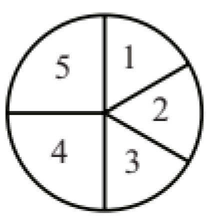
Probability (Week 9)

This week's problem set covers probability, particularly the topics of independent events, dependent events, geometric probability, conditional probability, and states. The problems span a wide range of difficulty, so attempt as many problems as you can. Enjoy!

If you are stuck on a problem or have questions, please email dengr557@gmail.com or michael.yh.voigt@gmail.com.
Once you have tried all the problems, check out the solutions to see how we solved them. Good luck!

Practice Problems

1. (2015 AMC 8) Each of two boxes contains three chips numbered 1,2,3. A chip is drawn randomly from each box and the numbers on the two chips are multiplied. What is the probability that their product is even?
2. (classic) Richard and Michael roll fair 6-sided die numbered 1 through 6. What is the probability that Richard rolls a higher number than Michael?
3. (2016 Mathcounts State) A spinner is divided into 5 sectors as shown. Each of the central angles of sectors 1 through 3 measures 60° while each of the central angles of sectors 4 and 5 measures 90°. If the spinner is spun twice, what is the probability that at least one spin lands on an even number? Express your answer as a common fraction.



4. (2017 AMC 8) A box contains five cards, numbered 1, 2, 3, 4, and 5. Three cards are selected randomly without replacement from the box. What is the probability that 4 is the largest value selected?
5. (2019 AMC 8) The faces of each of two fair dice are numbered 1, 2, 3, 5, 7, and 8. When the two dice are tossed, what is the probability that their sum will be an even number?
6. (2018 AMC 8) From a regular octagon, a triangle is formed by connecting three randomly chosen vertices of the octagon. What is the probability that at least one of the sides of the triangle is also a side of the octagon?

7. (2020 AMC 10A) A point is chosen at random within the square in the coordinate plane whose vertices are $(0,0), (2020,0), (2020,2020),$ and $(0,2020)$. The probability that the point is within d units of a lattice point is $\frac{1}{2}$. (A point (x,y) is a lattice point if x and y are both integers.) What is d to the nearest tenth?
8. (2020 Mathcounts Chapter) Ms. Pauling's chemistry class has 5 lab benches, each of which seats 2 students. If 6 students file into her otherwise empty classroom, and each student picks a random available open seat, what is the probability that at least one of the lab benches is completely empty? Express your answer as a common fraction.
9. (2020 AMC 10B) Ms. Carr asks her students to select 5 of the 10 books to read on her classroom reading list. Harold randomly selects 5 books from this list, and Betty does the same. What is the probability that there are exactly 2 books that they both select?
10. (2003 HMMT). Daniel and Scott are playing a game where a player wins as soon as he has two points more than his opponent. Both players start at par, and points are earned one at a time. If Daniel has a 60% chance of winning each point, what is the probability that he will win the game?

Challenge Problems

1. (2021 AMC 10B) A fair 6-sided die is repeatedly rolled until an odd number appears. What is the probability that every even number appears at least once before the first occurrence of an odd number?
2. (2021 AIME I) Zou and Chou are practicing their 100-meter sprints by running 6 races against each other. Zou wins the first race, and after that, the probability that one of them wins a race is ⅔ if they won the previous race but only ⅓ of they lost the previous race. The probability that Zou will win exactly 5 of the 6 races is m/n, where m and n are relatively prime positive integers. Find m+n.
3. (2020 AIME II) Let P be a point chosen uniformly at random in the interior of the unit square with vertices at $(0,0), (1,0), (1,1)$, and $(0,1)$. The probability that the slope of the line determined by P and the point (⅝,⅜)is greater than or equal to 1/2 can be written as m/n, where m and n are relatively prime positive integers. Find $m+n$.
4. (2021 AMC 10B) A square with side length 8 is colored white except for 4 black isosceles right triangular regions with legs of length 2 in each corner of the square and a black diamond with side length $2\sqrt{2}$ in the center of the square, as shown in the diagram. A circular coin with diameter 1 is dropped onto the square and lands in a random location where the coin is completely contained within the square, The probability that the coin will cover part of the black

region of the square can be written as $\frac{1}{196}(a + b\sqrt{2} + \pi)$, where a and b are positive integers. What is a+b?

5. (2014 AMC 10B) In a small pond there are eleven lily pads in a row labeled 0 through 10. A frog is sitting on pad 1. When the frog is on pad N, $0 < N < 10$, it will jump to pad N-1 with probability N/10 and to pad N+1 with probability 1 - N/10.. Each jump is independent of the previous jumps. If the frog reaches pad 0 it will be eaten by a patiently waiting snake. If the frog reaches pad 10 it will exit the pond, never to return. What is the probability that the frog will escape without being eaten by the snake?