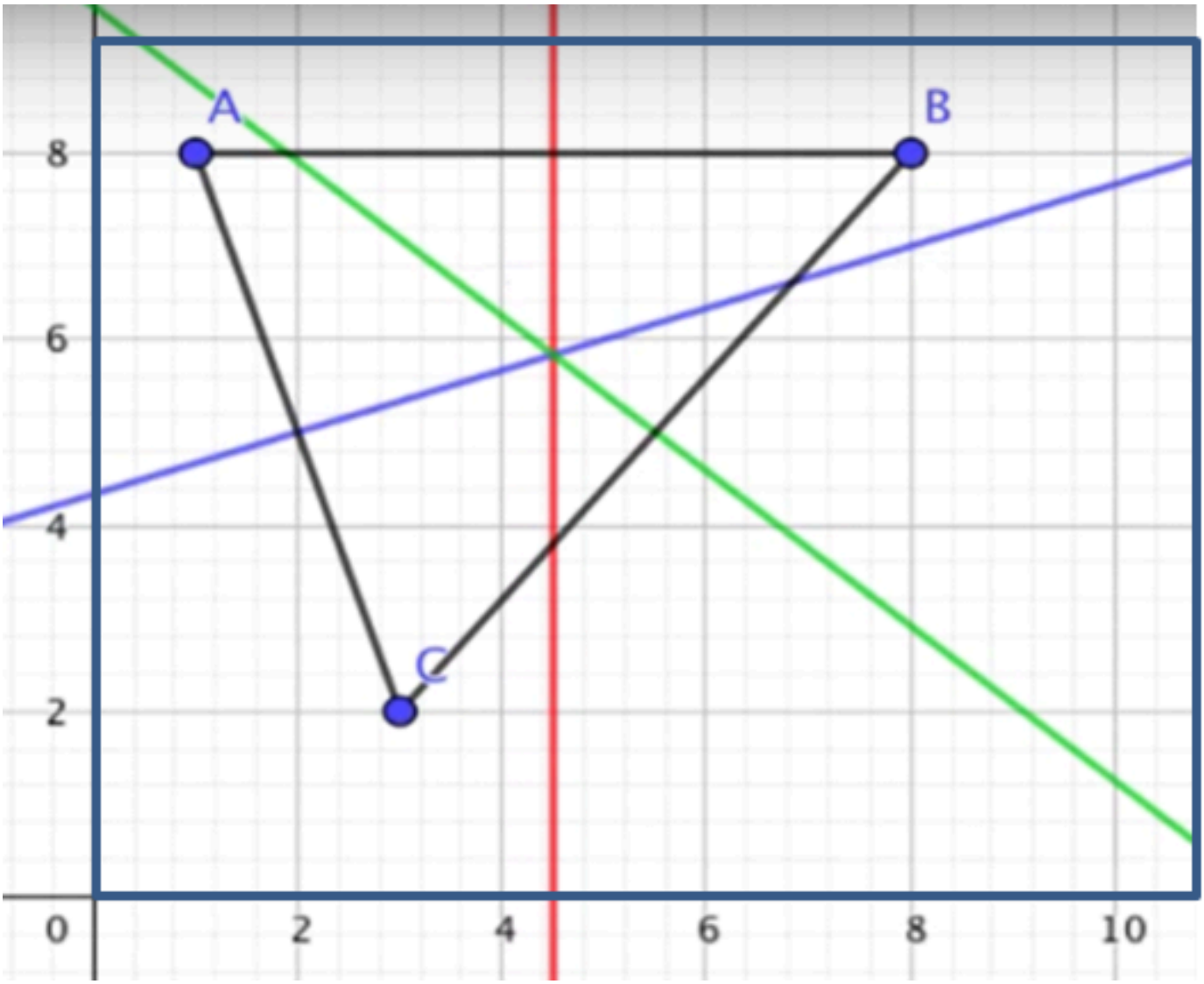
Voronoi Diagrams: Geometric Approach

Divide the waters around these three islands into Voronoi cells which fairly divide the waters by proximity to each island.

Inside this region there are 3 hospitals A, B, C. As all hospitals are equally good, ambulances are dispatched to the nearest one.

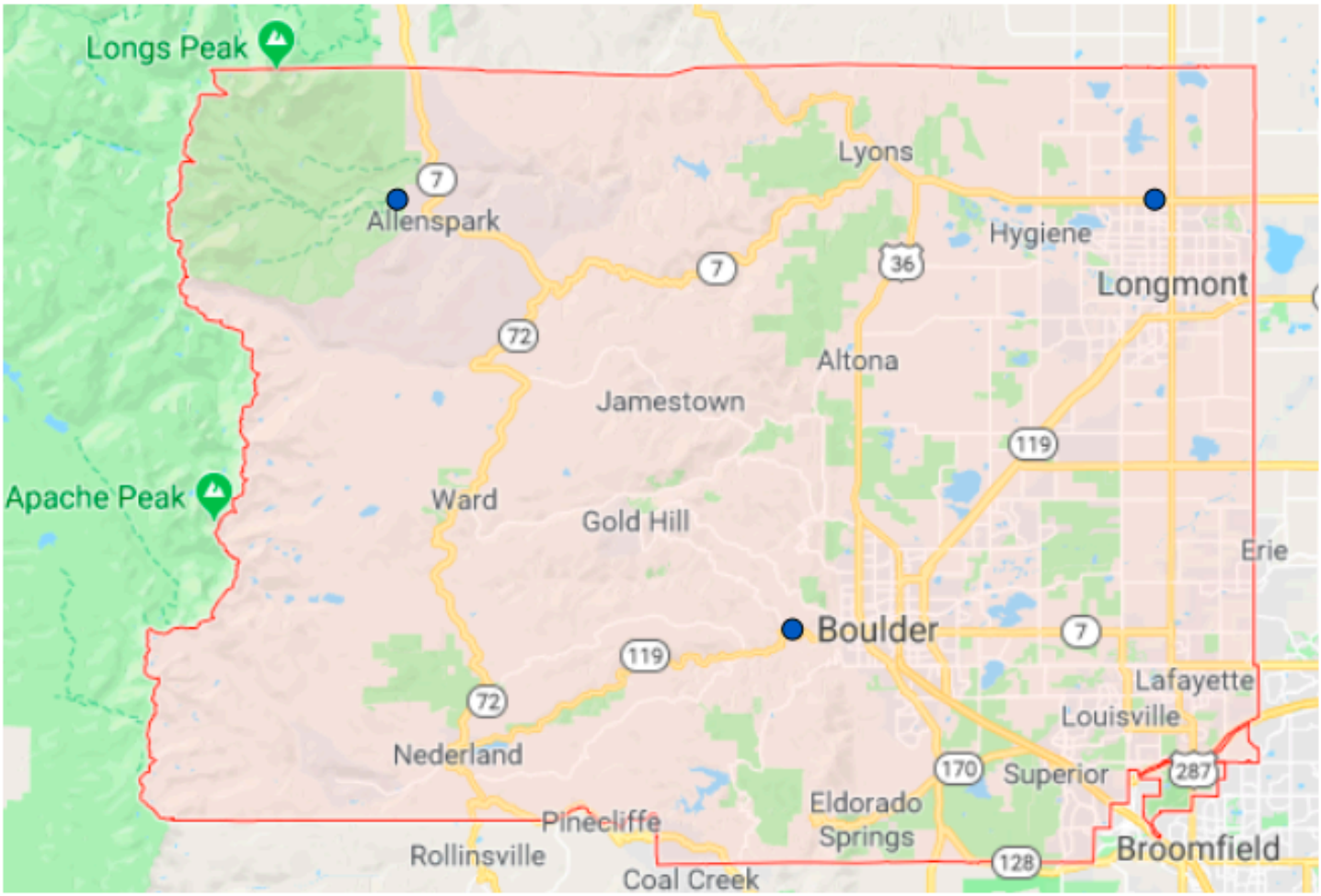
The three hospitals have been connected with segments and perpendicular bisectors for each of the three segments have been added to the diagram.

Using THREE colors, shade the the CELLS that surround each SEED in order to create a Voronoi Diagram that represents the regions closest to each of the three hospitals.



Voronoi Diagrams:
Geometric Construction Approach

Meteorologists in Boulder County, Colorado in the US track the cm of precipitation (rainfall and snowfall) received at three sites, marked on the map below:



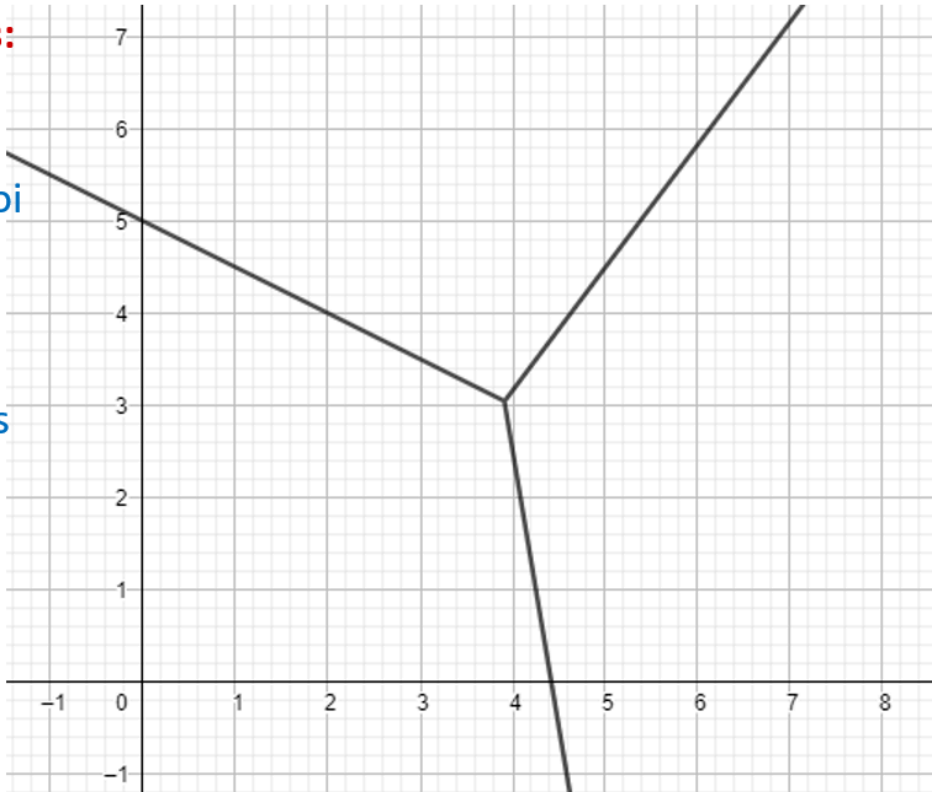
1. Which station would best predict the precipitation of Lyons? Nederland? Jamestown? Create a Voronoi diagram to support your answers.

2. The meteorologists get additional funding to place a fourth precipitation collection site.
a. If this station is to be within the area between the other three but as far as possible from all three, then where should it be placed?

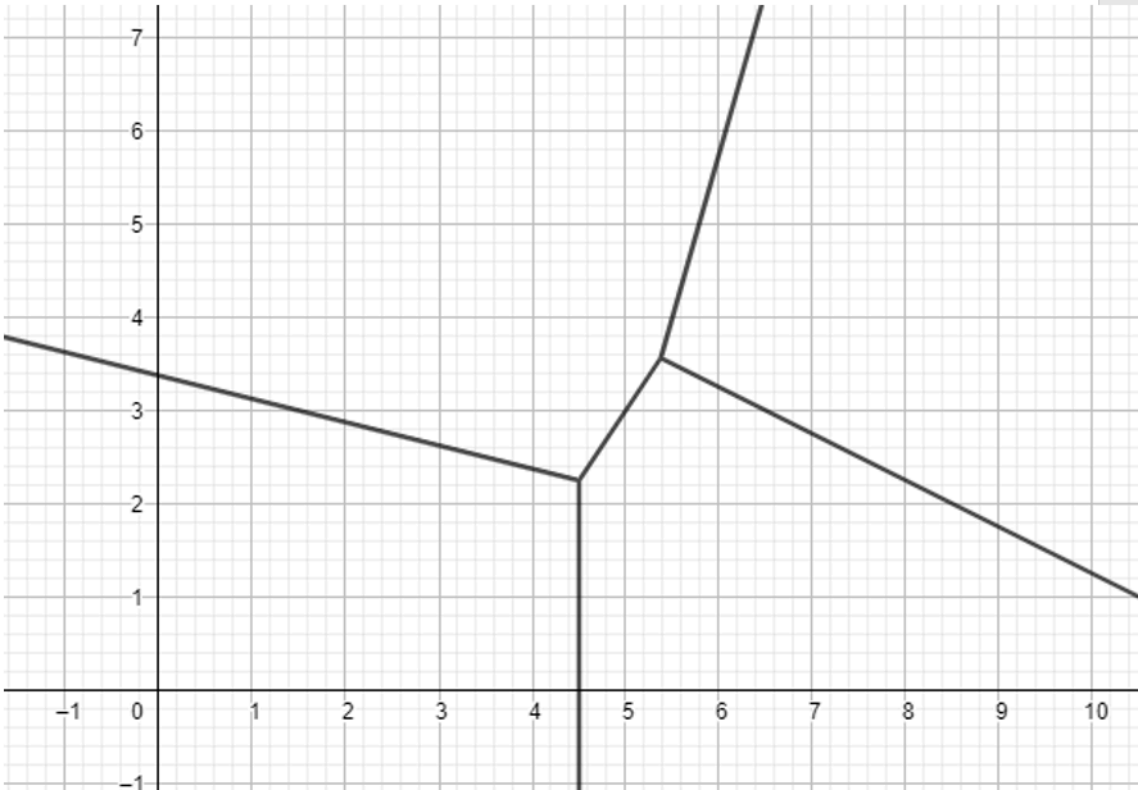
b. Where should they place this site if they want the most accurate data possible for the whole county?

Backwards:

Based on this Voronoi Diagram, where do you think the 3 seeds are ?

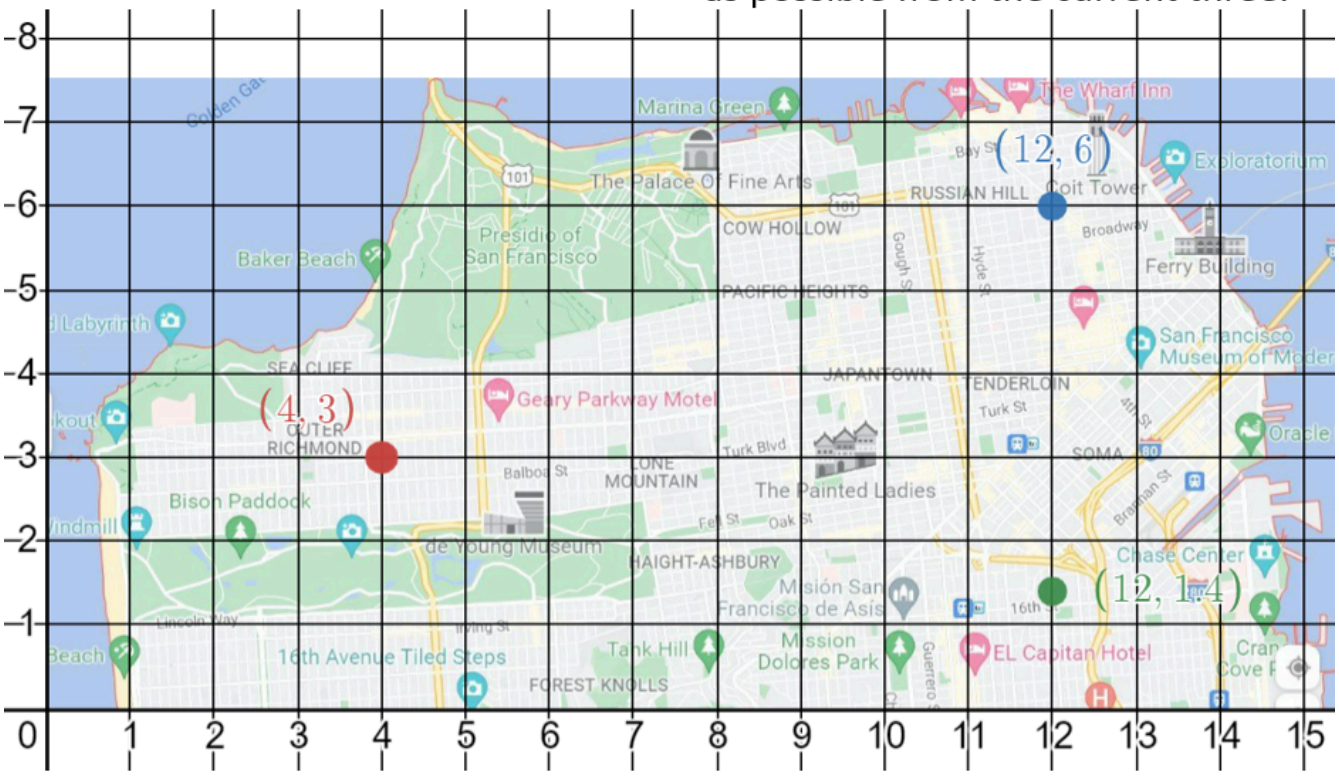


Backwards: Where are the 4 seeds ?



Voronoi Diagrams: Algebraic Approach

As an owner of three coffee shops in San Francisco, you want to open a fourth at a location that is as far as possible from the current three.



a. State the midpoint of AB.

b. State the equation of the perpendicular bisector of AB.

c. Find the midpoint of AC.

$$(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2})$$

d. Find slope of AC.

$$\frac{y_2 - y_1}{x_2 - x_1}$$

e. Create an equation for the perpendicular bisector of AC.

$$y - y_1 = m\,(x - x_1)$$

f. Repeat steps c. – e. in order to find the equation for the perpendicular bisector of BC.

g. Find the point that will be at the corner of all three cells (regions) of the Voronoi diagram.

h. Graph the lines to complete the Voronoi diagram and mark the proposed location, D.

Assessment Task: Applied Voronoi Diagrams

The purpose of this task is to practice the process of beginning a mathematical exploration. An exploration always begins with an interest or yours, an established context, and an aim (a question to be answered).

Unlike your mathematical exploration which will be very open-ended, this task is very structured.
Please follow the steps below carefully. Thank you! ☺

Steps:

PART 1:

1. Create a Google Document for this Voronoi Exploration task. Share this Document with me.

2. Decide on your **context**. What business or public institution, in Riga or Jurmala, do you want to investigate using Voronoi diagrams? Describe your context in several sentences.

3. Add an **aim** to the beginning of your report. When you write your true mathematical exploration, your aim will be completely your choice. In this practice exploration the aim should take this form: The aim of this

exploration is to find the point that is *furthest from the four plotted locations* of my business or institution in order to propose a new location for my business or institution.

4. Go to Google Maps and clip a portion of the map of Riga or Jurmala.

5. Go to **geogebra.org** and press "Start Calculator".

- Press the "+" button in order to import your clipplied map image
- Click on the map image and set partial transparency using the transparency tool () so that the grid can be seen through the map.
- Select **four** key locations of the business or public institution that you are aiming to investigate. Plot these points by typing in coordinates into Geogebra.
- Take a screenshot from Geogebra of your map and four coordinates. Import this screenshot into your Google doc report.
- Use the Voronoi tool (

) to create a Voronoi diagram. (Simply type "Voronoi" and then list your points in the parentheses).
- Clip an image of your GeoGebra voronoi diagram and import into your Google doc report.

Parts 2, 3 and 4 on next page

PART 2:

1. Now, **on grid paper**, that you will scan and include in your report, create this Voronoi diagram algebraically:

- Connect the four points to make a quadrilateral, using four segments
- Algebraically, find the midpoint of each segment
- Algebraically, find the slope of each segment
- Create a linear equation for the perpendicular bisector of each segment
- Graph these four perpendicular bisectors
- Create the Voronoi diagram

Do not forget to scan all this work you have done on grid paper and import it into your report.

2. Add your perpendicular bisector linear equations to your previously created Voronoi diagram in GeoGebra. Confirm that the graphed lines follow the edges of the Voronoi diagram cells exactly. Take a screenshot and include this in your report, as well.

PART 3:

On your grid paper, note the **two vertices** of your Voronoi diagram cells that are furthest from the four sites. There should be two points like this.

 By setting two perpendicular bisector equations equal to each other and solving, you should be able to find the exact locations of these vertices.

 Using these points, and the distance formula, find the distances from these vertices to your sites in order to decide which of the two points is furthest from your sites.

 Select the furthest point as a proposed new location for your business or institution.

PART 4:

1. Your business or institution likely has more than just four locations.

 Back in GeoGebra, add the locations of all (or many) other locations for your business or institution.

 Once the Voronoi diagram for all these sites has been created, snip an image of it and add it to your report.

2. Conclude your report by discussing what observations you can make by looking at this Voronoi diagram carefully. What can you say about the sizes of the cells, the locations, the parts of the city that have few sites, and so on.