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## MODULE 3

### Syllabus

**Shear Force and Bending Moment:** Type of beams, Loads and reactions, Relationship between loads, shear forces and bending moments, Shear force and bending moments of cantilever beams, Pin support and roller supported beams subjected to concentrated loads, uniformly distributed constant / varying loads.

**Stress in Beams:** Bending and shear stress distribution in rectangular, I and T section beams.

### Introduction:

In many engineering structures members are required to resist forces that are applied laterally or transversely to their axes. These types of members are termed as beams.

There are various ways to define the beams such as

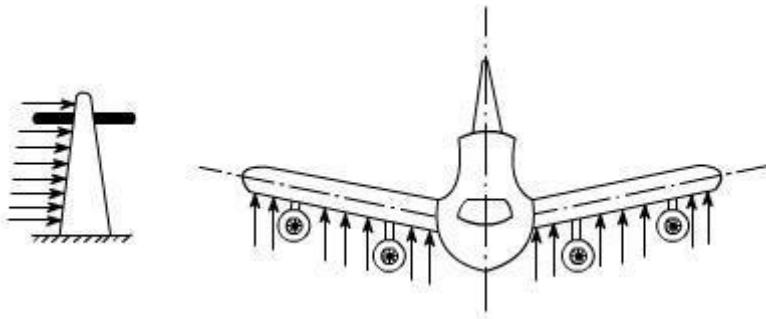
**Definition I:** A beam is a laterally loaded member, whose cross-sectional dimensions are small as compared to its length.

**Definition II:** A beam is nothing simply a bar which is subjected to forces or couples that lie in a plane containing the longitudinal axis of the bar. The forces are understood to act perpendicular to the longitudinal axis of the bar.

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### Examples of Beams:

Refer to the figures shown below that illustrates the beam

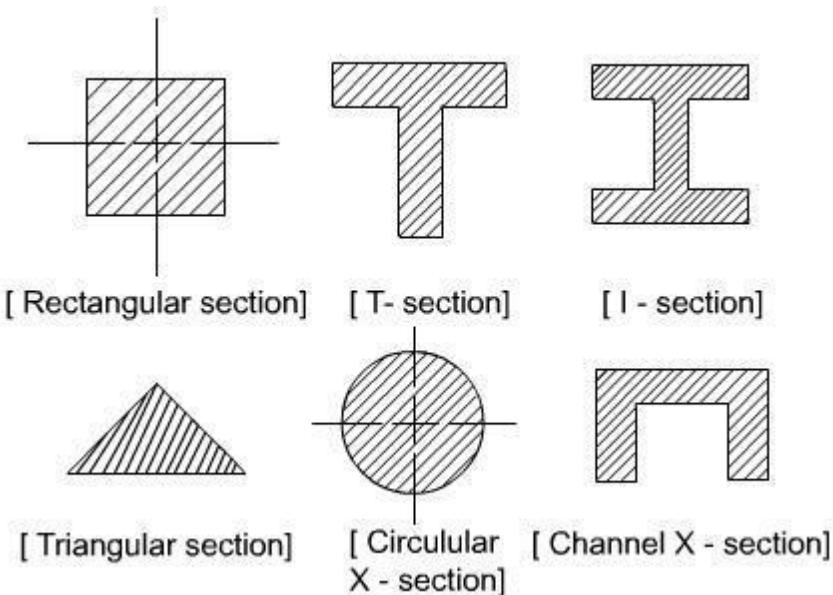


In the fig.1, an electric pole has been shown which is subject to forces occurring due to wind; hence it is an example of beam.

In the fig.2, the wings of an aero plane may be regarded as a beam because here the aerodynamic action is responsible to provide lateral loading on the member

### Geometric forms of Beams:

The Area of X-section of the beam may take several forms some of them have been shown below



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### Issues Regarding Beam:

Designer would be interested to know the answers to following issues while dealing with beams in practical engineering application

- At what load will it fail
- How much deflection occurs under the application of loads.

### Types of Beams / Classification of Beams:

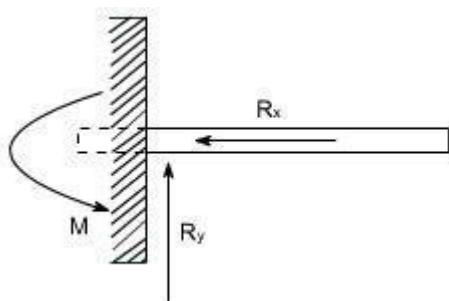
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Beams are classified on the basis of their geometry and the manner in which they are supported.

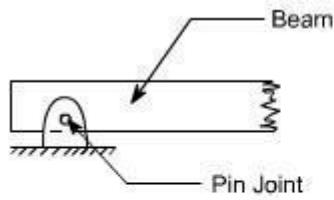
**Classification I:** The classification based on the basis of geometry normally includes features such as the shape of the X-section and whether the beam is straight or curved.

**Classification II:** Beams are classified into several groups, depending primarily on the kind of supports used. But it must be clearly understood why do we need supports. The supports are required to provide constraint to the movement of the beams or simply the supports resist the movements either in particular direction or in rotational direction or both. As a consequence of this, the reaction comes into picture whereas to resist rotational movements the moment comes into picture. On the basis of the support, the beams may be classified as follows:

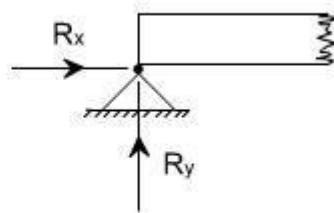
**Cantilever Beam:** A beam which is supported on the fixed support is termed as a cantilever beam: Now let us understand the meaning of a fixed support. Such a support is obtained by building a beam into a brick wall, casting it into concrete or welding the end of the beam. Such a support provides both the translational and rotational containment to the beam, therefore the reaction as well as the moments appears, as shown in the figure below



**Simply Supported Beam:** The beams are said to be simply supported if their supports create only the translational constraints:

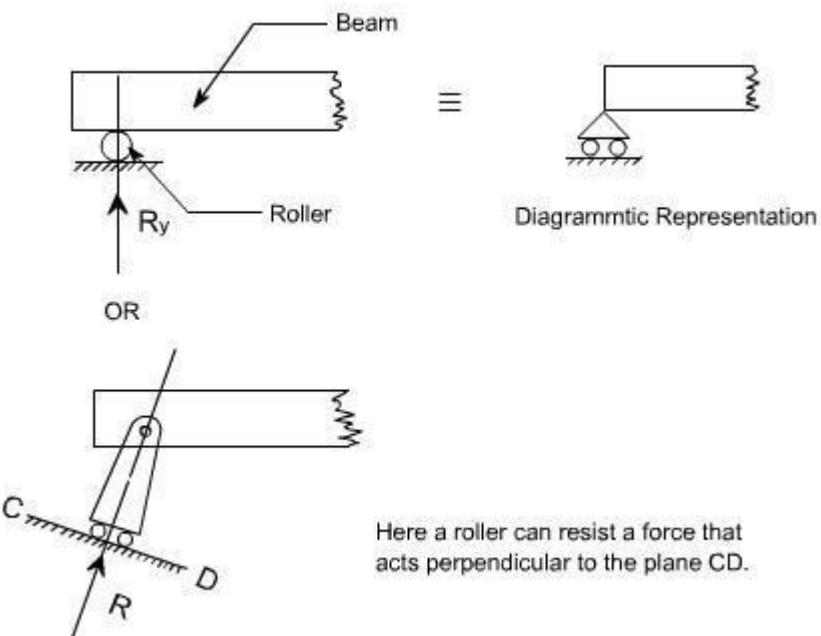


(a) Actual Representation



(b) Diagrammatic Representation

Some times the translational movement may be allowed in one direction with the help of rollers and can be represented like this



### **Statically Determinate or Statically Indeterminate Beams:**

The beams can also be categorized as statically determinate or else it can be referred as statically indeterminate. If all the external forces and moments acting on it can be determined from the equilibrium conditions alone then. It would be referred as a statically determinate beam, whereas in the statically indeterminate beams one has to consider deformation i.e. deflections to solve the problem.

#### **Types of loads acting on beams:**

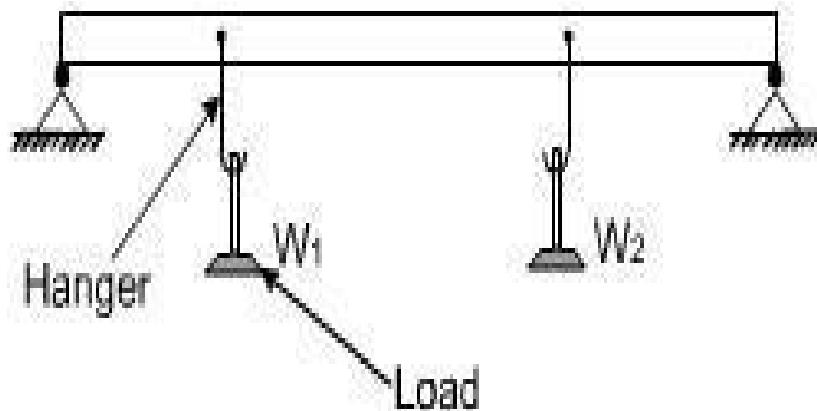
**(June-July 2018)**

A beam is normally horizontal whereas the external loads acting on the beams is generally in the vertical directions. In order to study the behaviors of beams under flexural loads. It becomes pertinent that one must be familiar with the various types of loads acting on the beams as well as their physical manifestations.

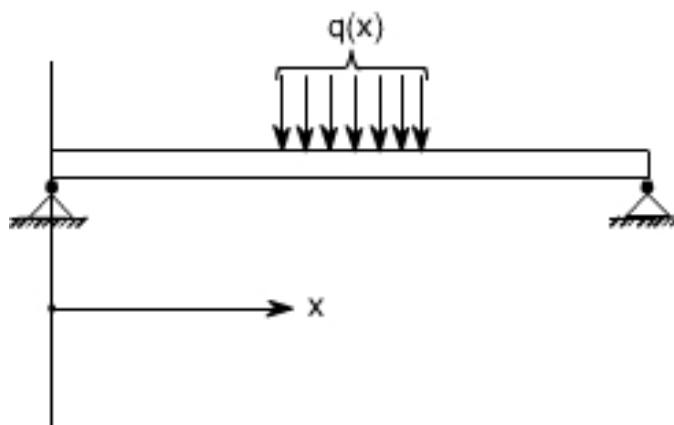
**A. Concentrated Load:** It is a kind of load which is considered to act at a point. By this we mean that the length of beam over which the force acts is so small in comparison to its total

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length that one can model the force as though applied at a point in two-dimensional view of beam. Here in this case, force or load may be made to act on a beam by a hanger or through other means

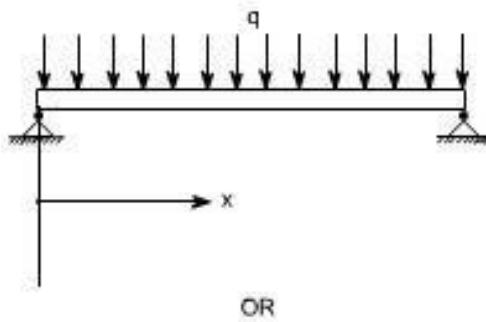


B. **Distributed Load:** The distributed load is a kind of load which is made to spread over a entire span of beam or over a particular portion of the beam in some specific manner

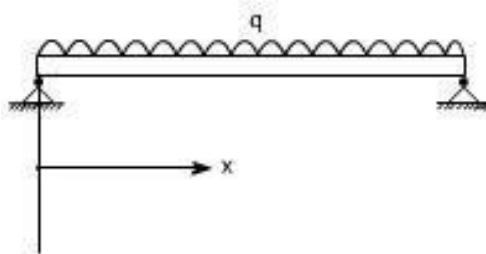


In the above figure, the rate of loading 'q' is a function of  $x$  i.e. span of the beam, hence this is a non-uniformly distributed load.

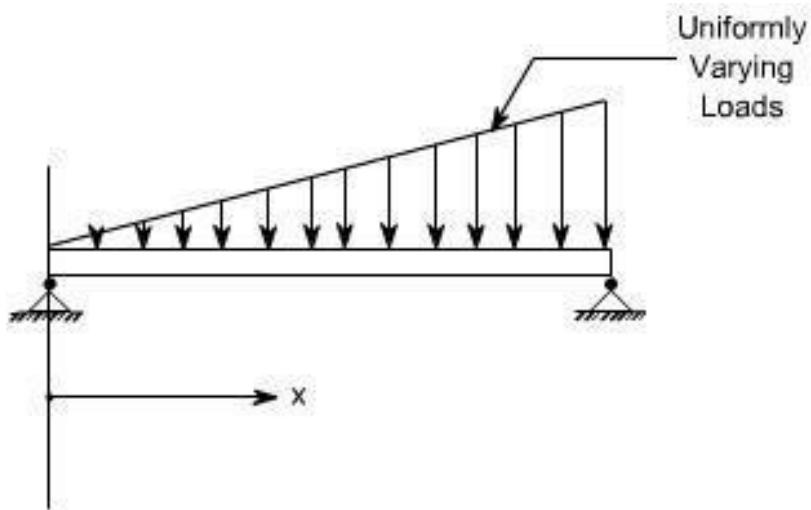
The rate of loading 'q' over the length of the beam may be uniform over the entire span of beam, then we call this as a uniformly distributed load (U.D.L). The U.D.L may be represented in either of the way on the beams



OR



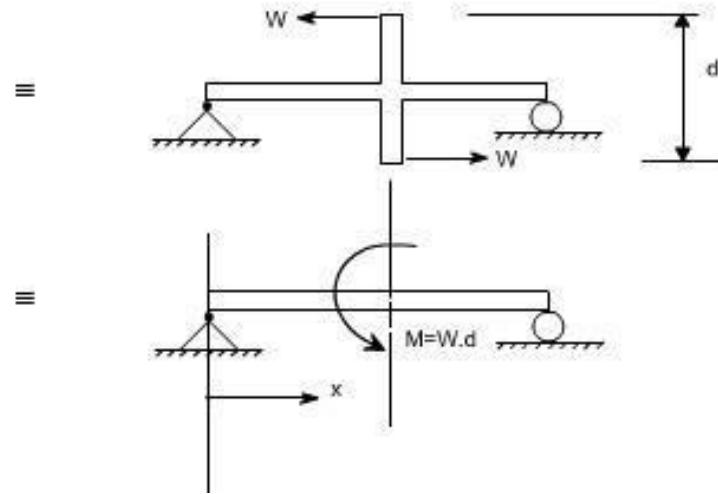
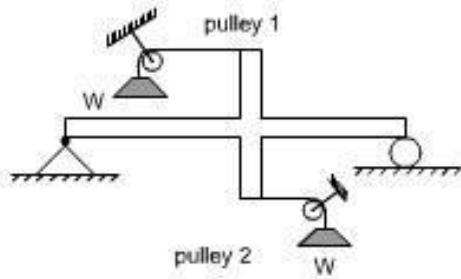
Some times the load acting on the beams may be the uniformly varying as in the case of dams or on inclind wall of a vessel containing liquid, then this may be represented on the beam as below:



The U.D.L can be easily realized by making idealization of the ware house load, where the bags of grains are placed over a beam.

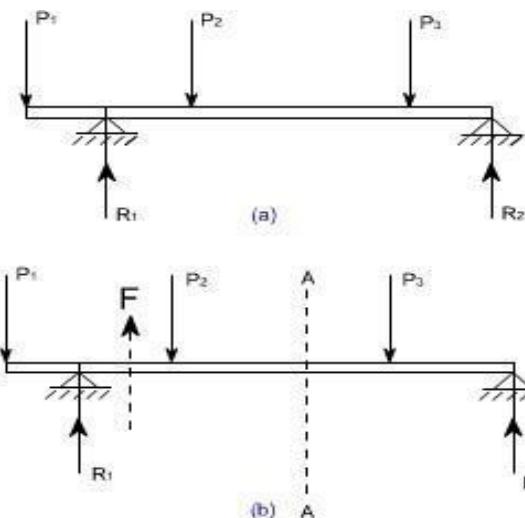
### Concentrated Moment:

The beam may be subjected to a concentrated moment essentially at a point. One of the possible arrangements for applying the moment is being shown in the figure below:



### Concept of Shear Force and Bending moment in beams:

When the beam is loaded in some arbitrarily manner, the internal forces and moments are developed and the terms shear force and bending moments come into pictures which are helpful to analyze the beams further. Let us define these terms



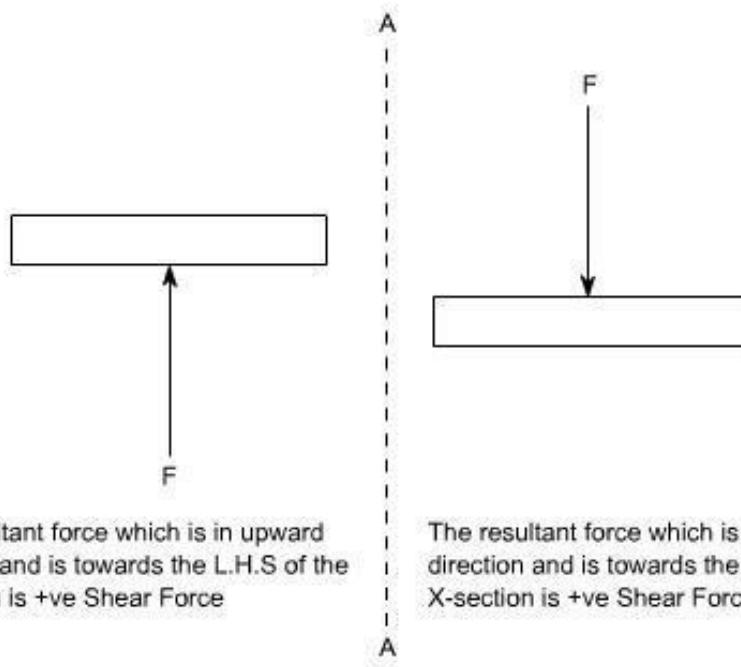
- Now let us consider the beam as shown in fig 1(a) which is supporting the loads  $P_1$ ,  $P_2$ ,  $P_3$  and is simply supported at two points creating the reactions  $R_1$  and  $R_2$  respectively.
- Now let us assume that the beam is to be divided into or imagined to be cut into two portions at a section AA.
- Now let us assume that the resultant of loads and reactions to the left of AA is  $F$  vertically upwards,
- And since the entire beam is to remain in equilibrium, thus the resultant of forces to the right of AA must also be  $F$  acting downwards.
- This force  $F$  is a shear force. The shearing force at any x-section of a beam represents the tendency for the portion of the beam to one side of the section to slide or shear laterally relative to the other portion.

Therefore, now we are in a position to define the shear force  $F$  as follows:

At any x-section of a beam, the shear force  $?F'$  is the algebraic sum of all the lateral components of the forces acting on either side of the x-section.

### Sign Convention for Shear Force:

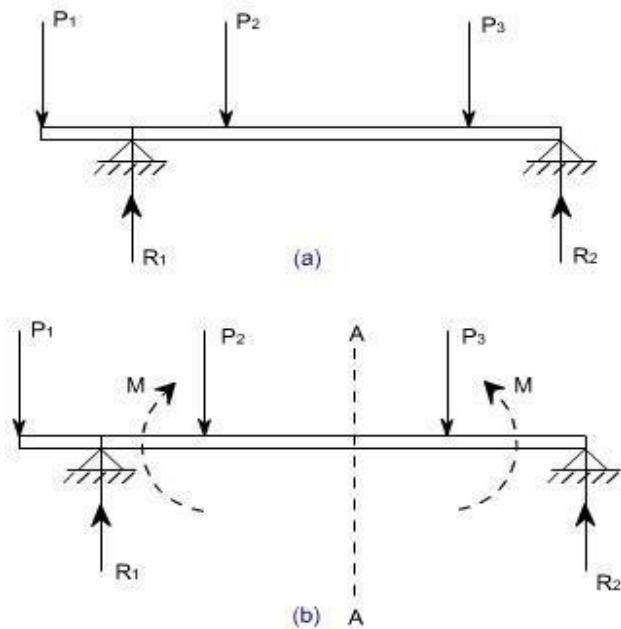
The usual sign conventions to be followed for the shear forces have been illustrated in figures 2 and 3.



**Fig 2: Positive Shear Force**

**Fig 3: Negative Shear Force**

### Bending Moment:



**Fig 4**

Let us again consider the beam which is simply supported at the two prints, carrying loads  $P_1$ ,  $P_2$  and  $P_3$  and having the reactions  $R_1$  and  $R_2$  at the supports Fig 4. Now, let us imagine that the beam is cut into two portions at the x-section AA. In a similar manner, as done for the case of shear force, if we say that the resultant moment about the section AA of all the loads and reactions to the left of the x-section at AA is  $M$  in C.W direction, then moment of forces to the right of x-section AA must be  $M$  in C.C.W. Then  $M$  is called as the Bending moment and is abbreviated as B.M. Now one can define the bending moment to be simply as the algebraic sum of the moments about an x-section of all the forces acting on either side of the section

### Sign Conventions for the Bending Moment:

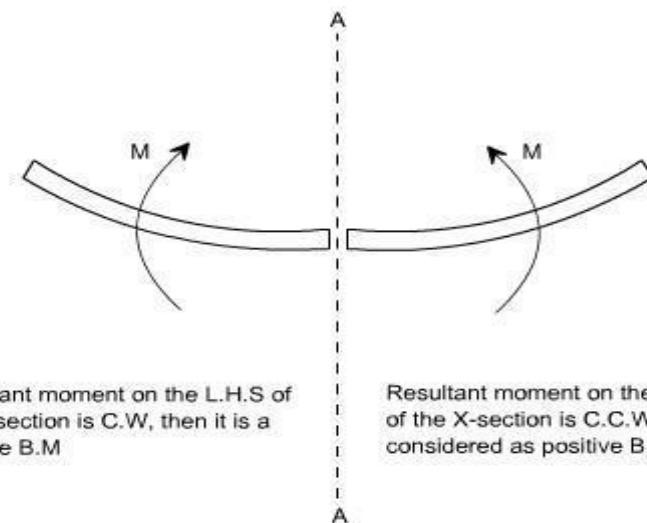
For the bending moment, following sign conventions may be adopted as indicated in Fig 5 and Fig 6.

### Bending Moment and Shear Force Diagrams:

The diagrams which illustrate the variations in B.M and S.F values along the length of the beam for any fixed loading conditions would be helpful to analyze the beam further.

Thus, a shear force diagram is a graphical plot, which depicts how the internal shear force 'F' varies along the length of beam. If  $x$  denotes the length of the beam, then  $F$  is function  $x$  i.e.  $F(x)$ .

Similarly a bending moment diagram is a graphical plot which depicts how the internal bending moment 'M' varies along the length of the beam. Again  $M$  is a function  $x$  i.e.  $M(x)$ .



**Fig 5: Positive Bending Moment**

**Fig 6: Negative Bending Moment**

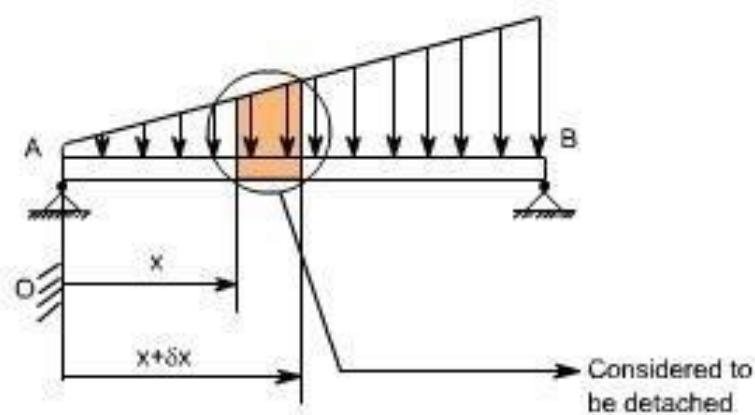
Some times, the terms 'Sagging' and Hogging are generally used for the positive and negative bending moments respectively.

#### Basic Relationship Between the Rate of Loading, Shear Force and Bending Moment:

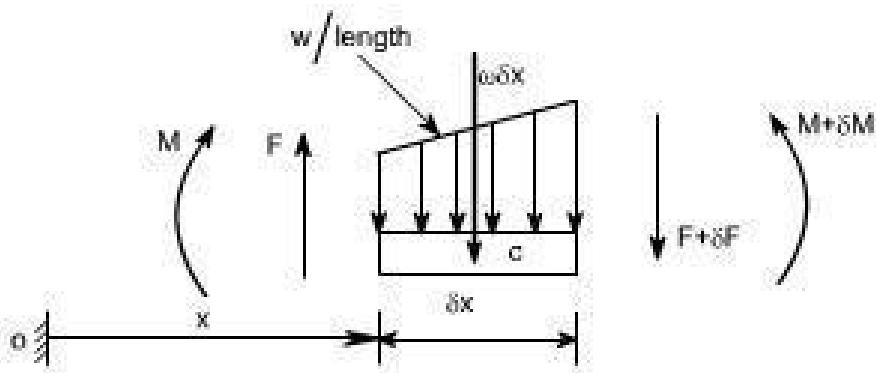
(VTU Dec-Jan 2018)

The construction of the shear force diagram and bending moment diagrams is greatly simplified if the relationship among load, shear force and bending moment is established.

Let us consider a simply supported beam AB carrying a uniformly distributed load  $w/\text{length}$ . Let us imagine to cut a short slice of length  $dx$  cut out from this loaded beam at distance 'x' from the origin '0'.



Let us detach this portion of the beam and draw its free body diagram.



The forces acting on the free body diagram of the detached portion of this loaded beam are the following

- The shearing force  $F$  and  $F + \delta F$  at the section  $x$  and  $x + \delta x$  respectively.
- The bending moment at the sections  $x$  and  $x + \delta x$  be  $M$  and  $M + \delta M$  respectively.
- Force due to external loading, if ' $w$ ' is the mean rate of loading per unit length then the total loading on this slice of length  $\delta x$  is  $w \cdot \delta x$ , which is approximately acting through the centre ' $c$ '. If the loading is assumed to be uniformly distributed then it would pass exactly through the centre ' $c$ '.

This small element must be in equilibrium under the action of these forces and couples. Now let us take the moments at the point ' $c$ '. Such that

$$\begin{aligned}
 M + F \cdot \frac{\delta x}{2} + (F + \delta F) \cdot \frac{\delta x}{2} &= M + \delta M \\
 \Rightarrow F \cdot \frac{\delta x}{2} + (F + \delta F) \cdot \frac{\delta x}{2} &= \delta M \\
 \Rightarrow F \cdot \frac{\delta x}{2} + F \cdot \frac{\delta x}{2} + \delta F \cdot \frac{\delta x}{2} &= \delta M \quad [\text{Neglecting the product of} \\
 &\quad \delta F \text{ and } \delta x \text{ being small quantities}]
 \end{aligned}$$

$$\Rightarrow F \cdot \delta x = \delta M$$

$$\Rightarrow F = \frac{\delta M}{\delta x}$$

Under the limits  $\delta x \rightarrow 0$

$$\boxed{F = \frac{dM}{dx}} \quad \dots \dots \dots \quad (1)$$

Resolving the forces vertically we get

$$w \cdot \delta x + (F + \delta F) = F$$

$$\Rightarrow w = -\frac{\delta F}{\delta x}$$

Under the limits  $\delta x \rightarrow 0$

$$\Rightarrow w = -\frac{dF}{dx} \text{ or } -\frac{d}{dx}\left(\frac{dM}{dx}\right)$$

$$\boxed{w = -\frac{dF}{dx} = -\frac{d^2M}{dx^2}} \quad \dots \dots \dots \quad (2)$$

**Conclusions:** From the above relations, the following important conclusions may be drawn

- From Equation (1), the area of the shear force diagram between any two points, from the basic calculus is the bending moment diagram

$$M = \int F \cdot dx$$

- The slope of bending moment diagram is the shear force, thus

$$F = \frac{dM}{dx}$$

- Thus, if  $F=0$ ; the slope of the bending moment diagram is zero and the bending moment is therefore constant.
- The maximum or minimum Bending moment occurs where

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$$\frac{dM}{dx} = 0$$

- The slope of the shear force diagram is equal to the magnitude of the intensity of the distributed loading at any position along the beam. The +ve sign is as a consequence of our particular choice of sign conventions