STATWAY® COLLEGE MODULE 4 v4.1

TABLE OF CONTENTS:

Preparation 4.1

INTRODUCTION

In Module 3 we used *sampling distributions of sample proportions* to estimate or test a claim about a population proportion. In this module, we will focus on *sample means* from random samples. We begin by examining the variability of sample means that are randomly obtained from a population. We will use these ideas to learn about **sampling distributions of sample means**.

Exploring Samples of Acorn Weights

In Texas, oak trees are important to the overall health of the ecosystem. Acorns are the seeds of oak trees. Many birds and small mammals eat the acorns that come from oak trees. A decline in the growth of new oak trees could have a serious impact on local animal and plant life.

Botanists (scientists who study plant life) are especially interested in knowing the weights of acorns in a region. Botanists use information about the weights of acorns to help them predict the future growth of oak trees.

A group of students in Austin, Texas gathered acorns to study their weights (this data is in the 4.1-Supplement: Acorn Mass Table). Take a moment to look at the table at the end of this Preparation.

The table is a random assortment of 400 acorn weights, measured in grams, ranging from 0.7g to 6.7g. In this Preparation, you will take two samples of nine randomly selected acorns from this list.

Collect the Sample

1 Note that the acorns are numbered 1-400. Use a table of random digits or a random number generator (e.g. www.random.org/integers) to select nine integers between 1 and 400. Write down the nine integers here:

Here is an example of nine randomly generated integers between 1 and 400:

56	15	177
106	59	9
394	174	263

Here are the acorn weights that correspond to the example above:

56 - **4.2g**	15 - **3.2g**	177 - **3.9g**
106 - **4.6g**	59 - **3.5g**	9 - **3.9g**
394 - **4.6g**	174 - **3.3g**	263 - **2.6g**

In the example above, the sample mean is $\frac{4.2+3.2+3.9+4.6+3.5+3.9+4.6+3.3+2.6}{9} \approx 3.76\text{g}$

2 From your responses to Question 1, find the corresponding acorn weights by looking them up on the 4.1-Supplement: Acorn Mass Table. Write them down in the table below:

3 Calculate the mean weight for your sample of nine acorns. Round the sample mean to two decimal places.

4 Repeat this process to find a second sample mean.

- Randomly generate nine integers between 1 and 400.
- Find the nine corresponding acorn weights, and write them down.
- Calculate the mean of the nine acorn weights. Round the sample mean to two decimal places.

Second sample mean:

4.1-S: Acorn Mass Table (Supplement)

Acorn Mass (in grams)															
#	**mass**	**#**	**mass**	**#**	**mass**	**#**	**mass**	**#**	**mass**	**#**	**mass**	**#**	**mass**	**#**	**mass**
1	3.4	**51**	4.5	**101**	5.2	**151**	4.9	**201**	3.5	**251**	0.7	**301**	4.6	**351**	4.9
2	3.5	**52**	3.7	**102**	5.6	**152**	3.7	**202**	3.7	**252**	2.6	**302**	4.2	**352**	2.4
3	3.4	**53**	4.3	**103**	3.9	**153**	3.3	**203**	2.6	**253**	2.6	**303**	3.4	**353**	3.8
4	3.4	**54**	5.3	**104**	2	**154**	4.5	**204**	3.8	**254**	2.7	**304**	3.9	**354**	3.4
5	2.7	**55**	4.6	**105**	4.3	**155**	4.1	**205**	3.4	**255**	2.1	**305**	3.1	**355**	4.4
6	4.6	**56**	4.2	**106**	4.6	**156**	3.7	**206**	4.5	**256**	2	**306**	3.3	**356**	4.3
7	5.3	**57**	2.7	**107**	4.8	**157**	4.2	**207**	4	**257**	2.6	**307**	3.6	**357**	5.1
8	5.3	**58**	4.4	**108**	6.1	**158**	3.7	**208**	5.1	**258**	2.5	**308**	4.2	**358**	3.4
9	3.9	**59**	3.5	**109**	3.3	**159**	4.2	**209**	3	**259**	3.7	**309**	4.1	**359**	5.1
10	5	**60**	4.8	**110**	4.5	**160**	5	**210**	1.6	**260**	2.5	**310**	4.6	**360**	2.6
11	3.7	**61**	5	**111**	2.5	**161**	3.6	**211**	3.2	**261**	0.8	**311**	3.2	**361**	5.4
12	4.7	**62**	4.3	**112**	4.4	**162**	4.5	**212**	5.4	**262**	4.4	**312**	3.5	**362**	2.7
13	2.3	**63**	2.6	**113**	3.4	**163**	4.1	**213**	4.9	**263**	2.6	**313**	4.9	**363**	2
14	3.1	**64**	3.6	**114**	4.2	**164**	3.4	**214**	2.7	**264**	3.8	**314**	4.2	**364**	3.6
15	3.2	**65**	4.6	**115**	4.8	**165**	3.8	**215**	2.9	**265**	5.2	**315**	3.5	**365**	2.1
16	6.6	**66**	4.6	**116**	3.8	**166**	3.7	**216**	3.6	**266**	2.6	**316**	4	**366**	2.3
17	6.1	**67**	3.3	**117**	2.6	**167**	4.2	**217**	3.1	**267**	3.5	**317**	3.8	**367**	3.3
18	5	**68**	3.8	**118**	4.3	**168**	5.3	**218**	3	**268**	3.3	**318**	4.7	**368**	3.2
19	3.4	**69**	5.4	**119**	3.2	**169**	4	**219**	4.5	**269**	2.6	**319**	3.6	**369**	4.3
20	3.8	**70**	4.4	**120**	4.2	**170**	3.5	**220**	3.7	**270**	3.3	**320**	3.5	**370**	3.7
21	3.4	**71**	1.2	**121**	4.2	**171**	3.7	**221**	3.8	**271**	5.4	**321**	2.9	**371**	3.6
22	3.2	**72**	4.7	**122**	3	**172**	3.9	**222**	3.8	**272**	2.6	**322**	4.9	**372**	3.9
23	4.1	**73**	6.3	**123**	3.2	**173**	2.9	**223**	3.2	**273**	3.7	**323**	5.3	**373**	3.3
24	4.4	**74**	2.5	**124**	3.8	**174**	3.3	**224**	3.3	**274**	4	**324**	4	**374**	4.4
25	4.2	**75**	3.4	**125**	3	**175**	4.1	**225**	2.7	**275**	4.2	**325**	2.9	**375**	4.8
26	3.1	**76**	5.1	**126**	5	**176**	4.5	**226**	3.7	**276**	3.6	**326**	3.9	**376**	4.8
27	3.5	**77**	2.3	**127**	5	**177**	3.9	**227**	3.9	**277**	2.9	**327**	3.5	**377**	3.7
28	0.9	**78**	5.6	**128**	6.3	**178**	3.8	**228**	3.8	**278**	0.7	**328**	3.3	**378**	5.8
29	5.9	**79**	5.7	**129**	3.3	**179**	3.3	**229**	3.1	**279**	4.4	**329**	2.8	**379**	3.7
30	4.1	**80**	6.7	**130**	3.5	**180**	3.4	**230**	3.7	**280**	2.7	**330**	2.9	**380**	4.3
31	3.3	**81**	4.2	**131**	5.1	**181**	3.9	**231**	3.5	**281**	4.3	**331**	3.5	**381**	2.4
32	4	**82**	4.1	**132**	5.2	**182**	3.5	**232**	3.6	**282**	4	**332**	3.1	**382**	3.5
33	4.6	**83**	4.7	**133**	4.1	**183**	3.3	**233**	2	**283**	3.6	**333**	4	**383**	2.6
34	5.4	**84**	5.7	**134**	5.1	**184**	2.3	**234**	2.7	**284**	1.4	**334**	3.5	**384**	4.6
35	4.2	**85**	5.1	**135**	3.1	**185**	3.9	**235**	4.7	**285**	3.5	**335**	4.3	**385**	5.7

Acorn Mass (in grams)													
36	4	**86**	4.2	**136**	4.6	**186**	3.5	**236**	0.9	**286**	3.3	**336**	3	**386**	4.5
37	3.2	**87**	4.9	**137**	3.9	**187**	2.5	**237**	2.7	**287**	3.7	**337**	2.7	**387**	3.4
38	5.7	**88**	1.4	**138**	4	**188**	3.5	**238**	4	**288**	2.3	**338**	2.1	**388**	4.1
39	2.5	**89**	4.6	**139**	4	**189**	5.1	**239**	3.4	**289**	3.6	**339**	3.7	**389**	3.2
40	4	**90**	3.5	**140**	5.1	**190**	3.8	**240**	3	**290**	5	**340**	2.3	**390**	3.8
41	5	**91**	5.3	**141**	4.2	**191**	2.4	**241**	3.5	**291**	2	**341**	4.1	**391**	4.1
42	2.9	**92**	5.1	**142**	4.4	**192**	3.2	**242**	3.7	**292**	4.6	**342**	3.9	**392**	2.4
43	4.3	**93**	2	**143**	2.5	**193**	2.6	**243**	4.3	**293**	2.9	**343**	3.3	**393**	3.1
44	3.7	**94**	4.8	**144**	4	**194**	2.8	**244**	4.6	**294**	4.5	**344**	4.2	**394**	4.6
45	5.1	**95**	4.6	**145**	2.5	**195**	4.4	**245**	3	**295**	3.5	**345**	4.2	**395**	3.3
46	2.2	**96**	4.6	**146**	4	**196**	4.1	**246**	3.3	**296**	3.2	**346**	1	**396**	4.9
47	4.4	**97**	4.9	**147**	3.3	**197**	3.9	**247**	1	**297**	2.6	**347**	3.1	**397**	3
48	5.1	**98**	4.9	**148**	3.4	**198**	3.6	**248**	1.2	**298**	3.6	**348**	1.9	**398**	2.6
49	3.8	**99**	4.2	**149**	3	**199**	4.5	**249**	2.6	**299**	3.8	**349**	3.7	**399**	2.2
50	4.4	**100**	4.3	**150**	5.7	**200**	2.7	**250**	2.9	**300**	4.4	**350**	3.8	**400**	3.1

4.1: Sampling Distributions of Sample Means

LEARNING GOALS

By the end of this collaboration, you should understand that:

- Sample means will vary from one sample to another when sampling from quantitative data.
- When the size of the random sample increases, the variability among sample means decreases.
- When the size of the random sample increases, the sample mean tends to provide a better estimate of the population mean.
- A sampling distribution of sample means is the collection of all possible sample means from random samples of the same size.
- The shape of the sampling distribution of sample means is normal if the population that is being sampled from is normal or if the sample size is 30 or more.
- The mean of a sampling distribution of sample means is the population mean.

$$\mu_{\bar{x}} = \mu$$

- The standard error of a sampling distribution of sample means is the population standard deviation divided by the square root of the sample size.

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

By the end of this collaboration, you should be able to:

- Use a simulation to describe the center, shape and spread of a distribution of sample means.
- Given population parameters and a sample size, determine the mean and standard error of a sampling distribution of sample means.
- Find the *Z*-score of a sample mean and the probability associated with a sample mean.

INTRODUCTION

In Module 3 we used *sampling distributions of sample proportions* to estimate or test a claim about a population proportion. In this module, we will focus on *sample means* from random samples. In Preparation 4.1, each of you calculated two sample means for two samples of nine randomly selected acorns. In Module 4 we will examine distributions of sample means. This collaboration begins by exploring the difference between comparing individuals in a population versus comparing sample means from a population.

TRY THESE

Comparing Individual Data Values Versus Sample Means

1 Let's start by comparing individual acorns. Each group member took two samples of nine individual acorns, which means you each selected 18 acorns. Between all of you, which individual acorn weight was the lowest?

2 Between all of you, which individual acorn weight was the greatest?

3 Calculate the range of the individually selected acorn weights in your combined samples. What is the difference between the greatest and lowest individual acorn weights in your combined samples?

Your group just compared the weight differences of individual acorns in the samples. Now we move to comparing sample means.

4 Each group member calculated two sample means. Between all of you, which sample mean was the lowest?

5 Between all of you, which sample mean was the greatest?

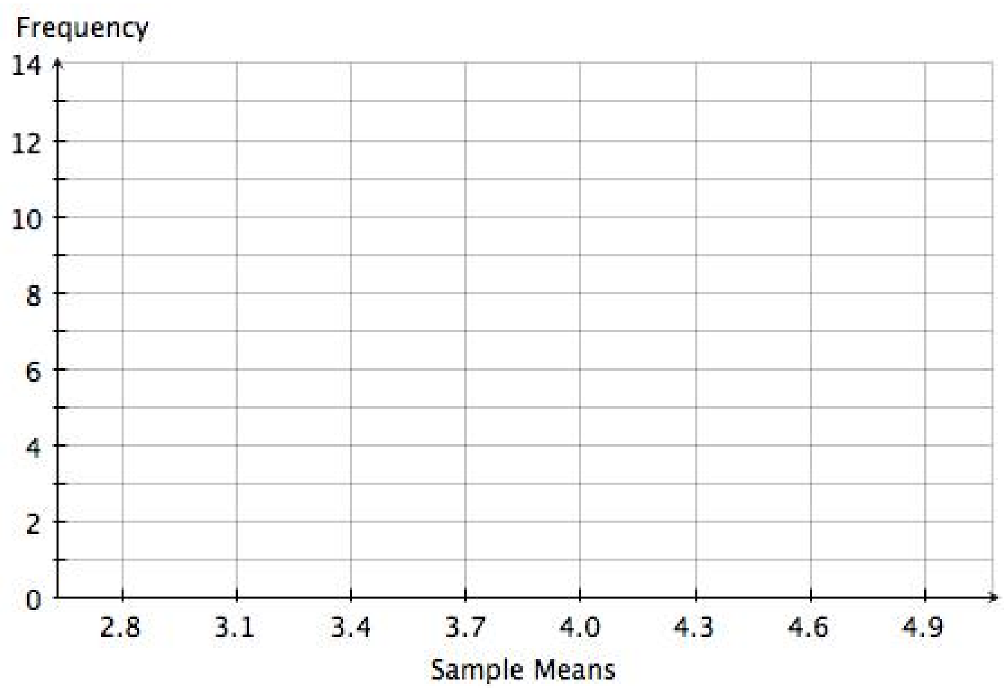
6 Calculate the range of the sample means. What is the difference between the greatest and lowest of the group's sample means?

There is significantly more variation between individual acorn weights than there is between sample means from random samples of 9 acorns. Sample means are closer together than individual weights. When we collect sample means, we use them to estimate the population mean, not individual weights.

7 Submit your sample means to your instructor who will compile a frequency distribution of sample means from all students in the class. Complete the table below using the class data. Recall that the frequency of a bin is the number of data values (sample means) in the bin. **Note**: If completing this problem online, follow the instructions given online.

Bin	Tally	Frequency
$2.8 \leq x < 3.1$		
$3.1 \leq x < 3.4$		
$3.4 \leq x < 3.7$		
$3.7 \leq x < 4.0$		
$4.0 \leq x < 4.3$		
$4.3 \leq x < 4.6$		
$4.6 \leq x < 4.9$		

8 Use the frequency distribution from Question 7 to create a histogram of the sample means. Each bin is represented by a vertical bar. The height of a bar is the frequency for the bin. Draw a bar for each bin. **Note**: If completing this problem online, follow the instructions given online.



9 The histogram you generated for Question 8 is a distribution of sample means. While samples vary, and sample means vary, there is only one population mean. How could you use this distribution to make an estimate for the mean of the population, that is, the mean of the 400 acorns? (*Hint*: Think about what you can see in the distribution that might help you to make a good estimate.)

What is your estimate of the mean weight of the *individual* acorns in the study?

10 Suppose we want to improve upon our estimate of the mean weight of *individual* acorns collected by the students in Austin, TX. How could we use a sampling process to establish a more accurate estimate?

11 Sample means are estimates of the population mean, so a distribution of sample means is a distribution of estimates. Any deviation that exists between a sample mean and the population mean is an error. This is why we call the standard deviation of all sample means the *standard error*. Estimate the standard error of the distribution of sample means from the previous problem.

In the previous problem, we used a distribution of sample means to estimate a population mean. We also estimated the standard error (standard deviation) of sample means using the Empirical Rule. The Central Limit Theorem below details how the sampling distribution of sample means relates to the population parameters and the size of random samples.

YOU NEED TO KNOW

The **Central Limit Theorem for Sample Means** states that:

> Given *any* population with mean μ and standard deviation σ, the sampling distribution of sample means (sampled with replacement) from random samples of size *n* will have a distribution that approaches normality with increasing sample size.

The mean and standard error of the sampling distribution are:

$$\mu_{\bar{x}} = \mu \text{ and } \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

The criteria for the approximate normality of a sampling distribution are that either the population from which we are sampling is normal, or the sample size is greater than 30. Very non-normal populations may require samples substantially larger than 30.

NEXT STEPS

Applying the Central Limit Theorem

We will now shift our attention from distributions of sample means to the sampling distribution of sample means. The sampling distribution of sample means is the distribution of *all* possible sample means from random samples of the same size.

When a sampling distribution of sample means is approximately normal, we can use its mean and standard error to find the *Z*-score of any particular sample mean. The *Z*-score of a sample mean $\bar{x}$ from a sample of size *n* is found by the formula:

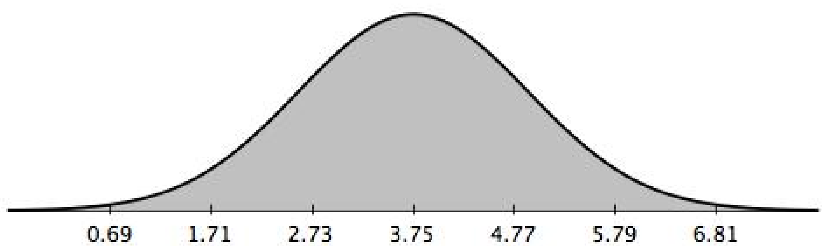
$$Z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

Earlier in this collaboration, we explored a population distribution of acorn weights from oak trees. Since we define these 400 acorns as the population, we can calculate the population mean and standard deviation. The population of acorn weights are normally distributed with a mean weight of 3.75 grams and a standard deviation of 1.02 grams.

The population parameters are: μ = 3.75 and σ = 1.02.

The population of acorn weights is displayed in the graph below.

Population of Acorn Weights



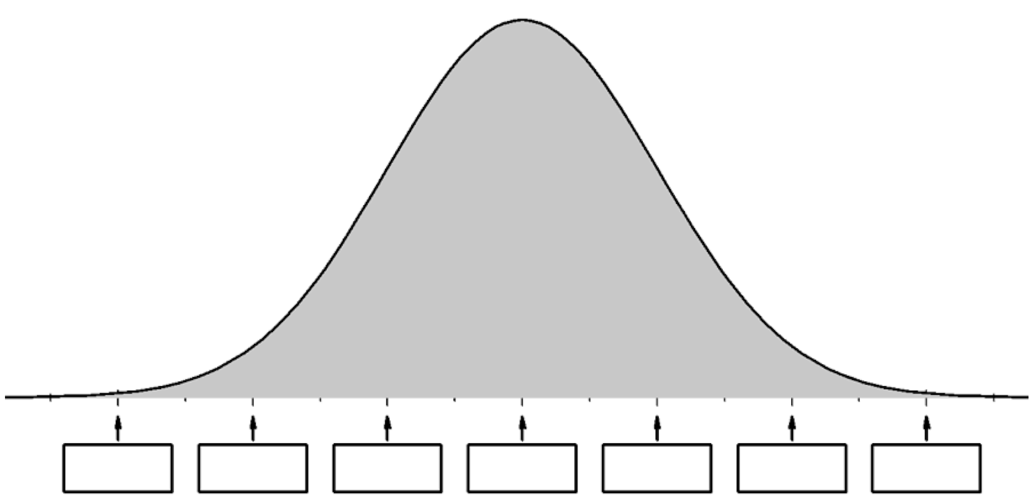
Suppose that we randomly select samples of 40 acorn weights. According to the Central Limit Theorem, since the population is normally distributed, sample means from random samples of size 40 will also be normally distributed.

12 The sample size is n = 40. Use the Central Limit Theorem to calculate the mean and standard error of the sampling distribution. Round the standard error to three decimal places.

A $\mu_{\bar{x}} = \mu =$ __________

B $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} =$ ________

13 We can use a normal curve to represent the sampling distribution of sample means. The boxes under the normal distribution below are one standard error apart, with the center box at the mean. Use the mean and standard error above to enter the correct values into the boxes. Use the standard error, rounded to 3 decimal places.



14 A particular random sample of size 40 has a mean weight of 4 grams. Plot this sample mean on the axis above. Find the *Z*-score for the sample mean, $\bar{x} = 4$. Round the *Z*-score to two decimal places.

15 Would you consider this sample mean to be unusual? Be sure to explain your answer.

16 If a sample of 40 acorns from this population is randomly selected, what is the probability that its mean weight $\bar{x}$ will be greater than 4 grams? Write your answer as a decimal rounded to three places.

LET'S SUMMARIZE

Please consider the following key points:

- The Central Limit Theorem for sample means states that a sampling distribution of sample means is approximately normal if the population from which we are sampling is normal, or the sample size is greater than 30.
- When the criteria for approximate normality are satisfied, the normal distribution may be used to determine probabilities about sample means.
- The mean and standard error (or standard deviation) for the sampling distribution of sample means are given by:

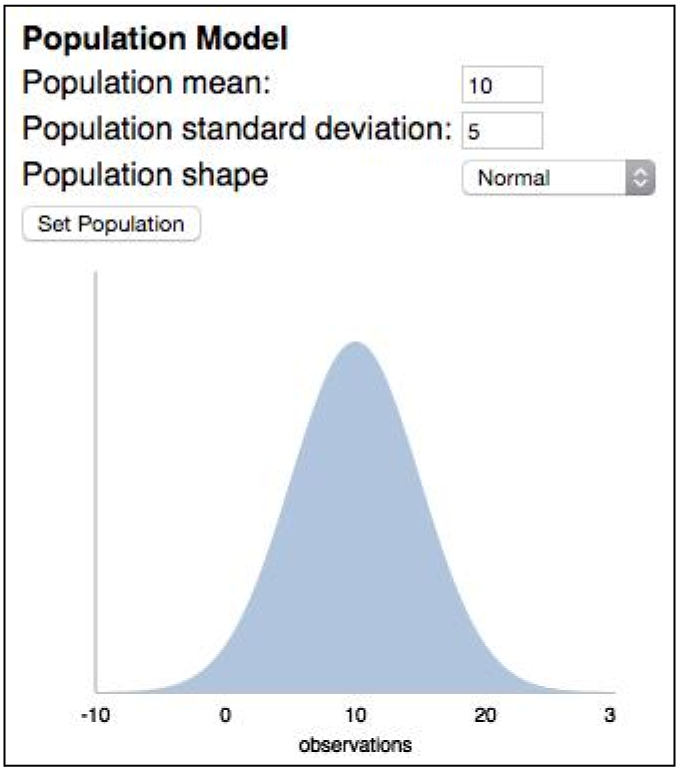
$$\text{Mean: } \mu_{\bar{x}} = \mu$$

$$\text{Standard error: } \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

Exercise 4.1

Go to the website: https://carnegiemathpathways.org/go/rossonesampling

This applet on this website simulates sampling distributions from specified populations. The population that is first displayed in this simulation is a normal distribution with a mean of 10 and standard deviation of 5. This is shown below.

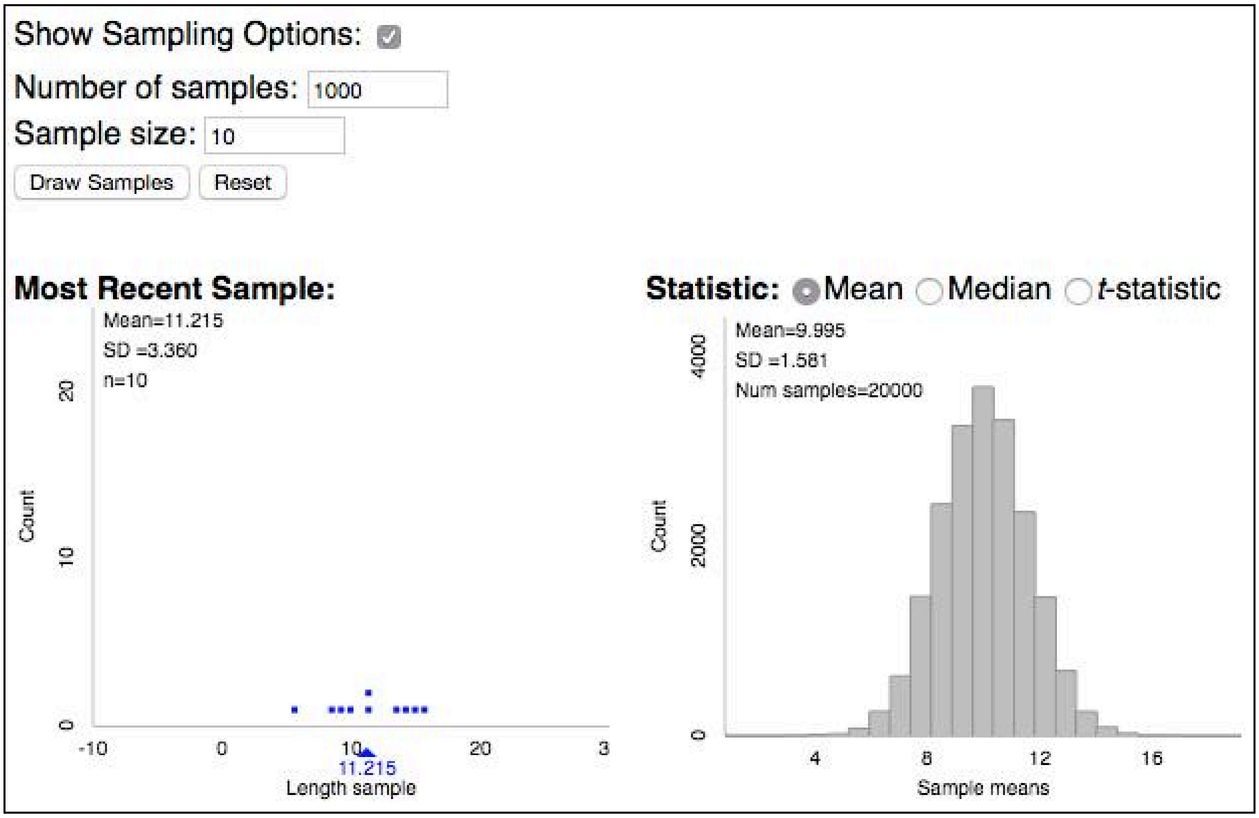


By selecting "Show Sampling Options", you can simulate random sampling of the population. The sample means of these simulated samples will be displayed in the "Statistics" plot.

To use the simulation:

- Select "Show Sampling Options"
- Set the Number of Samples to 1000
- Set the Sample size (n) to 10
- Click "Draw Samples" to plot the distribution of sample means in the plot on the right. The middle plot displays the most recent sample created by the simulation.

The screenshot below shows an example of simulation results for 1000 samples of size n = 10.



The histogram on the right shows the distribution of sample means calculated from each of the 1,000 samples of size 10. These sample means have a mean of 9.995 and a standard deviation of 1.58. Recall that the standard deviation of a distribution of sample means is called the standard error. The standard error of the sample means, in this case, is 1.58.

The dotplot on the left shows the most recent random sample created by the simulation. This simulated sample has a sample mean of 11.22 and a sample standard deviation of 3.36. The dotplot enables us to view the actual 10 values in the sample. To view a different sample, click on a sample mean in the histogram.

You may use the "Reset" button to clear the distribution at any time.

1 Run the simulation for the five sample sizes in the table below. Set "Number of samples" to 1000. Enter in each sample size into the input box labeled "Sample size." Then click "Draw Samples" to create the distribution of sample means.

Fill in the table below with the means and standard errors of the distribution of sample means. This will be the information from the histogram on the right in the simulation. You may round the mean and standard errors to two decimal places.

	Sample Size				
	2	**5**	**10**	**16**	**25**
Mean of Sample Means					
Standard Error					

2 Describe how increasing the sample size affects the mean of sample means and the standard error.

Sampling from a Skewed-Right Population

The Central Limit Theorem states that the sampling distribution of sample means, sampled with replacement from random samples of size *n,* from *any* population, will have a distribution that approaches normality with increasing sample size.

We can investigate this by sampling from a population with a skewed-right distribution. Let's simulate 1,000 random samples using three different sample sizes (n = 2, n = 10, n = 25).

- In the Population Model section, set "Population shape" to **Skewed right**.
- Set the Number of samples to 1000.
- Set the Sample size (n) to 2.
- Click "Draw Samples" to plot the distribution of sample means.

3 A What is the lowest sample mean in the distribution of sample means?

B What is the greatest sample mean in the distribution of sample means?

C What is the shape of the distribution of sample means?

D Now increase the sample size to 10.

- Keep the “Population shape” and number of samples as “Skewed right” and “1000”
- Change the Sample size (n) to 10.
- Click “Draw Samples” to plot the distribution of sample means.

What is the lowest mean in the distribution of sample means?

E What is the greatest sample mean in the distribution of sample means?

F What is the shape of the distribution of sample means?

G Now let’s up the sample size to 25.

- Keep the “Population shape” and number of samples as “Skewed right” and “1000”
- Change the Sample size (n) to 25.
- Click “Draw Samples” to plot the distribution of sample means.

What is the lowest sample mean in the distribution of sample means?

H What is the greatest sample mean in the distribution of sample means?

I What is the shape of the distribution of sample means?

J What happened to the range of the distribution of sample means as the sample size increased from n = 2 to n = 25?

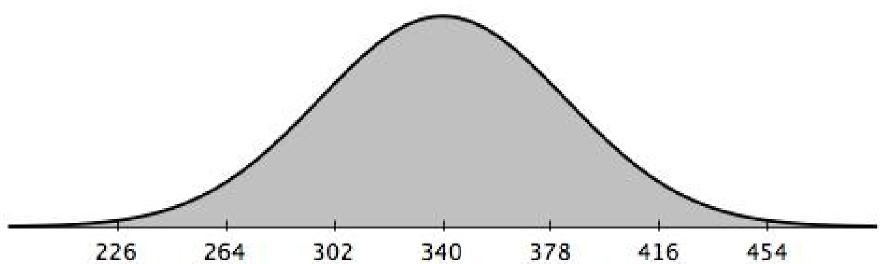
K The mean and standard deviation for the population were set as 10 and 5, respectively. Look at the mean and standard error for the distribution of sample means when n = 25. Write a statement that best describes the comparison between the population and the distribution of sample means.

Applying the Central Limit Theorem for Sample Means

African lions are magnificent animals which capture the interest and imagination of people throughout the world. In response to dire concerns about the low number of lions roaming in western and central Africa, in 2015, the U.S. Fish and Wildlife Service recognized African lions as an endangered species. Scientists fear that a lack of action to save African lions could result in the species going extinct.

African lions are impressively large animals. Biologists estimate that African lions have weights that typically range from 265 pounds to 420 pounds[1]. Let's assume that the weights of African lions are normally distributed with mean 340 pounds and standard deviation 38 pounds. This population of African lion weights is displayed in the graph below.

Population of African Lion Weights



Suppose that we randomly select samples of 30 African lion weights. According to the Central Limit Theorem, since the population is normally distributed, sample means from random samples of size 30 will also be normally distributed.

4 A The sample size is n = 30. Use the Central Limit Theorem to calculate the mean of the sampling distribution.

$\mu_{\bar{x}} = \mu =$ __________

[1] https://www.livescience.com/27404-lion-facts.html

B The sample size is n = 30. Use the Central Limit Theorem to calculate the standard error of the sampling distribution. Round the standard error to two decimal places.

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \_\_\_\_\_\_\_\_\_\_$$

C A particular random sample of size 30 has a mean weight of 325 pounds. Find the *Z*-score for the sample mean, $\bar{x} = 325$. Round the *Z*-score to two decimal places.

$$Z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \_\_\_\_\_\_\_\_\_\_$$

D Write a statement that best describes the meaning of this *Z*-score.

E If a sample of 30 African lions from this population is randomly selected, what is the probability that its mean weight $\bar{x}$ will be less than 325 pounds? Round your answer to three decimal places.

4.1 Monitor (survey)

Monitor your progress on learning the objectives for this unit. If you identify any objectives you need to review, go back through the unit’s activities. Your responses are not graded, but will be available for your instructor to see.

Rate how confident you are on a scale of 1–5 (1 = not confident and 5 = very confident).

Skill or Concept: I can …	**Rating from 1 to 5**
Use a simulation to describe the center, shape and spread of a distribution of sample means.	
Given population parameters and a sample size, determine the mean and standard error of a sampling distribution of sample means.	
Find the *Z*-score of a sample mean and the probability associated with a sample mean.	

Preparation 4.2

We begin Unit 4.2 with a description of **Student's *T*-distribution**. As you read the description below, take notes of questions that come up so you can share your questions and get support from your group.

Previously, we have seen that under certain conditions the *sampling distribution of sample means* is normally distributed. As with any normal distribution, we can calculate *Z*-scores if we know the *population standard deviation*. But what happens when we don't know the population standard deviation?

Often we do not know the population standard deviation. In this unit, we will confront this reality. Not knowing the population standard deviation influences the way we perform statistical inference regarding a population mean.

When σ is Unknown

In most situations we do not know the population standard deviation. In other words, σ *is unknown*. The only option available to us is to approximate σ with a *sample standard deviation*, s. To do this we need to substitute s for σ.

When we make this substitution, the standard error of the sampling distribution *is estimated by*

$$\frac{s}{\sqrt{n}}$$

The test statistic for a sample mean is

$$T = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

Note that these equations are very similar to equations introduced in Unit 4.1, but we use a sample standard deviation in place of the population standard deviation.

Since the sample standard deviation (s) varies from sample to sample and only approximates σ, it introduces additional variability in the test statistic. The test statistic actually varies according to what we call **Student's *T*-distribution**.

Language Tip

Note that instead of saying "the standard error of the sampling distribution *is given by*", we say it "*is estimated by*". We make this change because s is only an approximation of σ.

Language Tip

The *T*-distribution was developed by William Gosset in the early 1900's and published under Gosset's pseudonym "Student".

1 Which of these statements are accurate, according to what you've read so far? More than one answer can be correct.

(i) Student's *T*-Distribution is necessary when the population standard deviation is unknown.

(ii) *T* and *Z* are both test statistics that measure how many standard errors a sample mean is from the population mean.

(iii) *T* is a statistic, a value that measures something using sample data.

(iv) *T* is a parameter, a value that measures something using population data.

(v) Student's *T*-Distribution is another name for the Normal Distribution.

The *T*-Distribution

The *T-distribution* is used when

- the sampling distribution of sample means is normal, and
- a sample standard deviation (s) is used to estimate the population standard deviation (σ).

This test statistic is called a *T-statistic*. The *T*-statistic is an estimate of how many standard errors the sample mean is from the hypothesized population mean. The *T*-statistic is similar to a *Z*-score of a sample mean.

A *T*-distribution is a member of a *family* of continuous probability distributions. The width of a *T*-distribution depends on how much a sample standard deviation can vary. The amount of variability in a sample standard deviation depends on how many deviations *vary freely* when it is computed.

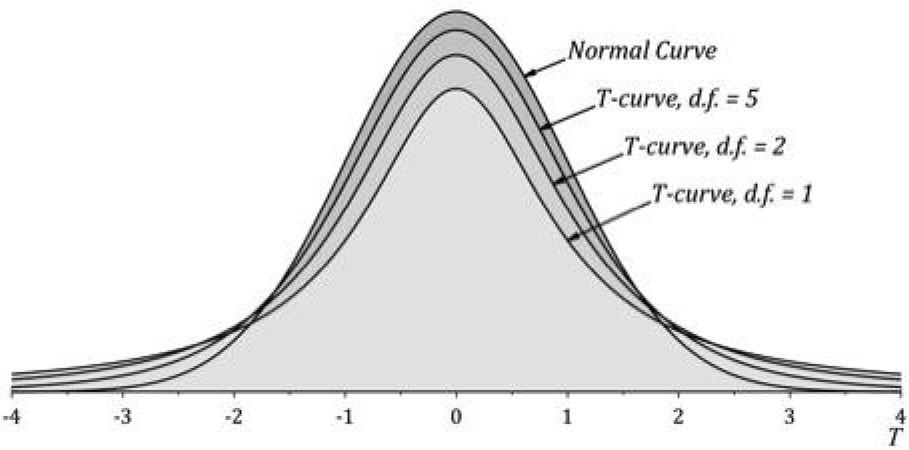
Language Tip
When variables *vary freely*, they are random and unpredictable.

The sample standard deviation is computed using the formula below:

$$s = \sqrt{\frac{\Sigma(x - \bar{x})^2}{n - 1}}$$

The deviations from the mean $(x - \bar{x})$ are averaged in a sample standard deviation. When the deviations are added together, they always add to zero. Because of this, the last deviation summed in this average is not *free*—it is always the value that makes the resulting sum zero. There are n deviations from the mean, but only $n - 1$ of these are free deviations.

The variability of standard deviations depends on the number of free deviations in the sample standard deviation, $n - 1$. This quantity is known as the **degrees of freedom (*d.f.*)**. Technically speaking, each degree of freedom defines a uniquely associated *T*-distribution.



It is important to know that the fewer the degrees of freedom, the more the sample standard deviation varies. In other words, the smaller the sample size, the more the sample standard deviation varies. So, the smaller the sample size, the greater the variability in a *T*-distribution.

T-distributions *have* the following characteristics:

- *T*-distributions are bell-shaped and symmetric with a mean of 0.
- Each *T*-distribution depends on the *degrees of freedom*, *d.f.*
- *T*-distributions have heavier tails and narrower peaks than the standard normal distribution.
- The area under each *T*-distribution curve is 1.
- As the degrees of freedom increase, the tails become thinner.
- As the degrees of freedom increase, the *T*-distribution approaches the standard normal distribution.
- When making inferences about a population mean, the degrees of freedom are equal to the sample size minus 1 (*d.f.* = $n - 1$).

2 Check any of the statements below that are true:

(i) Student's *T*-Distribution, unlike the Normal Distribution, changes shape based on the sample size.

(ii) Student's *T*-Distribution looks more and more like the Normal Distribution as sample size increases.

(iii) Student's *T*-Distribution gets wider in the tails as the sample size increases.

(iv) The Empirical Rule (68-95-99.7) applies to the *T*-Distribution as well.

(v) Degrees of freedom is another name for sample size.

Write down any questions you have about Student's *T*-distribution. You can reference your questions in the 4.2 Collaboration to make sure they get addressed.

4.2: Confidence Intervals for a Population Mean

LEARNING GOALS

By the end of this collaboration, you should understand that:

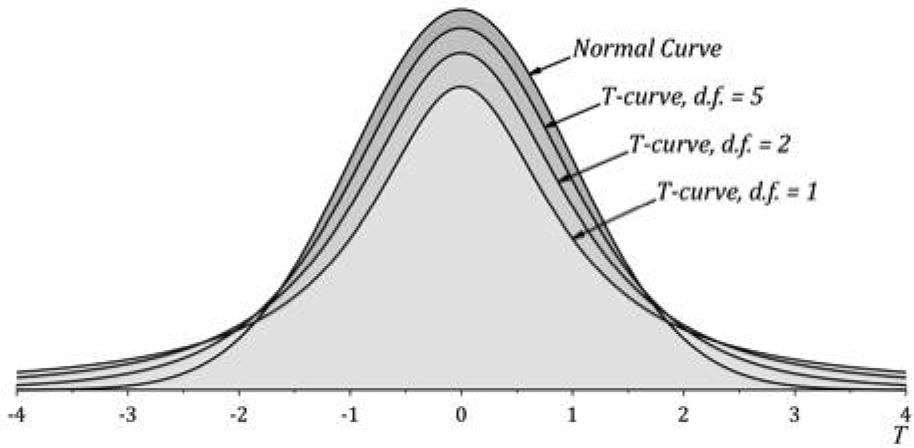
- When the population standard deviation is unknown, and a random sample is sufficiently large or drawn from a normal population, it is appropriate to use *Student's T-distribution* to make inferences.
- While the *T*-statistic is similar to the *Z*-statistic in form, it uses a sample standard deviation (s) in place of an unknown population standard deviation (σ). This makes the *T*-distribution wider than the standard normal distribution, due to the added variability of the sample standard deviation.
- The *T*-distribution is a family of continuous probability distributions that are differentiated by their *degrees of freedom*.
- When making inferences about a population mean with an unknown population standard deviation, the degrees of freedom are equal to the sample size minus 1 (*d.f.* = $n - 1$).
- *T*-distribution curves are similar in shape to the standard normal distribution, but with narrower peaks and heavier tails. As the degrees of freedom increase, *T*-distributions approach normality.
- Increasing the sample size will lead to a narrower confidence interval.
- Increasing the confidence level will lead to a wider confidence interval.

By the end of this collaboration, you should be able to:

- Identify the appropriate *T*-distribution associated with a given sample size by finding the correct number of degrees of freedom.
- Calculate the *T*-statistic using the formula: $T = \frac{\overline{x} - \mu}{\frac{s}{\sqrt{n}}}$
- Interpret the value of a *T*-statistic in context and determine whether or not it is unusual.
- Determine the *T*-distribution critical value for a given sample size and level of confidence.
- Construct 90%, 95%, and 99% confidence intervals for the population mean.
- Use the criteria for approximate normality of the sampling distribution of sample means to determine when the *T* confidence interval procedure is appropriate.

INTRODUCTION

Recall the following summary and illustration from the 4.2 Preparation:



T-distributions *have* the following characteristics:

- *T*-distributions are bell-shaped and symmetric with a mean of 0.
- Each *T*-distribution depends on the *degrees of freedom*, *d.f.*
- *T*-distributions have heavier tails and narrower peaks than the standard normal distribution.
- The area under each *T*-distribution curve is 1.
- As the degrees of freedom increase, the tails become thinner.
- As the degrees of freedom increase, the *T*-distribution approaches the standard normal distribution.
- When making inferences about a population mean, the degrees of freedom are equal to the sample size minus 1 ($d.f. = n - 1$).

Take a moment to check in as a group, and discuss any group member's questions that came up in the Preparation 4.2.

TRY THESE 1

Researchers are very worried about how quickly sea ice is melting. If sea ice continues to melt, it is possible that polar bears will become an endangered species. That means that eventually there may be no more polar bears anywhere in the world. Polar bears use the sea ice for hunting and making their dens. Also, as the ice melts and erodes, there are smaller seal populations. Seals are the polar bears' main source of food. This further endangers the polar bear population.

Biologists regularly visit Arctic regions to track the health and numbers of polar bears. One important measurement is the weight of adult male polar bears.

Biologists estimate that the average weight of an adult male polar bear is approximately 475 kilograms (1050 pounds). For the questions below, assume that polar bear weights are normally distributed.

1 Suppose that two random samples of five polar bears are drawn from two different areas in Alaska. The weights of the polar bears in kilograms (kg) are displayed in the table below:

Sample A	466	520	512	513	498
Sample B	493	482	431	450	452

A The two samples have the same sample size. What is the appropriate number of degrees of freedom for this sample size? (Remember that degrees of freedom = $n - 1$.)

B Calculate the mean for Sample A. Round to one decimal place.

C Calculate the standard deviation for Sample A. Round to two decimal places.

D Calculate the *T*-statistic for Sample A. Assume that the population mean weight (μ) is 475 kg. Round the value to two decimal places.

E Calculate the mean for Sample B. Round to one decimal place.

F Calculate the standard deviation for Sample B. Round to two decimal places.

G Calculate the *T*-statistic for Sample B. Assume that the population mean weight (μ) is 475 kg. Round the value to two decimal places.

2 A Which *T*-statistic indicates that the sample mean is below the population mean μ = 475?

B Which *T*-statistic indicates a sample mean that is the closest (in estimated standard errors) to the population mean μ = 475?

C Based on the sample statistics, which sample(s) should biologists and conservationists be most worried about? Explain. (**Note**: Conservationists are people who are interested in conserving or saving endangered animals or plant life.)

NEXT STEPS

We now begin the process of making inferences about the *mean* of a population. We use the *T*-distribution to compute margins of error and confidence intervals for a population mean.

Confidence Interval for a Population Mean

In Module 3, we computed confidence intervals for population proportions using sample proportions and *critical values*. Since we did not know the population proportion (p), the margin of error in a sample proportion ($\widehat{p}$) was computed using the formula

$$E = Z_c \cdot \text{estimated standard error}$$

$$E = Z_c \cdot \sqrt{\frac{\widehat{p}(1-\widehat{p})}{n}}$$

Remember:

- The margin of error is equal to a *Z* critical value multiplied by the estimated standard error for sample proportions.
- The sample proportion is a *point estimate*. The resulting confidence interval (an *interval* estimate) is

$$(\widehat{p} - E, \widehat{p} + E)$$

To compute the margin of error in a sample mean, we multiply a *T* critical value (not *Z*) by the estimated standard error for sample means, $\frac{s}{\sqrt{n}}$. Because the shape of the *T*-Distribution changes as sample size changes, the *T* critical value is based on a *T*-distribution with $n - 1$ degrees of freedom.

$$E = T_c \cdot estimated\ standard\ error$$

$$E = T_c \cdot \frac{s}{\sqrt{n}}$$

When estimating a population mean:

- The sample mean is the *point estimate* for the population mean. The resulting confidence interval is

$$(\overline{x} - E, \overline{x} + E)$$

- Another way to write the confidence interval for a population mean is

$$\overline{x} \pm E$$

Understanding T-Distribution Critical Values

There are several important things to remember about the *T*-distribution:

- The proportion of the *T*-distribution contained between $-T_C$ and T_C is equal to the confidence level.
- The *T*-distribution varies with degrees of freedom.
- *T* critical values depend on both the confidence level *and* degrees of freedom.

Below, you will find a partial table of *T*-distribution critical values. When we use the table, or technology, it is possible to find *T* critical values for different combinations of confidence levels and degrees of freedom.

Critical Values for the *T*-Distribution

Confidence Level

d.f.	90%	95%	98%	99%
1	6.31	12.71	31.82	63.66
2	2.92	4.30	6.97	9.93
3	2.35	3.18	4.54	5.84
4	2.13	2.78	3.75	4.60
5	2.02	2.57	3.37	4.03
…	…	…	…	…
9	1.83	2.26	2.82	3.25
10	1.81	2.23	2.76	3.17
11	1.80	2.20	2.72	3.11
12	1.78	2.18	2.68	3.06
13	1.77	2.16	2.65	3.01
14	1.76	2.15	2.62	2.98
15	1.75	2.13	2.60	2.95

16	1.75	2.12	2.58	2.92
….	…	…	…	…
40	1.68	2.02	2.42	2.70

3 Use the T-distribution critical value table to find the critical value T_c that corresponds to the 90% confidence level and a sample size of n = 11. Round the critical value to two decimal places.

TRY THESE 2

The questions below are designed to help you think about the relationships between:

- Degrees of freedom
- Margins of error in confidence intervals
- Sample size
- Precision of confidence intervals
- Critical values and confidence intervals

To answer the questions, use the margin of error formula and the T-distribution table above.

4 For a given confidence level, as the degrees of freedom increase, what happens to the critical values?

5 For a given confidence level, as the degrees of freedom increase, what happens to the margins of error in the confidence intervals?

6 Larger degrees of freedom result from larger sample sizes. What does this imply about the relationship between the sample size and the width of the confidence interval?

7 For a given degrees of freedom, what happens to the critical values as the confidence level increases?

8 For a given degrees of freedom, what happens to the margins of error in the confidence intervals as the confidence level increases?

9 What do your answers (in Questions 7–8) imply about the relationship between the confidence level and the width of the confidence interval?

TRY THESE 3

How Long Did it Take to Get Here?

10 What is the average commute time for students at a local college? This may not seem very important, but for the administration and staff who schedule classes and plan activities, travel time is an important factor. A college administrator found that in a random sample of 16 students, the mean commute was $\bar{x} = 22.5$minutes and the standard deviation was s = 4.2 minutes. Assume that the commute times are normally distributed. Construct a 95% confidence interval for μ, the mean commute time for all students at the college.

A Determine the critical value T_c that corresponds to the 95% confidence level. Round the critical values to two decimal places.

B Compute the margin of error (E). Round your answer to three decimal places.

C Calculate the lower limit of the confidence interval, $\bar{x} - E$. Round your answer to three decimal places.

D Calculate the upper limit of the confidence interval, $\bar{x} + E$. Round your answer to three decimal places.

E Interpret your confidence interval. What does the confidence interval say about the commute time for all students at the college?

YOU NEED TO KNOW

Steps for Computing a Confidence Interval for a Population Mean

(1) Verify that the normality criteria for sample means are met: either n > 30, or the population from which we are sampling is normal.

(2) Determine the critical value (T_c) that corresponds to the chosen confidence level and the degrees of freedom.

(3) Compute the margin of error:

$$E = T_c \cdot \frac{s}{\sqrt{n}}$$

(4) Construct the confidence interval:

$$(\bar{x} - E, \bar{x} + E)$$

(5) Interpret the confidence interval in context. In theory, the confidence level is the proportion of intervals that contain the population mean, µ. In practice, our interval is based on sample data. The confidence level represents how confident we are that our interval contains the population mean.

Exercise 4.2

Are Snakes Left-Handed or Right-Handed?

Sometimes a snake will coil, or curl up, so that its left side is pointing in. Sometimes a snake will coil so that its right side is pointing in.

In the following experiment, several cottonmouth snakes were observed over a period of time.[2] The observers recorded the proportion of time that each one of the snakes coiled to the left.

Four different categories of snakes were observed: adult females (n = 15), adult males (n = 5), juvenile females (n = 5), and juvenile males (n = 5). Assume that the measure "proportion of left-handed coils" is normally distributed.

The table below shows the proportion of left-handed coils that was recorded for each of the snakes:

Adult Females	0.582	0.585	0.550	0.554	0.609
	0.545	0.544	0.600	0.638	0.656
	0.600	0.696	0.424	0.493	0.491
Adult Males	0.563	0.556	0.522	0.541	0.395
Juvenile Females	0.512	0.556	0.565	0.417	0.429
Juvenile Males	0.486	0.492	0.475	0.464	0.493

1 The observers wanted to study the adult females first:

A How many degrees of freedom are associated with the sample size of adult females?

B Find the sample mean for the adult females. Round your answer to four decimal places.

C Find the sample standard deviation for the adult females. Round your answer to four decimal places.

---

[2] Eric D. Roth, "'Handedness' in Snakes? Lateralization of Coiling Behaviour in a Cottonmouth, *Agkistrodon Piscivorus Leucostoma*, Population," *Animal Behaviour* 66 (2003): 337-41.

D We will assume that the population of adult female snakes have **no preference** for coiling right or left. If this is true, the proportion of left-handed coils for these snakes is 0.50. This means that 50% of coils are to the left, and 50% are to the right. Assuming that $\mu = 0.5$, calculate the *T*-statistic for the sample of adult females. Round your answer to three decimal places.

E What does this test statistic say about the coiling preferences of the population of adult females?

2 The observers then studied the adult males.

A How many degrees of freedom are associated with the sample of adult males?

B Find the sample mean for the adult males. Round your answer to four decimal places.

C Find the sample standard deviation for the adult males. Round your answer to four decimal places.

D We will assume that the population of adult male snakes have no preference for coiling right or left. If this is true, the proportion of left-handed coils for these snakes is 0.50. Assuming that $\mu = 0.5$, calculate the *T*-statistic for the sample of adult males. Round your answer to three decimal places.

E What does this test statistic say about the coiling preferences of the sample of adult males?

3 Finally, the observers studied the juvenile males.

A How many degrees of freedom are associated with the sample of juvenile males?

B Find the sample mean for the juvenile males. Round your answer to three decimal places.

C Find the sample standard deviation for the juvenile males. Round your answer to four decimal places.

D We will assume that the population of juvenile male snakes have no preference for coiling right or left. If this is true, the proportion of left-handed coils for these snakes is 0.50. Assuming that $\mu = 0.5$, calculate the *T*-statistic for the sample of juvenile males. Round your answer to three decimal places.

E What does this test statistic say about the coiling preferences of the sample of juvenile males?

Constructing Confidence Intervals for Population Means

4 Suppose that researchers measured the resting pulse rates of men who exercise according to a particular bicycle exercise regimen. The table below shows pulse rates (in heartbeats per minute) from such men. These values were taken from a random sample of men who exercise according to this regimen. Construct a 90% confidence interval for the mean resting pulse rate for all adult males.

65	82	87	64	77
89	77	73	78	88
79	83	85	87	82
91	87	85	87	73
88	78	74	78	88
82	77	59	97	84
87	85	87	83	79
73	83	85	87	91
99				

A In this problem, are the criteria for sampling distribution normality met? Why or why not?

B Determine the sample mean. Round to one decimal place.

C Determine the sample standard deviation. Round your answer to two decimal places.

D Determine the critical value that corresponds to the 90% confidence level.

E Compute the margin of error (E) for a 90% confidence interval. Round your answer to three decimal places.

F Construct a 90% confidence interval for the population mean by first calculating the lower limit, $\overline{x} - E$. Round your answer to three decimal places.

G Calculate the upper limit of the confidence interval, $\overline{x} + E$. Round your answer to three decimal places.

H Write a statement that interprets your confidence interval in context.

5 Logging companies harvest wood by cutting down trees in forests. Some people believe that logging affects the behavior of black bears. In particular, people believe that logging changes the size of the bears' *home range*. The home range is the area that the bears use on a daily basis. Researchers studied Canadian black bears that lived in a forest where there was active logging.[3] The researchers put radio-collars (tracking devices) on 12 female black bears. Then the researchers measured the spring and early summer home ranges (in square kilometers) of the female black bears. The sizes of the 12 home ranges are listed below. You should assume that the home range of female black bears in this logged forest is normally distributed.

39.9	23.5	42.1	29.4	34.4	40.9	27.9	22.3	13.0	20.1	13.3	8.6

A Are the normality criteria met for the sampling distribution of sample means?

B Determine the sample mean. Round your answer to two decimal places.

C Determine the sample standard deviation. Round your answer to two decimal places.

D Determine the critical value that corresponds to the 95% confidence level.

E Compute the margin of error (E) for a 95% confidence interval. Round your answer to two decimal places.

F Construct a 95% confidence interval for the population mean by first calculating the lower limit, $\bar{x} - E$. Round your answer to three decimal places.

[3] Vincent Brodeur et al., "Habitat Selection by Black Bears in an Intensively Logged Boreal Forest," *Canadian Journal of Zoology* 10 (2008): 1307-16.

G Calculate the upper limit of the confidence interval, $\bar{x} + E$. Round your answer to two decimal places.

H What does your 95% confidence interval tell you about the mean home range of female black bears in the Canadian logged forest?

I Researchers have been studying the impact of logging in forests for a long time. They have found that the typical home range of female black bears in forests with no logging is 20 square kilometers. Think about the 95% confidence interval that you constructed. Could the mean home range of female black bears in the Canadian logged forest be the same as the mean home range for female black bears in non-logged forests? Why or why not?

4.2 Monitor (survey)

Monitor your progress on learning the objectives for this unit. If you identify any objectives you need to review, go back through the unit's activities. Your responses are not graded, but will be available for your instructor to see.

Rate how confident you are on a scale of 1–5 (1 = not confident and 5 = very confident).

Skill or Concept: I can …	**Rating from 1 to 5**
Identify the appropriate T-distribution associated with a given sample size by finding the correct number of degrees of freedom.	
Calculate the T-statistic using the formula: $T = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$	
Interpret the value of a T-statistic in context and determine whether or not it is unusual.	
Construct 90%, 95%, and 99% confidence intervals for the population mean.	

4.3: Hypothesis Tests for Population Means 1

LEARNING GOALS

By the end of this collaboration, you should understand that:

- Hypothesis testing is appropriate when a decision is desired regarding an unknown population parameter.
- The *T*-distribution can be used for a hypothesis test for a population mean when the population standard deviation is unknown.
- When the *P*-value is less than the level of significance, the sample mean is statistically significant.
- A Type I error occurs when we reject a null hypothesis that is actually true.
- A Type II error occurs when we fail to reject the null hypothesis, even though it is false.
- A confounding variable is a variable in an experiment which may impact the response variable and must be controlled.

By the end of this collaboration, you should be able to:

- Define appropriate null and alternative hypotheses.
- Compute the *T*-test statistic.
- Find *P*-values for left-, right-, and two-tailed tests using technology and interpret the *P*-value in context.
- Use a *P*-value and level of significance to reach a decision about the null and alternative hypotheses and the underlying research question.
- Describe the Type I and Type II error for a given test.
- Assuming an error was made, determine whether it was a Type I or Type II error.
- Identify potential confounding variables in an experiment.

INTRODUCTION

In earlier units, we learned that there are two basic forms of statistical inference: confidence intervals and hypothesis tests. In Unit 4.2, we constructed confidence intervals for a population mean. Now, we will test the hypothesis that a population mean is equal to a specified value. We use the four-step hypothesis testing process introduced in Module 3.

Monarch's Magnetic Fields

Some people believe that the earth's magnetic field guides some animals as they travel over great distances. However, it is not clear how these animals detect the earth's magnetic field. The monarch butterfly is an example of an animal that may travel this way. Monarch butterflies cannot survive a long cold winter, so they migrate long distances south in the fall. Monarch butterflies fly to the same winter roosts year after year. Often they fly to the same trees each winter. It is not known how the monarch butterflies locate their winter homes. One possibility is that they have some magnetic material in their bodies that allows them to sense a magnetic field.

Biologists wanted to determine whether monarch butterflies have some magnetic material in their bodies.[4] They used a tool called a *magnetometer* for this test. The magnetometer measures magnetic units called *nano-emus*. Unfortunately, the magnetometer itself creates some magnetism. The magnetometer creates about 20 nano-emus of magnetism. (A nano-emu is 10^{-9} magnetic units.) In order to demonstrate that monarch butterflies have some magnetic material in their bodies, the biologists needed to be able to show that the monarch butterflies' mean magnetism is greater than 20 nano-emus.

For this test, we will use a 5% level of significance. We will assume that the population of magnetic intensities in monarch butterflies is normally distributed.

Step 1: Determine the Hypotheses

The null hypothesis states that the population mean is equal to a specific value:

H_0: μ= *assumed value*

The alternative hypothesis is one of the following inequalities:

H_a: μ< *assumed value*	For a left-tailed test
H_a: μ> *assumed value*	For a right-tailed test
H_a: μ≠ *assumed value*	For a two-tailed test

1 Let μ be the mean magnetic intensity for all monarch butterflies.

A State the null hypothesis.

B State the alternative hypothesis.

[4]Douglas Jones and Bruce MacFadden, "Induced Magnetization in the Monarch Butterfly, *Danaus Plexippus* (Insecta, Lepidoptera)" *Journal of Experimental Biology* 96 (1982): 1-9.

Step 2: Collect the Data

2 The biologists examined a random sample of 16 monarch butterflies. The measured magnetic intensity (in nano-emus) of each monarch butterfly is given in the table below.

Magnetic Strength of Monarch Butterflies (in nano-emus)

48.6	32.8	53.2	32.3	17.3	12.2	36.6	29.8
12.4	21.9	26.4	49.0	14.5	7.5	40.2	29.0

A Are the criteria for the approximate normality of the sampling distribution of sample means satisfied? Explain why or why not.

B Calculate the sample mean. Round your answer to two decimal places.

C Calculate the sample standard deviation. Round your answer to three decimal places.

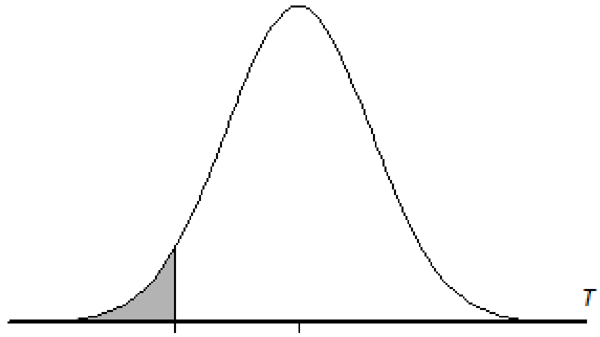
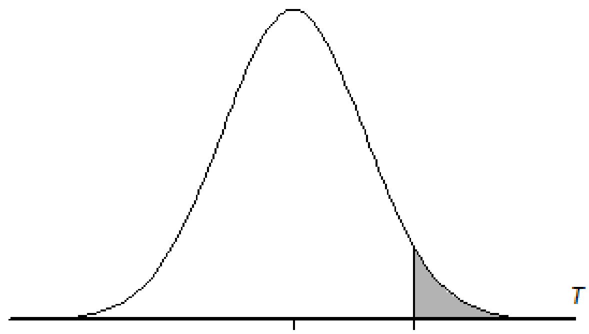
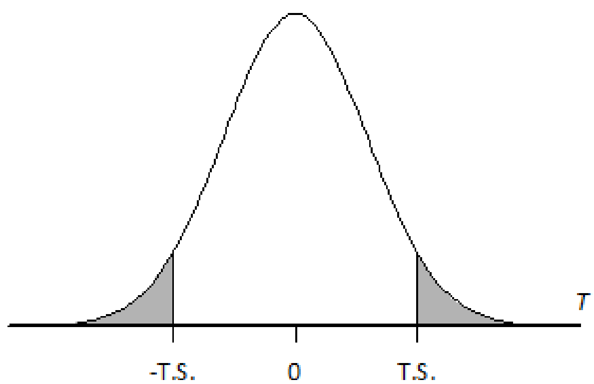
Step 3: Assess the Evidence

In this step, we calculate the test statistic and the *P*-value. The test statistic varies according to a *T*-distribution when two criteria are met:

- The sampling distribution of sample means is normal, and
- We use a *sample* standard deviation to standardize the sample mean.

The formula for the test statistic is: $T = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$

We will use technology or tables to find *P*-values. Recall that each *T*-distribution depends on the *degrees of freedom*, *d.f*. When finding a *P*-value, you must specify the degrees of freedom of the *T*-distribution which you are using. The degrees of freedom is the sample size minus 1 (*d.f.* = $n - 1$).

For a left-tailed test, the *P*-value is the area to the left of the test statistic.	
For a right-tailed test, the *P*-value is the area to the right of the test statistic.	
For a two-tailed test, the *P*-value is *twice* the area to the right of a positive test statistic, or *twice* the area to the left of a negative test statistic.	

3 We can use the T-distribution in this case since the normality criteria for the sampling distribution are met.

A Calculate the test statistic for the observed sample mean. Round your answer to two decimal places.

B Is the test statistic unusual? Be sure to explain your answer.

C Use the test statistic to find the *P*-value, rounded three places after the decimal.

Step 4: State a Conclusion

4 Do the data provide enough evidence to conclude that at the 5% significance level, the mean magnetic intensity for monarch butterflies is greater than 20 nano-emus?

A How does the *P*-value compare to the significance level?

B What should we conclude about the null and alternative hypotheses?

C Is the observed sample mean ($\bar{x} = 28.98$ nano-emus) statistically significant? What does this mean?

D State a conclusion in the context of the problem. Be sure to explain what the conclusion means in relation to the magnetic intensity of monarch butterflies.

Interpreting the Results

5 It is possible that the conclusion was incorrect. If so, what type of error was made, and what does that mean in the context of this hypothesis test?

NEXT STEPS

Mean Lifespan of Batteries

Imagine that a company sells portable walkie-talkie radios to construction crews. The batteries for these radios last for an average of 55 hours. The purchasing manager for this company receives a brochure in the mail that advertises a new brand of batteries. This new brand of batteries is cheaper than the brand that the company currently uses. However, the purchasing manager is concerned that the cheaper batteries may have a shorter average battery life than the current brand. (*Note*: The number of hours that batteries last is called their *battery life*.) The pricing manager installs 40 randomly selected batteries of the cheaper brand in the company's walkie-talkie radios. He finds that the mean battery life for the sample is 52 hours, with a standard deviation of 10 hours. He wants to perform a statistical test at the 1% level of significance to determine whether the cheaper batteries have a shorter average battery life span than the average life span of the brand of batteries the company currently uses.

6 **Step 1: Determine the Hypotheses**

A State the null hypothesis.

B State the alternative hypothesis.

7 **Step 2: Collect the Data**

We have been provided the sample size, n = 40, the sample mean, $\bar{x} = 52$**,** and the sample standard deviation, s = 10.

Are the criteria for the approximate normality of the sampling distribution of sample means satisfied? Explain why or why not.

8 **Step 3: Assess the Evidence**

A Calculate the test statistic for the observed sample mean. Round your answer to two decimal places.

B Sketch the *T*-distribution and identify the position of the observed test statistic. Shade the area that represents the *P*-value. Use the test statistic to find the *P*-value, rounded three places after the decimal.

9 **Step 4: State a Conclusion**

A How does the *P*-value compare to the significance level?

B What should we conclude about the null and alternative hypotheses?

C State your conclusion to summarize this hypothesis test in the context of the problem.

Designing an Experiment: Confounding Variables

In experiments, it is important to consider the potential *confounding variables,* which could influence how we must interpret the causal relationship. A **confounding variable** is a variable that has an effect on the response variable, but was not properly controlled for in the experiment.

For example, suppose a student living in a dorm wanted to do an experiment to see if offering a free t-shirt would get more people to come to his social activism club. He put up large signs on the third and fourth floors of his dorm advertising a free t-shirt and free pizza for those who attended the next meeting. He did not put up any signs on the first and second floors. He found that more students from the third and fourth floors than the first and second floors attended the next meeting.

After talking to a Statway student, he realized he could not conclude that offering a free t-shirt increased attendance, because there were confounding variables in his experiment.

10 What are two potential confounding variables in this experiment?

Researchers should attempt to *control* for such confounding variables by carefully designing their experiment.

11 How might the student have controlled for the two confounding variables that you noted in your answer for Question 10?

LET'S SUMMARIZE

Please consider the following key points:

- A hypothesis test consists of four steps: (1) Determine the hypotheses, (2) Collect the data, (3) Assess the evidence, and (4) State a conclusion.
- In a hypothesis test for a population mean, we use a sample mean from a random sample to test a claim made about a population mean.
- In a hypothesis test we make an assumption that the null hypothesis is true to create a sampling distribution. We then use the sampling distribution to determine the likelihood of observing a sample statistic like the one we found. This likelihood is called the *P*-value.
- When the *P*-value is less than the level of significance, we conclude that the observed sample statistic is statistically significant. This indicates that the sample statistic is significantly different than the assumed population parameter (and quite improbable), so we reject the null hypothesis in favor of the alternative hypothesis.

Exercise 4.3

Quality Control

Quality control is an important aspect of the manufacturing process. Suppose a local company manufactures soda-dispensing machines that fill bottles with 600 ml of soda. Over time, the dispensing machines can become less precise. That is, older machines may dispense too much or too little soda into the bottles. When something becomes less precise, we say that it has lost its precision. When this occurs, the machine requires an adjustment. In this case, a soda dispensing machine will need adjustment if the mean amount of soda dispensed is different than 600 ml.

The owner of one soda dispensing machine believes that her machine has lost precision. She measures the amount of soda dispensed at eight random times throughout the day. The eight amounts of soda her machines dispensed during the day are listed below:

600.15	599.92	599.85	599.92	599.81	600.14	600.04	599.98

At 5% level of significance, is there sufficient evidence to conclude that this machine needs an adjustment? Research has indicated that the amounts of soda dispensed from the machine are approximately normal in their distribution.

Complete the steps below.

Step 1: Determine the Hypotheses

1 State the null hypothesis.

2 State the alternative hypothesis.

Step 2: Collect the Data

3 Are the criteria for the approximate normality of the sampling distribution of sample means satisfied?

4 Use technology to calculate the sample mean. Round your answer to two decimal places.

5 Use technology to calculate the sample standard deviation. Round your answer to three decimal places.

Step 3: Assess the Evidence

6 At the 5% level of significance, is there sufficient evidence to conclude that the mean amount of soda dispensed by the machine is different than 600 ml?

Calculate the test statistic for the observed sample mean. Round your answer to two decimal places.

7 Use the test statistic to find the *P*-value, rounded three places after the decimal.

Step 4: State a Conclusion

8 How does the *P*-value compare to the significance level?

9 What should we conclude about the null and alternative hypotheses?

10 State your conclusion for this hypothesis test in the context of the problem.

The Impact of Light Aerobic Exercise on Weight Loss

A graduate student conducted an experiment to examine how light aerobic exercise contributes to weight loss among college students. The researcher recruited 50 college students to participate in the experiment. Twenty-five students were assigned to engage in light aerobic exercise activities for 30 minutes, three times a week. The other twenty-five students were assigned to engage in the same exercise activities for 1 hour, five times a week. The researcher identified a group of six exercise activities that burned a similar amount of calories and gave participants the option to choose which of the six exercises they wanted to perform. The researcher weighed participants prior to the beginning of the study and 4 weeks later, then computed the weight loss for each participant. The results of the experiment are shown in the table below. The sample statistics summarize the weight loss (in pounds) for each group of participants.

Weight Loss for Group 1 (Exercised for 30 minutes, 3 times a week)	Weight Loss for Group 2 (Exercised for 60 minutes, 5 times a week)
$\bar{x} = 1.7$lb, $s = 0.8$ lb, $n = 25$	$\bar{x} = 1.9$lb, $s = 1.2$ lb, $n = 25$

11 What is the explanatory variable in this experiment?

12 What is the response variable in this experiment?

13 What are some possible confounding variables in this experiment?

We can use the sample data from Group 1 to perform a hypothesis test. Let's assume the weight losses of the 25 participants in Group 1 represent the weight losses of all college students who exercise in a similar fashion to Group 1 over a 4-week period. At 5% level of significance, do the sample statistics from Group 1 provide sufficient evidence to conclude that the mean weight loss of all college students who perform the same light aerobic exercise activities over a 4-week period is less than 2 pounds? Let's assume that weight losses are approximately normally distributed.

Step 1: Determine the Hypotheses

14 State the null hypothesis.

15 State the alternative hypothesis.

Step 2: Collect the Data

16 Are the criteria for the approximate normality of the sampling distribution of sample means satisfied?

17 What is the sample mean?

18 What is the sample standard deviation?

Step 3: Assess the Evidence

19 Calculate the test statistic for the observed sample mean. Round your answer to two decimal places.

20 Use the test statistic to find the *P*-value, rounded three places after the decimal.

Step 4: State a Conclusion

21 How does the *P*-value compare to the significance level?

22 What should we conclude about the null and alternative hypotheses?

23 State your conclusion for this hypothesis test in the context of the problem.

4.3 Monitor (survey)

Monitor your progress on learning the objectives for this unit. If you identify any objectives you need to review, go back through the unit's activities. Your responses are not graded, but will be available for your instructor to see.

Rate how confident you are on a scale of 1–5 (1 = not confident and 5 = very confident).

Skill or Concept: I can …	**Rating from 1 to 5**
Define appropriate null and alternative hypotheses.	
Compute the *T*-test statistic.	
Find *P*-values for left-, right-, and two-tailed tests using technology and interpret the *P*-value in context.	
Describe the Type I and Type II error for a given test.	
Identify potential confounding variables in an experiment.	

T-Distribution p -Values

Degrees of Freedom

$T.S.$	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	40	50	60	70	80	90	100	∞
0.0	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500
0.1	.532	.535	.537	.537	.538	.538	.538	.539	.539	.539	.539	.539	.539	.539	.539	.539	.539	.539	.539	.539	.539	.539	.539	.539	.539	.539	.540	.540	.540	.540	.540	.540	.540	.540	.540	.540	.540	.540
0.2	.563	.570	.573	.574	.575	.576	.576	.577	.577	.577	.577	.578	.578	.578	.578	.578	.578	.578	.578	.578	.578	.578	.578	.578	.579	.579	.579	.579	.579	.579	.579	.579	.579	.579	.579	.579	.579	.579
0.3	.593	.604	.608	.610	.612	.613	.614	.614	.615	.615	.615	.615	.616	.616	.616	.616	.616	.616	.616	.616	.616	.617	.617	.617	.617	.617	.617	.617	.617	.617	.617	.617	.617	.618	.618	.618	.618	.618
0.4	.621	.636	.642	.645	.647	.649	.650	.650	.651	.651	.652	.652	.652	.652	.653	.653	.653	.653	.653	.653	.653	.654	.654	.654	.654	.654	.654	.654	.654	.654	.654	.655	.655	.655	.655	.655	.655	.655
0.5	.648	.667	.674	.678	.681	.683	.684	.685	.686	.686	.687	.687	.687	.688	.688	.688	.688	.688	.689	.689	.689	.689	.689	.689	.689	.689	.689	.690	.690	.690	.690	.690	.691	.691	.691	.691	.691	.692
0.6	.672	.695	.705	.710	.713	.715	.716	.717	.718	.719	.720	.720	.721	.721	.721	.722	.722	.722	.722	.722	.723	.723	.723	.723	.723	.723	.723	.723	.723	.724	.724	.724	.725	.725	.725	.725	.725	.726
0.7	.694	.722	.733	.739	.742	.745	.747	.748	.749	.750	.751	.751	.752	.752	.753	.753	.753	.754	.754	.754	.754	.754	.755	.755	.755	.755	.755	.755	.755	.755	.756	.756	.757	.757	.757	.757	.757	.758
0.8	.715	.746	.759	.766	.770	.773	.775	.777	.778	.779	.780	.780	.781	.782	.782	.782	.783	.783	.783	.783	.784	.784	.784	.784	.784	.785	.785	.785	.785	.785	.786	.786	.787	.787	.787	.787	.787	.788
0.9	.733	.768	.783	.791	.795	.799	.801	.803	.804	.805	.806	.807	.808	.808	.809	.809	.810	.810	.810	.811	.811	.811	.811	.812	.812	.812	.812	.812	.812	.812	.813	.814	.814	.814	.815	.815	.815	.816
1.0	.750	.789	.805	.813	.818	.822	.825	.827	.828	.830	.831	.832	.832	.833	.833	.834	.834	.835	.835	.835	.836	.836	.836	.836	.837	.837	.837	.837	.837	.837	.838	.839	.839	.840	.840	.840	.840	.841
1.1	.765	.807	.824	.834	.839	.843	.846	.848	.850	.851	.853	.854	.854	.855	.856	.856	.857	.857	.858	.858	.858	.858	.859	.859	.859	.859	.860	.860	.860	.860	.861	.862	.862	.862	.863	.863	.863	.864
1.2	.779	.824	.842	.852	.858	.862	.865	.868	.870	.871	.872	.873	.874	.875	.876	.876	.877	.877	.878	.878	.878	.879	.879	.879	.879	.880	.880	.880	.880	.880	.881	.882	.883	.883	.883	.883	.884	.885
1.3	.791	.838	.858	.868	.875	.879	.883	.885	.887	.889	.890	.891	.892	.893	.893	.894	.895	.895	.895	.896	.896	.897	.897	.897	.897	.898	.898	.898	.898	.898	.900	.900	.901	.901	.901	.902	.902	.903
1.4	.803	.852	.872	.883	.890	.895	.898	.901	.903	.904	.906	.907	.908	.908	.909	.910	.910	.911	.911	.912	.912	.912	.913	.913	.913	.913	.914	.914	.914	.914	.915	.916	.917	.917	.917	.918	.918	.919
1.5	.813	.864	.885	.896	.903	.908	.911	.914	.916	.918	.919	.920	.921	.922	.923	.924	.924	.925	.925	.925	.926	.926	.926	.927	.927	.927	.927	.928	.928	.928	.929	.930	.931	.931	.931	.931	.932	.933
1.6	.822	.875	.896	.908	.915	.920	.923	.926	.928	.930	.931	.932	.933	.934	.935	.935	.936	.937	.937	.937	.938	.938	.938	.939	.939	.939	.939	.940	.940	.940	.941	.942	.943	.943	.943	.943	.944	.945
1.7	.831	.884	.906	.918	.925	.930	.934	.936	.938	.940	.941	.943	.944	.944	.945	.946	.946	.947	.947	.948	.948	.948	.949	.949	.949	.950	.950	.950	.950	.950	.952	.952	.953	.953	.954	.954	.954	.955
1.8	.839	.893	.915	.927	.934	.939	.943	.945	.947	.949	.950	.952	.953	.953	.954	.955	.955	.956	.956	.957	.957	.957	.958	.958	.958	.958	.959	.959	.959	.959	.960	.961	.962	.962	.962	.962	.963	.964
1.9	.846	.901	.923	.935	.942	.947	.950	.953	.955	.957	.958	.959	.960	.961	.962	.962	.963	.963	.964	.964	.964	.965	.965	.965	.966	.966	.966	.966	.966	.967	.968	.968	.969	.969	.970	.970	.970	.971
2.0	.852	.908	.930	.942	.949	.954	.957	.960	.962	.963	.965	.966	.967	.967	.968	.969	.969	.970	.970	.970	.971	.971	.971	.972	.972	.972	.972	.972	.973	.973	.974	.975	.975	.975	.976	.976	.976	.977
2.1	.859	.915	.937	.948	.955	.960	.963	.966	.967	.969	.970	.971	.972	.973	.974	.974	.975	.975	.975	.976	.976	.976	.977	.977	.977	.977	.977	.978	.978	.978	.979	.980	.980	.980	.981	.981	.981	.982
2.2	.864	.921	.942	.954	.961	.965	.968	.971	.972	.974	.975	.976	.977	.977	.978	.979	.979	.979	.980	.980	.980	.981	.981	.981	.981	.982	.982	.982	.982	.982	.983	.984	.984	.984	.985	.985	.985	.986
2.3	.870	.926	.948	.959	.965	.969	.973	.975	.977	.978	.979	.980	.981	.981	.982	.982	.983	.983	.984	.984	.984	.984	.985	.985	.985	.985	.985	.985	.986	.986	.987	.987	.988	.988	.988	.988	.988	.989
2.4	.874	.931	.952	.963	.969	.973	.976	.978	.980	.981	.982	.983	.984	.985	.985	.986	.986	.986	.987	.987	.987	.987	.988	.988	.988	.988	.988	.988	.989	.989	.989	.990	.990	.991	.991	.991	.991	.992
2.5	.879	.935	.956	.967	.973	.977	.980	.982	.983	.984	.985	.986	.987	.987	.988	.988	.989	.989	.989	.989	.990	.990	.990	.990	.990	.991	.991	.991	.991	.991	.992	.992	.992	.993	.993	.993	.993	.994
2.6	.883	.939	.960	.970	.976	.980	.982	.984	.986	.987	.988	.988	.989	.990	.990	.990	.991	.991	.991	.991	.992	.992	.992	.992	.992	.992	.993	.993	.993	.993	.994	.994	.994	.994	.995	.995	.995	.995
2.7	.887	.943	.963	.973	.979	.982	.985	.987	.988	.989	.990	.990	.991	.991	.992	.992	.992	.993	.993	.993	.993	.994	.994	.994	.994	.994	.994	.994	.994	.994	.995	.995	.996	.996	.996	.996	.996	.997
2.8	.891	.946	.966	.976	.981	.984	.987	.988	.990	.991	.991	.992	.993	.993	.993	.994	.994	.994	.994	.995	.995	.995	.995	.995	.995	.995	.995	.995	.996	.996	.996	.996	.997	.997	.997	.997	.997	.997
2.9	.894	.949	.969	.978	.983	.986	.989	.990	.991	.992	.993	.993	.994	.994	.995	.995	.995	.995	.995	.996	.996	.996	.996	.996	.996	.996	.996	.996	.997	.997	.997	.997	.997	.998	.998	.998	.998	.998
3.0	.898	.952	.971	.980	.985	.988	.990	.992	.993	.993	.994	.995	.995	.995	.996	.996	.996	.996	.996	.997	.997	.997	.997	.997	.997	.997	.997	.997	.997	.997	.998	.998	.998	.998	.998	.998	.998	.999
3.1	.901	.955	.973	.982	.987	.989	.991	.993	.994	.994	.995	.995	.996	.996	.996	.997	.997	.997	.997	.997	.997	.997	.998	.998	.998	.998	.998	.998	.998	.998	.998	.998	.999	.999	.999	.999	.999	.999
3.2	.904	.957	.975	.984	.988	.991	.993	.994	.995	.995	.996	.996	.997	.997	.997	.997	.997	.998	.998	.998	.998	.998	.998	.998	.998	.998	.998	.998	.998	.998	.999	.999	.999	.999	.999	.999	.999	.999
3.3	.906	.960	.977	.985	.989	.992	.993	.995	.995	.996	.997	.997	.997	.997	.998	.998	.998	.998	.998	.998	.998	.998	.998	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999
3.4	.909	.962	.979	.986	.990	.993	.994	.995	.996	.997	.997	.997	.998	.998	.998	.998	.998	.998	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999
3.5	.911	.964	.980	.988	.991	.994	.995	.996	.997	.997	.998	.998	.998	.998	.998	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999
3.6	.914	.965	.982	.989	.992	.994	.996	.997	.997	.998	.998	.998	.998	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999
3.7	.916	.967	.983	.990	.993	.995	.996	.997	.998	.998	.998	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999
3.8	.918	.969	.984	.990	.994	.996	.997	.997	.998	.998	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999
3.9	.920	.970	.985	.991	.994	.996	.997	.998	.998	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999
4.0	.922	.971	.986	.992	.995	.996	.997	.998	.998	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999
4.1	.924	.973	.987	.993	.995	.997	.998	.998	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999
4.2	.926	.974	.988	.993	.996	.997	.998	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999
4.3	.927	.975	.988	.994	.996	.998	.998	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999
4.4	.929	.976	.989	.994	.997	.998	.998	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999
4.5	.930	.977	.990	.995	.997	.998	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999
4.6	.932	.978	.990	.995	.997	.998	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999
4.7	.933	.979	.991	.995	.997	.998	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999
4.8	.935	.980	.991	.996	.998	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999
4.9	.936	.980	.992	.996	.998	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999
5.0	.937	.981	.992	.996	.998	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999

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T-Distribution p -Values

	Degrees of Freedom																																					
T.S.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	40	50	60	70	80	90	100	∞
-0.0	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500
-0.1	.468	.465	.463	.463	.462	.462	.462	.461	.461	.461	.461	.461	.461	.461	.461	.461	.461	.461	.461	.461	.461	.461	.461	.461	.461	.461	.461	.461	.461	.461	.460	.460	.460	.460	.460	.460	.460	.460
-0.2	.437	.430	.427	.426	.425	.424	.424	.423	.423	.423	.423	.422	.422	.422	.422	.422	.422	.422	.422	.422	.422	.422	.422	.422	.422	.422	.422	.422	.421	.421	.421	.421	.421	.421	.421	.421	.421	.421
-0.3	.407	.396	.392	.390	.388	.387	.386	.386	.386	.385	.385	.385	.385	.384	.384	.384	.384	.384	.384	.384	.384	.384	.383	.383	.383	.383	.383	.383	.383	.383	.383	.383	.383	.383	.383	.382	.382	.382
-0.4	.379	.364	.358	.355	.353	.352	.351	.350	.349	.349	.348	.348	.348	.348	.347	.347	.347	.347	.347	.347	.347	.347	.346	.346	.346	.346	.346	.346	.346	.346	.346	.345	.345	.345	.345	.345	.345	.345
-0.5	.352	.333	.326	.322	.319	.317	.316	.315	.315	.314	.314	.313	.313	.312	.312	.312	.312	.312	.311	.311	.311	.311	.311	.311	.311	.311	.311	.311	.310	.310	.310	.310	.310	.309	.309	.309	.309	.309
-0.6	.328	.305	.295	.290	.287	.285	.284	.283	.282	.281	.280	.280	.279	.279	.279	.279	.278	.278	.278	.278	.278	.277	.277	.277	.277	.277	.277	.277	.277	.277	.276	.276	.275	.275	.275	.275	.275	.274
-0.7	.306	.278	.267	.261	.258	.255	.253	.252	.251	.250	.249	.249	.248	.248	.247	.247	.247	.246	.246	.246	.246	.246	.246	.245	.245	.245	.245	.245	.245	.245	.244	.244	.243	.243	.243	.243	.243	.242
-0.8	.285	.254	.241	.234	.230	.227	.225	.223	.222	.221	.220	.220	.219	.219	.218	.218	.217	.217	.217	.217	.216	.216	.216	.216	.216	.216	.215	.215	.215	.215	.214	.214	.213	.213	.213	.213	.213	.212
-0.9	.267	.232	.217	.210	.205	.201	.199	.197	.196	.195	.194	.193	.192	.192	.191	.191	.190	.190	.190	.189	.189	.189	.189	.189	.188	.188	.188	.188	.188	.188	.187	.186	.186	.186	.185	.185	.185	.184
-1.0	.250	.211	.196	.187	.182	.178	.175	.173	.172	.170	.169	.169	.168	.167	.167	.166	.166	.165	.165	.165	.164	.164	.164	.164	.163	.163	.163	.163	.163	.163	.162	.161	.161	.160	.160	.160	.160	.159
-1.1	.235	.193	.176	.167	.161	.157	.154	.152	.150	.149	.147	.147	.146	.145	.144	.144	.143	.143	.143	.142	.142	.142	.141	.141	.141	.141	.141	.140	.140	.140	.139	.138	.138	.138	.137	.137	.137	.136
-1.2	.221	.177	.158	.148	.142	.138	.135	.132	.130	.129	.128	.127	.126	.125	.124	.124	.123	.123	.122	.122	.122	.122	.121	.121	.121	.121	.120	.120	.120	.120	.119	.118	.117	.117	.117	.117	.117	.115
-1.3	.209	.162	.142	.132	.125	.121	.117	.115	.113	.111	.110	.109	.108	.107	.107	.106	.106	.105	.105	.104	.104	.104	.103	.103	.103	.103	.102	.102	.102	.102	.101	.100	.099	.099	.099	.099	.098	.097
-1.4	.197	.148	.128	.117	.110	.106	.102	.100	.098	.096	.095	.093	.093	.092	.091	.090	.090	.089	.089	.088	.088	.088	.087	.087	.087	.087	.086	.086	.086	.086	.085	.084	.083	.083	.083	.083	.082	.081
-1.5	.187	.136	.115	.104	.097	.092	.089	.086	.084	.082	.081	.080	.079	.078	.077	.077	.076	.076	.075	.075	.074	.074	.074	.073	.073	.073	.073	.072	.072	.072	.071	.070	.069	.069	.069	.069	.068	.067
-1.6	.178	.125	.104	.092	.085	.080	.077	.074	.072	.070	.069	.068	.067	.066	.065	.065	.064	.064	.063	.063	.062	.062	.062	.061	.061	.061	.061	.060	.060	.060	.059	.058	.057	.057	.057	.057	.056	.055
-1.7	.169	.116	.094	.082	.075	.070	.067	.064	.062	.060	.059	.057	.057	.056	.055	.054	.054	.053	.053	.052	.052	.052	.051	.051	.051	.051	.050	.050	.050	.050	.048	.048	.047	.047	.047	.046	.046	.045
-1.8	.161	.107	.085	.073	.066	.061	.057	.055	.053	.051	.050	.049	.048	.047	.046	.045	.045	.044	.044	.044	.043	.043	.043	.042	.042	.042	.042	.041	.041	.041	.040	.039	.038	.038	.038	.038	.037	.036
-1.9	.154	.099	.077	.065	.058	.053	.050	.047	.045	.043	.042	.041	.040	.039	.038	.038	.037	.037	.036	.036	.036	.035	.035	.035	.035	.034	.034	.034	.034	.034	.032	.032	.031	.031	.031	.030	.030	.029
-2.0	.148	.092	.070	.058	.051	.046	.043	.040	.038	.037	.035	.034	.033	.033	.032	.031	.031	.030	.030	.030	.029	.029	.029	.029	.028	.028	.028	.028	.028	.027	.026	.026	.025	.025	.024	.024	.024	.023
-2.1	.142	.085	.063	.052	.045	.040	.037	.035	.033	.031	.030	.029	.028	.027	.027	.026	.026	.025	.025	.024	.024	.024	.023	.023	.023	.023	.023	.022	.022	.022	.021	.020	.020	.020	.019	.019	.019	.018
-2.2	.136	.079	.058	.046	.040	.035	.032	.030	.028	.026	.025	.024	.023	.023	.022	.021	.021	.021	.020	.020	.020	.019	.019	.019	.019	.018	.018	.018	.018	.018	.017	.016	.016	.016	.015	.015	.015	.014
-2.3	.131	.074	.053	.042	.035	.031	.028	.025	.024	.022	.021	.020	.019	.019	.018	.018	.017	.017	.017	.016	.016	.016	.015	.015	.015	.015	.015	.015	.014	.014	.013	.013	.013	.012	.012	.012	.012	.011
-2.4	.126	.069	.048	.037	.031	.027	.024	.022	.020	.019	.018	.017	.016	.015	.015	.015	.014	.014	.013	.013	.013	.013	.012	.012	.012	.012	.012	.012	.012	.011	.011	.010	.010	.009	.009	.009	.009	.008
-2.5	.121	.065	.044	.033	.027	.023	.021	.019	.017	.016	.015	.014	.013	.013	.012	.012	.012	.011	.011	.011	.010	.010	.010	.010	.010	.009	.009	.009	.009	.009	.008	.008	.008	.007	.007	.007	.007	.006
-2.6	.117	.061	.040	.030	.024	.020	.018	.016	.014	.013	.012	.012	.011	.011	.010	.010	.009	.009	.009	.009	.008	.008	.008	.008	.008	.008	.007	.007	.007	.007	.006	.006	.006	.006	.005	.005	.005	.005
-2.7	.113	.057	.037	.027	.021	.018	.015	.014	.012	.011	.010	.010	.009	.009	.008	.008	.008	.007	.007	.007	.007	.006	.006	.006	.006	.006	.006	.006	.006	.006	.005	.005	.004	.004	.004	.004	.004	.003
-2.8	.109	.054	.034	.024	.019	.016	.013	.012	.010	.009	.009	.008	.007	.007	.007	.006	.006	.006	.006	.005	.005	.005	.005	.005	.005	.005	.005	.005	.004	.004	.004	.004	.003	.003	.003	.003	.003	.003
-2.9	.106	.051	.031	.022	.017	.014	.012	.010	.009	.008	.007	.007	.006	.006	.005	.005	.005	.005	.005	.004	.004	.004	.004	.004	.004	.004	.004	.004	.003	.003	.003	.003	.003	.002	.002	.002	.002	.002
-3.0	.102	.048	.029	.020	.015	.012	.010	.008	.007	.007	.006	.005	.005	.005	.004	.004	.004	.004	.004	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.002	.002	.002	.002	.002	.002	.002	.001
-3.1	.099	.045	.027	.018	.013	.011	.009	.007	.006	.006	.005	.005	.004	.004	.004	.003	.003	.003	.003	.003	.003	.003	.002	.002	.002	.002	.002	.002	.002	.002	.002	.002	.001	.001	.001	.001	.001	.001
-3.2	.096	.043	.025	.017	.012	.009	.007	.006	.005	.005	.004	.004	.003	.003	.003	.003	.003	.002	.002	.002	.002	.002	.002	.002	.002	.002	.002	.002	.002	.002	.001	.001	.001	.001	.001	.001	.001	.001
-3.3	.094	.040	.023	.015	.011	.008	.007	.005	.005	.004	.003	.003	.003	.003	.002	.002	.002	.002	.002	.002	.002	.002	.002	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001
-3.4	.091	.038	.021	.014	.010	.007	.006	.005	.004	.003	.003	.003	.002	.002	.002	.002	.002	.002	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001
-3.5	.089	.036	.020	.012	.009	.006	.005	.004	.003	.003	.002	.002	.002	.002	.002	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001
-3.6	.086	.035	.018	.011	.008	.006	.004	.003	.003	.002	.002	.002	.002	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001
-3.7	.084	.033	.017	.010	.007	.005	.004	.003	.002	.002	.002	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001
-3.8	.082	.031	.016	.010	.006	.004	.003	.003	.002	.002	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001
-3.9	.080	.030	.015	.009	.006	.004	.003	.002	.002	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001
-4.0	.078	.029	.014	.008	.005	.004	.003	.002	.002	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001
-4.1	.076	.027	.013	.007	.005	.003	.002	.002	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001
-4.2	.074	.026	.012	.007	.004	.003	.002	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001
-4.3	.073	.025	.012	.006	.004	.002	.002	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001
-4.4	.071	.024	.011	.006	.003	.002	.002	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001
-4.5	.070	.023	.010	.005	.003	.002	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001
-4.6	.068	.022	.010	.005	.003	.002	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001
-4.7	.067	.021	.009	.005	.003	.002	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001
-4.8	.065	.020	.009	.004	.002	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001
-4.9	.064	.020	.008	.004	.002	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001
-5.0	.063	.019	.008	.004	.002	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001

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4.4: Hypothesis Tests for Population Means 2

LEARNING GOALS

By the end of this collaboration, you should understand that:

- Hypothesis testing is appropriate when a decision is desired regarding an unknown population parameter.
- The *T*-distribution can be used for a hypothesis test of a population mean when the criteria for the approximate normality are met for the sampling distribution of sample means.
- When the *P*-value is less than the level of significance, it tells us that the sample mean is statistically significant.
- A Type I error occurs when we reject a null hypothesis that is actually true.
- A Type II error occurs when we fail to reject the null hypothesis, even though it is false. A hypothesis test uses statistical data to test a claim about a population parameter.
- A statistical result cannot be generalized to populations that are not represented well by the sample.
- Larger sample sizes make a given difference more statistically significant.
- Real-world significance is different from statistical significance.

By the end of this collaboration, you should be able to:

- Apply the four-step hypothesis testing procedure to test a claim about a population mean when the population standard deviation is unknown.
- Interpret the results of a hypothesis test.
- Decide when a statistical result can be applied to a larger population.
- Distinguish between statistical and real-world significance.

INTRODUCTION

In this collaboration, we will use the four-step hypothesis testing process to test claims made about population means. We will revisit key hypothesis-testing ideas which were introduced in Module 3, the difference between statistical significance and practical significance, and generalizing results of a hypothesis test.

TRY THESE

Household Credit Card Debt

Economists use average household credit card debt to gauge the financial well-being of families in the United States. Excessive credit card debt can lead to financial challenges and prevent individuals and families from saving money or investing money for future expenses.

For several years, average household credit card debt has been on the rise. In 2022, Americans owned more than 841 billion dollars in credit card debt. This equates to a mean credit card debt of $5,221.[5]

Suppose we are interested in comparing the mean credit card debt of households in a city in the United States against the national average. In a random sample of 24 households in the city, the mean credit card debt per household was $5,440, with a sample standard deviation of $510. We will assume that the credit card debts of households in the city are normally distributed. At the 1% level of significance, can we conclude that the mean credit card debt of households in the city is greater than the national average?

Step 1: Determine the Hypotheses

1 A State the null hypothesis.

B State the alternative hypothesis.

Step 2: Collect the Data

2 The problem statement above provided information about the sample obtained for this test.

A Does the sample satisfy the criteria for the approximate normality of the sampling distribution of sample means?

B What is the sample mean?

C Is the sample mean consistent with the alternative hypothesis? Be sure to explain your answer.

[5] https://www.fool.com/the-ascent/research/credit-card-debt-statistics/

Step 3: Assess the Evidence

3 In this step we calculate the test statistic and find the *P*-value.

A Calculate the test statistic for the observed sample mean. Round the test statistic to two decimal places.

B Use the test statistic to find the *P*-value. Round the *P*-value to three decimal places.

Step 4: State a Conclusion

4 Use the results from Step 3 to make a conclusion about the hypotheses.

A Is the sample mean statistically significant? Why or why not?

B State a conclusion in the context of the problem.

5 It is possible that the decision you made in the hypothesis test above is incorrect. Identify the type of error (Type I or II) that was possible, and what must be true about the population mean if this error occurred.

NEXT STEPS

Statistical vs. Practical (Real-World) Significance

In the previous hypothesis test, we concluded that the sample mean of $5,440 was not statistically significant. The sample did not provide sufficient evidence to conclude that the mean debt of households in the city was greater than the national average.

6 The sample mean of \$5,440 differed from the national average of \$5,221. Do you think the difference between the observed sample mean and the national average is significant in a real-world sense? Explain.

7 How would our assessment of statistical significance change if the sample mean of \$5440 was observed from a random sample of n = 100 households in the city? Assuming the sample standard deviation is still s = \$510, determine whether the sample mean of \$5,440 is now statistically significant. Would the conclusion of the hypothesis test change? Explain your answer.

8 Why does the increase in sample size result in such a drastic change in the result of the hypothesis test? Explain your answer.

YOU NEED TO KNOW

- With very large samples, even extremely small differences can be statistically significant.
- With this in mind, remember that statistical significance is different from *real-world significance*. Statistical significance is measured with probability. Real-world significance is measured through our system of personal values, and is therefore difficult to measure.

NEXT STEPS

Generalizing Results

9 The hypothesis test from Questions 1 - 4 was based on data from 24 randomly selected households in a city in the United States. The mean household debt per household was $5,440, with a sample standard deviation of $510. We concluded that the sample results based on this small sample were not statistically significant. Can we generalize the results of the hypothesis test to other cities in the United States? Explain.

10 Under what conditions would it be appropriate to generalize the results of the hypothesis test?

YOU NEED TO KNOW

Statistical inference should never be generalized to a population that is not represented by the sample being used.

LET'S SUMMARIZE

Please consider the following key points:

- A hypothesis test consists of four steps: determine the hypotheses, collect the data, assess the evidence, and state a conclusion.
- With very large samples, even extremely small differences can be statistically significant.
- Statistical significance is different from real-world significance. Statistical significance is measured with probability. Real-world significance is measured through our system of personal values, and is therefore difficult to measure.
- Statistical inference should never be generalized to a population that is not represented by the sample being used.

Exercise 4.4

Mean Speed of Vehicles on a Street

Do cars on your street drive faster than the published speed limit? A student in a statistics class was interested in using sample data to assess whether the mean speed of cars traveling on his street is greater than 30 miles per hour. He lives nearby a radar speed sign which informs drivers on the speed that they are traveling. On a weekday, during the morning hours between 8 am and 10 am, he randomly collected a sample of 20 speeds. The sample had a mean speed of 34.6 miles per hour with a standard deviation of 8.4 miles per hour. We can assume that vehicle speeds on this street during weekday mornings are approximately normally distributed. At the 5% level of significance, can we conclude that the average speed of vehicles on the street during weekday mornings is greater than the speed limit?

1 **Step 1: Determine the Hypotheses**

A State the null hypothesis.

B State the alternative hypothesis.

Step 2: Collect the Data

C Are the criteria for the approximate normality of the sampling distribution of sample means satisfied?

D What is the sample mean?

E What is the sample standard deviation?

2 **Step 3: Assess the Evidence**

A Calculate the test statistic for the observed sample mean. Round your answer to two decimal places.

B Use the test statistic to find the *P*-value, rounded three places after the decimal.

3 **Step 4: State a Conclusion**

A How does the *P*-value compare to the significance level?

B What should we conclude, regarding the null and alternative hypotheses?

C Write a statement that best describes the conclusion in the context of this scenario.

D Is it possible that the decision you made in the hypothesis test above is incorrect? Identify the type of error (Type I or II) that was possible, and what must be true about the population mean if this error occurred.

E What must be true about the population mean if a Type I error occurred?

F Would it be appropriate to generalize the results of this test to the mean speed of vehicles on the same street during weekday evening hours? Explain.

Mean Cost of Public Higher Education

The National Center for Education Statistics[6] reports that the mean annual total cost of undergraduate education at 2- and 4-year public colleges in the U.S. in the 2020-2021 academic year was $13,300. The total cost includes the cost of tuition and fees. Suppose we find that a random sample of 35 public colleges in the U.S. in the 2021-2022 academic year has a mean annual total cost of $13,900 with a standard deviation of $1,860. We can assume that annual total costs of undergraduate education at public institutions are approximately normally distributed.

4 Use the sample statistics to construct a 95% confidence interval to estimate the mean annual total cost of undergraduate education at public colleges in the U.S. in the 2021-2022 academic year. Complete the steps below.

A Determine the critical value that corresponds to the 95% confidence level. Round your critical value to two decimal places.

B Compute the margin of error (E) for a 95% confidence interval. Round the margin of error to two decimal places. The formula for the margin of error is: $E = T_c \cdot \frac{s}{\sqrt{n}}$

C Construct a 95% confidence interval for the population mean. What is the lower bound of the confidence interval? Round your answer to two decimal places.

[6] https://nces.ed.gov/fastfacts/display.asp?id=76

D What is the upper bound of the interval? Round your answer to two decimal places.

E Write a statement that best describes the meaning of the confidence interval in the context of the problem.

F Recall that The National Center for Education Statistics reports that the mean annual total cost of undergraduate education at public colleges in the U.S. in the 2020-2021 academic year was $13,300. Is $13,300 included in the 95% confidence interval? What does that signify?

You will now perform a hypothesis test to investigate the same question. Perform a hypothesis test at the 5% level of significance to determine whether the sample provides evidence that the mean annual total cost of undergraduate education at public colleges in the U.S. in 2021-2022 has changed since 2020-2021.

The data from the 2020-2021 study and 2021-2022 sample are summarized in the table below.

Public Colleges in 2020-2021	Sample of Public Colleges in 2021-2022
Mean Tuition = $13,300	$\bar{x}$ =13,900 s = 1,860 n = 35

5 **Step 1: Determine the Hypotheses**

A State the null hypothesis.

B State the alternative hypothesis.

Step 2: Collect the Data

C Are the criteria for the approximate normality of the sampling distribution of sample means satisfied?

D What is the sample mean?

E What is the sample standard deviation?

6 **Step 3: Assess the Evidence**

A Calculate the test statistic for the observed sample mean. Round your answer to two decimal places.

B Use the test statistic to find the *P*-value. Round your answer to three decimal places.

7 **Step 4: State a Conclusion**

A How does the *P*-value compare to the significance level?

B What should we conclude regarding the null and alternative hypotheses?

C Write a statement that best describes the conclusion in the context of this scenario.

8 In Question 4, you created a confidence interval to investigate whether or not the mean tuition at public colleges had changed from 2020-2021 to 2021-2022. In Question 5, you conducted a hypothesis test to investigate whether or not the mean tuition at public colleges had changed from 2020-2021 to 2021-2022. Which statement below best matches the results of these two inference methods?

(i) The results of the confidence interval and the hypothesis test match. They both show a change in the mean tuition for public colleges.

(ii) The results of the confidence interval and the hypothesis test match. Neither one proves a change in the mean tuition for public colleges.

(iii) The results of the confidence interval and the hypothesis test do not match.

4.4 Monitor (survey)

Monitor your progress on learning the objectives for this unit. If you identify any objectives you need to review, go back through the unit's activities. Your responses are not graded, but will be available for your instructor to see.

Rate how confident you are on a scale of 1–5 (1 = not confident and 5 = very confident).

Skill or Concept: I can …	**Rating from 1 to 5**
Apply the four-step hypothesis testing procedure to test a claim about a population mean when the population standard deviation is unknown.	
Interpret the results of a hypothesis test.	
Decide when a statistical result can be applied to a larger population.	
Distinguish between statistical and real-world significance.	