Chain Rule and Implicit Differentiation Applications Writing Task

In this assignment, you will analyze composite functions and implicitly defined relationships using the differentiation techniques covered in this module. You'll explore how these methods apply to modeling growth, change, and related rates in practical scenarios.

The Assignment

Part 1: Analyzing a Complex Growth Model

You are working as a research analyst modeling the spread of information through social media. The number of people who have seen a particular viral post after *t* hours can be modeled by:

$$P(t) = \frac{M}{1 + Ae^{-kt}}$$

Where:

- *M* represents the maximum possible reach (total relevant audience size)
- *A* and *k* are positive constants that determine the initial conditions and growth rate
- *t* is time in hours since posting

This type of function is known as a logistic function and is commonly used in modeling population growth with limiting factors.

1. Using the chain rule and other differentiation techniques as needed, find *P'(t)*, the rate at which the post is spreading at time *t*.
2. Find the value of *t* where the rate of spread is at its maximum (Hint: this occurs when *P''(t)* = 0).
3. If *M* = 100,000, *A* = 999, and *k* = 0.5, determine:
    - The initial number of people who have seen the post (*P(0)*)
    - The maximum rate of spread (maximum value of *P'(t)*)
    - The time when this maximum rate occurs
4. Interpret the meaning of the second derivative *P''(t)* in this context. What does it tell us about the spreading behavior of the post?

Part 2: Implicit Differentiation and Related Rates

An engineering firm is designing a spherical storage tank that expands and contracts slightly with temperature changes. The relationship between the temperature *T* (in degrees Celsius) and the radius *r* (in meters) can be modeled by the equation:

$$r^3 + T^2 r - 100 = 0$$

1. Use implicit differentiation to find $\frac{dr}{dT}$, the rate of change of the radius with respect to temperature.
2. If the current radius is 4 meters and the temperature is 10°C, find the rate at which the radius changes when the temperature increases.
3. The volume of the tank is given by $V = \frac{4}{3}\pi r^3$. Find $\frac{dV}{dT}$, the rate of change of volume with respect to temperature.
4. If the temperature is increasing at a rate of 2°C per hour at the moment when T = 10°C and r = 4 meters, how fast is the volume changing at that moment?

Part 3: Applications and Extensions

1. Identify another real-world scenario (different from the ones in Parts 1 and 2) where:
   - The chain rule would be necessary to model rates of change
   - Implicit differentiation would be more appropriate than explicit differentiation
2. For your chosen scenario:
   - Write the mathematical equation(s) that model the situation
   - Explain what each variable represents in the real-world context
   - Demonstrate how you would apply the appropriate differentiation technique
   - Describe what the derivative represents physically in your context
3. Discuss a limitation of your mathematical model and how the calculus concepts we've studied might help address or identify these limitations.

Your submission should include:

- Clear, step-by-step solutions showing all derivatives
- Proper use of the chain rule, product/quotient rules, and implicit differentiation as appropriate
- Correct application of relevant formulas
- Thoughtful interpretations of the meaning of derivatives in context
- Proper units in your answers where applicable
- Well-explained connections between the mathematical concepts and real-world applications

This assignment is worth 20 points. Your work will be assessed on mathematical accuracy, proper application of calculus techniques, clear explanations of your process, and the correctness of your final answers.

Rubric:

Criteria	Proficient	Developing	Not Evident	Points
Growth Model Analysis	Correctly applies the chain rule and other differentiation techniques to find $P'(t)$ and $P''(t)$. Accurately determines maximum spread rate and time. Interprets derivatives with clear understanding of their physical meaning.	Most derivatives are correct with minor errors. Identification of key points contains minor errors or incomplete interpretation. Understanding of the physical meaning of derivatives is present but limited.	Multiple significant errors in derivatives. Maximum rate or time incorrectly calculated. Little or no connection made between derivatives and their physical meaning.	___/6
Implicit Differentiation Application	Correctly applies implicit differentiation to find $\frac{dr}{dT}$. Accurately calculates related rates including $\frac{dV}{dT}$. Shows clear work and correct units throughout.	Applies implicit differentiation with minor errors. Most related rate calculations contain the correct approach but may have algebraic errors. Some units may be missing or incorrect.	Major errors in implicit differentiation or related rates calculation. Work is disorganized or difficult to follow. Units consistently missing or incorrect.	___/6

Original Application and Extension	Creates a realistic, well-explained scenario that appropriately demonstrates the need for chain rule or implicit differentiation. Model demonstrates sophisticated understanding of the calculus concepts and their applications. Limitations analysis shows deep insight.	Scenario is plausible but may be oversimplified. Model demonstrates basic understanding of calculus concepts with some application. Limitations analysis is present but lacks depth.	Scenario is unrealistic or inappropriately modeled. Model shows significant misconceptions about calculus concepts. Limitations analysis is missing or superficial.	___/5
Mathematical Communication	Work is exceptionally organized and clear. Proper mathematical notation used throughout. Explanations thoroughly connect mathematical results to real-world context. All steps are shown with detailed reasoning.	Work is mostly organized with some notation errors. Explanations connect most results to context but may lack clarity in some areas. Most steps are shown but some reasoning may be unclear.	Work is disorganized or difficult to follow. Significant notation errors. Minimal connection between mathematical results and context. Steps are missing or reasoning is unclear.	___/3
Total				___/20