1. quadratic; $3x^2$

2. Factor a polynomial by grouping when there are pairs of terms that have a common monomial factor.

3. It is written as a product of unfactorable polynomials with integer coefficients.

4. The Factor Theorem states that a polynomial has a factor $x - k$ if and only if $f(k) = 0$. When trying to factor a polynomial, the Factor Theorem can be used to determine if $x - k$ is a factor.

5. $x(x - 6)(x + 4)$

6. $4k^3(k - 5)(k + 5)$

7. $3p^3(p - 8)(p + 8)$

8. $2m^4(m - 8)(m - 4)$

9. $q^2(2q - 3)(q + 6)$

10. $r^4(3r + 4)(r - 5)$

11. $w^8(5w - 2)(2w - 3)$

12. $v^7(3v + 2)(6v + 7)$

13. $(x + 4)(x^2 - 4x + 16)$

14. $(y + 8)(y^2 - 8y + 64)$

15. $(g - 7)(g^2 + 7g + 49)$

16. $(c - 3)(c^2 + 3c + 9)$

17. $3h^6(h - 4)(h^2 + 4h + 16)$

18. $9n^3(n - 9)(n^2 + 9n + 81)$

19. $2t^4(2t + 5)(4t^2 - 10t + 125)$

20. $135z^8(z - 2)(z^2 + 2z + 4)$

21. $x^2 + 9$ is not a factorable binomial because it is not the difference of two squares; $3x^3 + 27x = 3x(x^2 + 9)$

22. The polynomial is not completely factored;

$$\begin{aligned} x^9 + 8x^3 &= (x^3)^3 + (2x)^3 \\ &= (x^3 + 2x)[(x^3)^2 - (x^3)(2x) + (2x)^2] \\ &= (x^3 + 2x)(x^6 - 2x^4 + 4x^2) \\ &= x^3(x^2 + 2)(x^4 - 2x^2 + 4) \end{aligned}$$

23. $(y^2 + 6)(y - 5)$

24. $(m^2 + 7)(m - 1)$

25. $(3a^2 + 8)(a + 6)$

26. $(2k^2 + 5)(k - 10)$

27. $(x - 2)(x + 2)(x - 8)$

28. $(z - 3)(z + 3)(z - 5)$

29. $(2q + 3)(2q - 3)(q - 4)$

30. $(4n - 1)(4n + 1)(n + 2)$

31. $(7k^2 + 3)(7k^2 - 3)$

32. $(2m^2 - 5)(2m^2 + 5)$

33. $(c^2 + 5)(c^2 + 4)$

34. $(y^2 - 7)(y^2 + 4)$

35. $(4z^2 + 9)(2z + 3)(2z - 3)$

36. $(9a^2 + 16)(3a - 4)(3a + 4)$

37. $3r^2(r^3 + 5)(r^3 - 4)$

38. $4n^2(n^5 - 6)(n^5 - 2)$

39. factor

40. not a factor

41. not a factor

42. not a factor

43. factor

44. factor

45.

−4	1	−1	−20	0
		−4	20	0
	1	−5	0	0

$g(x) = x(x + 4)(x - 5)$

46.

5	1	−5	−9	45
		5	0	−45
	1	0	−9	0

$t(x) = (x - 5)(x - 3)(x + 3)$

47.

$$\begin{array}{r|rrrrr} 6 & 1 & -6 & 0 & -8 & 48 \\ & & 6 & 0 & 0 & -48 \\ \hline & 1 & 0 & 0 & -8 & 0 \end{array}$$

$f(x) = (x - 6)(x - 2)(x^2 + 2x + 4)$

48.

$$\begin{array}{r|rrrrr} -4 & 1 & 4 & 0 & -64 & -256 \\ & & -4 & 0 & 0 & 256 \\ \hline & 1 & 0 & 0 & -64 & 0 \end{array}$$

$s(x) = (x + 4)(x - 4)(x^2 + 4x + 16)$

49.

$$\begin{array}{r|rrrr} -7 & 1 & 0 & -37 & 84 \\ & & -7 & 49 & -84 \\ \hline & 1 & -7 & 12 & 0 \end{array}$$

$r(x) = (x + 7)(x - 3)(x - 4)$

50.

$$\begin{array}{r|rrrr} -2 & 1 & -1 & -24 & -36 \\ & & -2 & 6 & 36 \\ \hline & 1 & -3 & -18 & 0 \end{array}$$

$h(x) = (x + 2)(x - 6)(x + 3)$

51. D; The x-intercepts of the graph are 2, 3, and -1.

52. C; The x-intercepts of the graph are 0, -2, -1, and 2.

53. A; The x-intercepts of the graph are -2, -3, and 1.

54. B; The x-intercepts of the graph are 0, 2, 1, and -2.

55. The model makes sense for $x > 6.5$; When factored completely, the volume is $V = x(2x - 13)(x - 3)$. For all three dimensions of the box to have positive lengths, the value of x must be greater than 6.5.

56. The model makes sense for $x > 3$; When factored completely, the volume is $V = (x - 1)(x - 3)(3x - 5)$. For all three dimensions of the cage to have positive lengths, the value of x must be greater than 3.

57. $a^4(a + 6)(a - 5)$; A common monomial can be factored out to obtain a factorable trinomial in quadratic form.

58. $(2m - 7)(4m^2 + 14m + 49)$; The difference of two cubes pattern can be used because the expression is of the form $a^3 - b^3$.

59. $(z - 3)(z + 3)(z - 7)$; Factoring by grouping can be used because the expression contains pairs of monomials that have a common factor. Difference of two squares can be used to factor one of the resulting binomials.

60. $2p^2(p^3 - 2)(p^3 - 4)$; A common monomial can be factored out to obtain a factorable trinomial in quadratic form where $u = p^3$.

61. $(4r + 9)(16r^2 - 36r + 81)$; The sum of two cubes pattern can be used because the expression is of the form $a^3 + b^3$.

62. $5x^3(x - 4)(x + 2)$; A common monomial can be factored out to obtain a factorable trinomial in quadratic form.

63. $(4n^2 + 1)(2n - 1)(2n + 1)$; The difference of two squares pattern can be used to factor the original expression and one of the resulting binomials.

64. $(3k^2 + 1)(3k - 8)$; Factoring by grouping can be used because the expression contains pairs of monomials that have a common factor.

65. **a.** no; $7z^4(2z + 3)(z - 2)$

b. no; $n(2 - n)(n + 6)(3n - 11)$

c. yes

66. 1 million

67. 0.7 million

68. *Sample answer:* 4; $\frac{x^3 - 3x^2 - 4x}{x - 4} = x^2 + x$

69. *Sample answer:* Factor Theorem and synthetic division; Calculations without a calculator are easier with this method because the values are lesser.

70. no; $f(x)$ may be factorable by factors other than $x - a$.

71. $k = 22$

7	2	−13	−22	105
		14	7	−105
	2	1	−15	0

72. $y = x(x + 2)(x - 2)$; Because 0, -2, and 2 are the x-intercepts, $(x - 0)$, $(x + 2)$, and $(x - 2)$ must be factors.

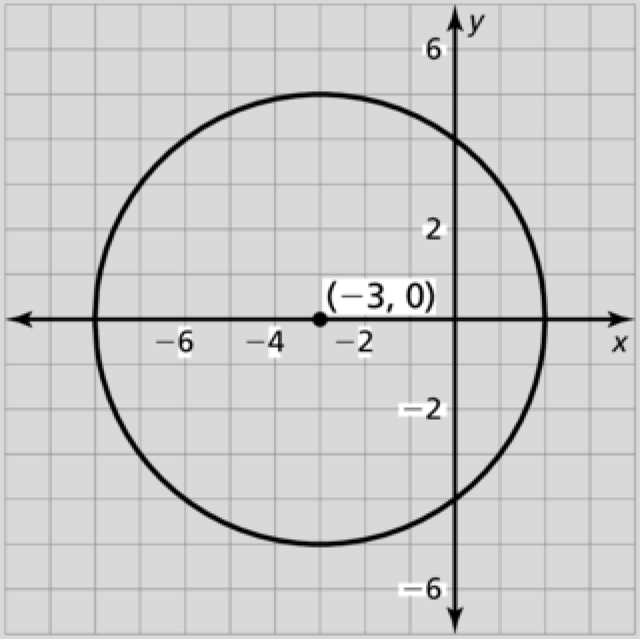
73. **a.** $(c - d)(c + d)(7a + b)$

b. $(x^n - 1)(x^n - 1)$

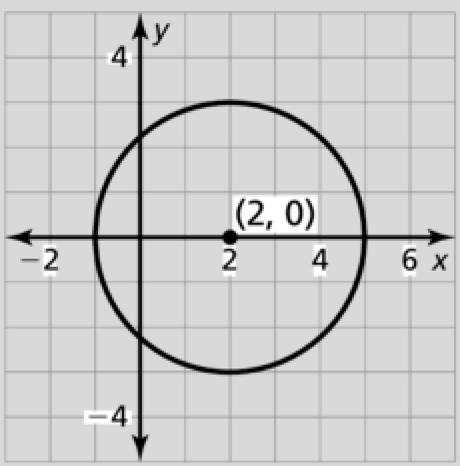
c. $(a^3 - b^2)(ab + 1)^2$

74. no; The function shows no x-intercepts. At least one x-intercept is needed to determine the factor used in synthetic division.

75. a. $(x + 3)^2 + y^2 = 5^2$; The center of the circle is $(-3, 0)$ and the radius is 5.



b. $(x - 2)^2 + y^2 = 3^2$; The center of the circle is $(2, 0)$ and the radius is 3.



76. **a.** If the volume of the missing block is included, the volume of the diagram is a^3 because the length, width, and height are all a. Because the length, width, and height of the missing piece are b, the volume of the missing block is b^3. Subtracting the volume of the missing block from the entire volume gives $a^3 - b^3$.

b. I: $a^2(a - b)$, II: $ab(a - b)$, III: $b^2(a - b)$

c. $a^3 - b^3 = a^2(a - b) + ab(a - b) + b^2(a - b)$

$= (a - b)(a^2 + ab + b^2)$

77. $x = 6$ and $x = -5$

78. $x = 9$ and $x = -4$

79. $x = \frac{5}{3}$ and $x = 2$

80. $x = \frac{1}{9}$ and $x = 3$

81. $x = 18$ and $x = -6$

82. $x = 4 \pm 3\sqrt{3}$

83. $x = -3$ and $x = -7$

84. $x = \dfrac{-9 \pm \sqrt{85}}{2}$