

Differential Equations (Applications to Finance)

1a) How much money will you have in 10 years if you make a one-time deposit of \$100 into a money market account that pays 8% interest compounded continuously?

b) How long will it take for the original \$100 to double?

2) You start a savings plan in which you make regular deposits that amount to \$1200 per year into an account that pays 6% interest compounded continuously? How much money will be in the account after 10 years?

3) You borrow \$20,000 for college. After graduation, you have 10 years to pay the loan off (assume the interest is deferred until graduation). If the interest rate is fixed at a continuous 7%, what will your regular yearly payments be? (*Of course, most loan payments are monthly, so divide your answer by 12 to get an idea of what your monthly payment will be.)

4) Now you are planning on buying your first home. After a down payment, you will need to finance (borrow) \$300,000 at a continuous interest rate of 4%, and you have 30 years to pay off the loan. What will be your yearly payments? (*Of course, most loans payments are monthly, so divide your answer by 12 to get an idea of what your monthly payment will be)

This model is easily extended to situations involving deposits or withdrawals in addition to the accrual of interest, dividends, or annual capital gains. If we assume that the deposits or withdrawals take place, at a constant rate k , then the value $S(t)$ is modeled by the initial value problem

$$\frac{dS}{dt} = rS + k \quad ; \quad S(0) = S_0.$$

- (2) A young person with no initial capital invests k dollars per year at an annual rate of return r . Assume that the investments are made continuously and that the return is compounded continuously.
- (a) Determine the sum $S(t)$ accumulated at any time t .

$$\left(S(t) = S_0 e^{rt} + \frac{k}{r} (e^{rt} - 1) \right)$$
- (b) If $r = 7.5\%$ determine k so that \$1 million will be available for retirement in 40 years.
 $(k = \$3929.68)$
- (c) If $k = \$2000/\text{year}$, determine the return rate r that must be obtained to have \$1 million available in 40 years.
 $(r = .09773)$
- (3) A certain college graduate borrows \$8000 to buy a car. The lender charges interest at an annual rate of 10%. Assuming that the interest is compounded continuously and the borrower makes payments continuously at a constant annual rate k , determine the payment rate k that is required to pay off the loan in 3 years. Also determine how much interest is paid during the 3 year period.
 $(k = \$3086.63 \text{ and this results in paying } \$1259.91 \text{ in interest over the 3 year period.})$
- (4) Suppose one opens an individual retirement account (IRA) at age 25 and makes annual investments of \$2000 thereafter in a continuous manner. Assuming a rate of return of 8%, what will be the balance in the IRA at age 65?
 $(\$58831.25)$
- (5) Since 1957 the stock market has averaged 10.56% per year. Suppose that money invested into the stock market earns interest compounded continuously at an interest rate of 10.56% per year. If \$1000 is initially invested, and we invest \$500 per year, assuming that the additional deposits are made continuously, how much money would we have invested after 40 years? How much of the amount in the stock market after 40 years is from the interest alone?
 $(\$319,795.86 \text{ is invested after 40 years. } \$298,795.86 \text{ was made from the interest.})$