# Calculus II | Group Work Answer Key

Answer key • Thursday, October 29, 2026 • Section 6.4

## 1. Who Is Correct?

Student B. Rotation about the y-axis gives r = x (horizontal distance). $y' = 2x$.
$S = 2\pi\int_0^1 x\sqrt{(1 + 4x^2)}\,dx$. Let $u = 1 + 4x^2$, $du = 8x\,dx$.
$S = (\pi/4)\int_1^5 u^{(1/2)}\,du = (\pi/6)(5\sqrt{5} - 1)$.

## 2. Predict Before Calculating

Since $y = x + 2 > x$ on [0,1], the x-axis radius is larger pointwise; arc-length factors match. $y' = 1$.
$S_x = 2\pi\sqrt{2}\int_0^1 (x + 2)\,dx = 5\pi\sqrt{2}$.
$S_y = 2\pi\sqrt{2}\int_0^1 x\,dx = \pi\sqrt{2}$. Thus $S_x > S_y$.

## 3. Find the Error

The derivative must be squared: $(y')^2 = (2x)^2 = 4x^2$, not 2x.
$S = 2\pi\int_1^2 x\sqrt{(1 + 4x^2)}\,dx = (\pi/6)[(1 + 4x^2)^{(3/2)}]_1^2$
$= (\pi/6)(17\sqrt{17} - 5\sqrt{5})$.

## 4. Design Your Own Curve

Many valid answers. Example: $f(x) = x^2 + 2$ (nonlinear and nonnegative on [0,1]).
$f' = 2x$; $S_x = 2\pi\int_0^1 (x^2 + 2)\sqrt{(1 + 4x^2)}\,dx$; $S_y = 2\pi\int_0^1 x\sqrt{(1 + 4x^2)}\,dx$.
Because $x^2 + 2 > x$ on [0,1] and $\sqrt{(1 + 4x^2)} > 0$, $S_x > S_y$ without evaluation.
$S_y = (\pi/6)(5\sqrt{5} - 1)$. $S_x$ need not be evaluated; it is beyond a straightforward u-substitution.

**Nonlinear vs linear**

**1. What is the difference?**

A straight line has a **constant slope**, while a nonlinear curve generally has a slope that changes as you move along it.

- **Straight line:** y = x + 2 → y' = 1 (constant slope)
- **Nonlinear curve:** y = x² + 2 → y' = 2x (changing slope)

**2. Do we use different surface area formulas?**

No. Both straight lines and nonlinear curves can be revolved around an axis to generate a surface. The same formula applies to both:

**Surface Area: $S = 2\pi \int r\,ds$**

where:

- r = distance from the curve to the axis of rotation
- ds = a small piece of arc length

For y = f(x):

**$ds = \sqrt{(1 + (y')^2)}\,dx$**

Therefore:

**$S = 2\pi \int_a^b r(x)\,\sqrt{(1 + (y')^2)}\,dx$**

**3. How does this affect integration?**

**Straight line: y = x + 2**

- y' = 1

- $ds = \sqrt{(1 + 1^2)}\, dx = \sqrt{2}\, dx$
- The arc length factor is constant, so integration is usually simpler.

**Nonlinear curve: $y = x^2 + 2$**

- $y' = 2x$
- $ds = \sqrt{(1 + 4x^2)}\, dx$
- The arc length factor changes with x, so integration may be more complicated.

However, a nonlinear curve does not always produce a difficult integral. Sometimes the expressions simplify.

**4. Does a straight line produce the same surface area about both axes?**

Not necessarily!

Consider $y = x + 2$ on $0 \le x \le 1$.

**About the x-axis:**

- Radius: $r = y = x + 2$
- $S_x = 2\pi \int_0^1 (x + 2)\sqrt{2}\, dx$
- $\mathbf{S_x = 5\pi\sqrt{2}}$

**About the y-axis:**

- Radius: $r = x$
- $S_Y = 2\pi \int_0^1 x\sqrt{2}\, dx$
- $\mathbf{S_Y = \pi\sqrt{2}}$

Therefore, $\mathbf{S_x > S_Y}$.

Even though we are revolving the same straight line, the surface areas are different because the distances from the axes are different.

**5. What does "curve" mean in calculus?**

In calculus, the word *curve* can include straight lines. A straight line is a special case of a curve.

- **Choose a curve:** A straight line or nonlinear curve may be used.
- **Choose a nonlinear function:** The graph must not be a straight line.

**Key Takeaway**

The important distinction is not simply whether the graph is straight or curved, but how far it is from the axis of rotation and how its arc length is calculated.

Always ask yourself:

1. **What is the radius?** How far is the curve from the axis of rotation?
2. **What is the arc length factor?** How does the slope affect the length of the curve?

**Same formula. Different radii and slopes. Different surface areas.**

## 5. Same Area, Different Position?

For both, $y' = 1$ and y-axis radius $r = x$.
$S_f = S_g = 2\pi\int_0^2 x\sqrt{2}\, dx = 4\pi\sqrt{2}$.

About the x-axis, radii differ: $f(x) = x$, $g(x) = x + 3$. Therefore areas differ.
Optional check: $S_x,f = 4\pi\sqrt{2}$; $S_x,g = 16\pi\sqrt{2}$.

### 6. Curved Surface Challenge

$y = \sqrt{(4 - x^2)}$ is the upper semicircle of radius 2; on [0,1] it is a portion of the circle.
$y' = -x/\sqrt{(4 - x^2)}$. Since $y > 0$ on [0,1],
$S = 2\pi\int_0^1 \sqrt{(4 - x^2)}\ \sqrt{[1 + x^2/(4 - x^2)]}\ dx$
$= 2\pi\int_0^1 \sqrt{(4 - x^2)} \cdot 2/\sqrt{(4 - x^2)}\ dx = 2\pi\int_0^1 2\ dx = 4\pi$.
Extending to [0,2] gives $8\pi$, twice as much. At $x = 2$, the derivative is undefined, but the improper endpoint is integrable by continuity of the simplified integrand. This is the surface area of a hemisphere of radius 2.