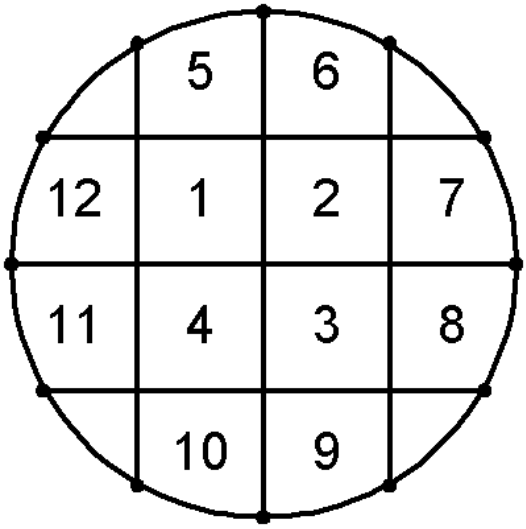
Moderation Study
November 2017

Math B-2

“Calculus”

Calculus Mastery Passage

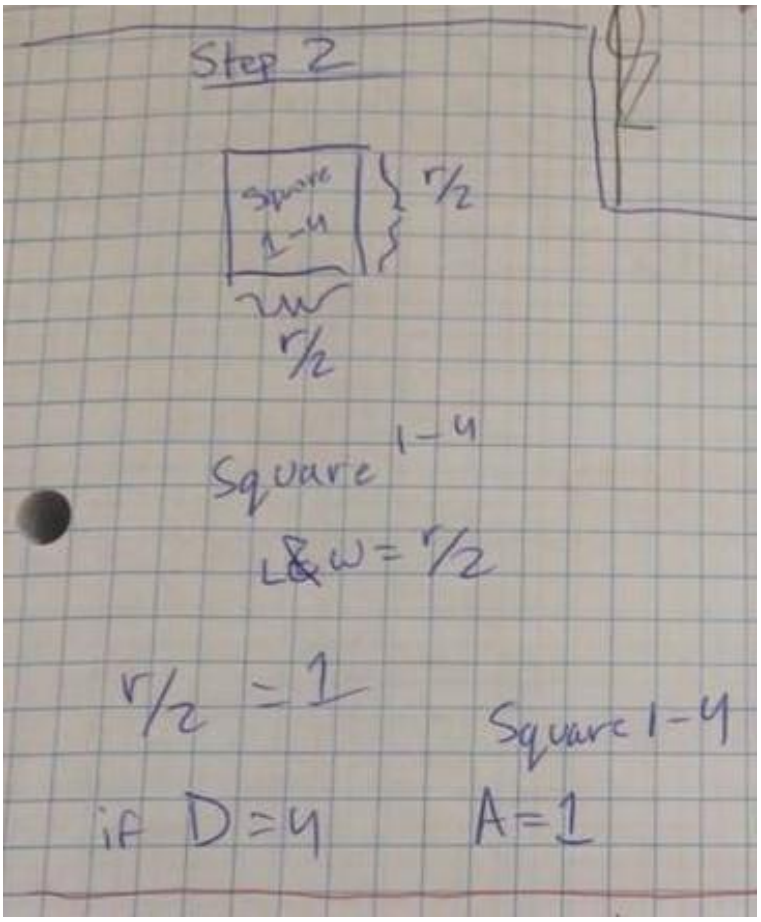
For my calculus mastery passage, I had to solve a problem from the real world using mathematics strategies. My problem was “We watched a pizza-cooking contest on the Food Network on TV. The master chefs who presented their creations cut their pizzas in an unusual way (see picture), creating 4 squares in the center, 8 slightly rounded pieces, and four smaller “triangle-like” pieces. Where do you need to cut the pizza so that your 12 larger pieces are the same size? Suppose the pizza’s diameter is D.”



In this problem I decided to graph out the pizza shape because I thought it would help me to figure out the area of the pieces. The pizza has been cut in six places, two crossing cuts across the diameter, and then four cuts made at a quarter of the way down on the diameters. With this information I created a step by step plan:

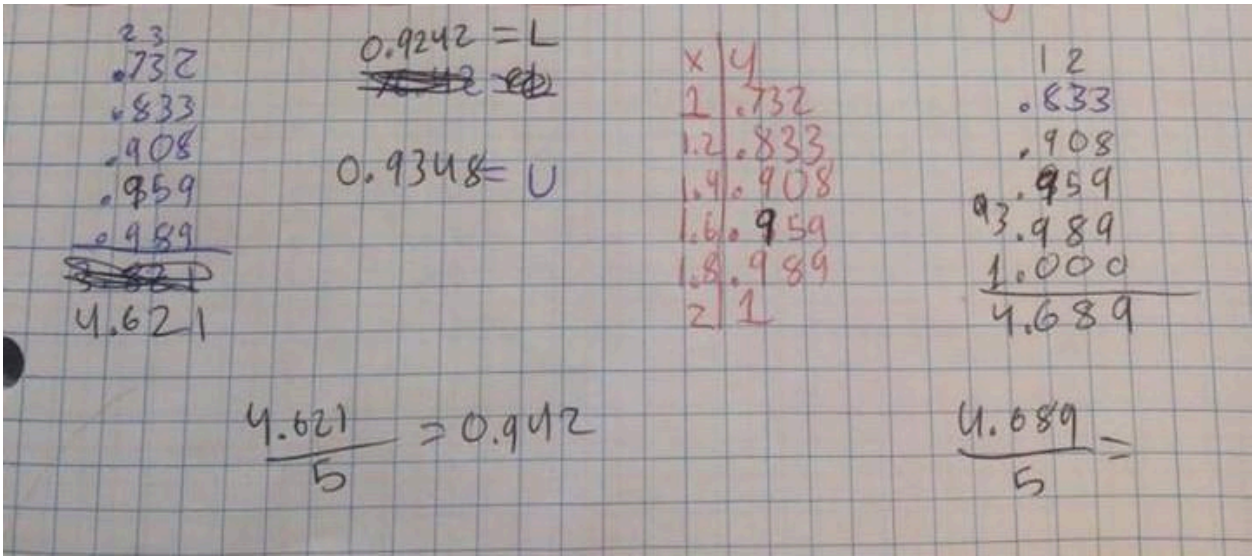
1. Find the values of D, r, r/2
2. Find the area of squares 1-4
3. Find the area of shapes 5-12 by:
    a. Figuring out the lengths of the upper and lower bound rectangles
    b. Multiplying the upper and lower lengths by the width
    c. Then find the average area
4. Then I find the difference of the areas
5. Take half the difference and add to the smaller area to find the proposed area
6. Then we take the square root of that number to find the length and width of square 1
7. Subtract that from the original cutting point to see how far the new cutting point is
8. Then add this number to the larger length of shape 5
9. Then find the smaller length of shape 5
10. Then find the lower bound, upper bound, and average areas

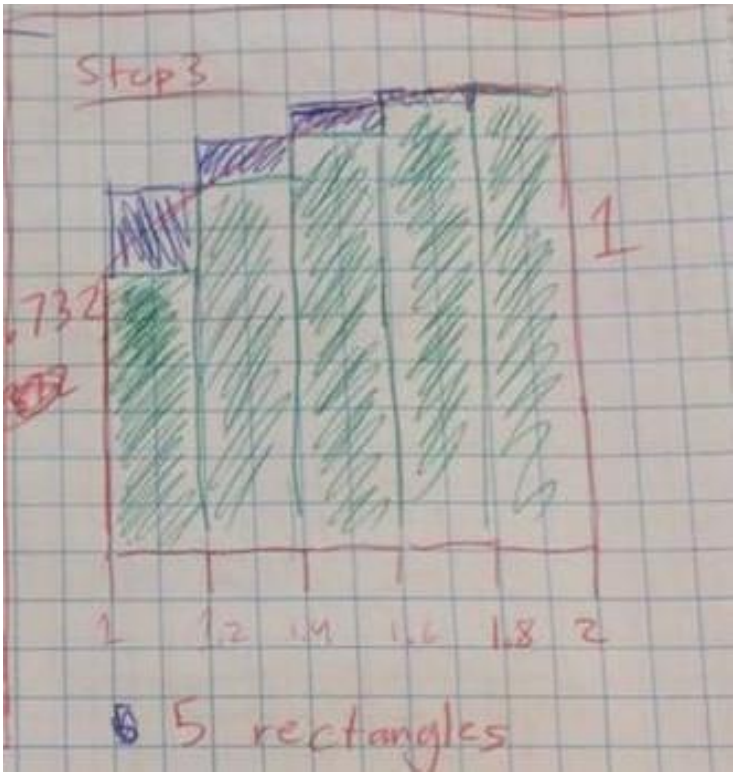
11. Step 1: I started off by using 4 as my value for D. This means that the radius of the circle is 2 and that r/2 or half of the radius is 1.



Step 2: To find the area of squares 1-4 was pretty easy, since they're squares. I know that the length and width is equal to r/2 which is 1 in this case. This would make the area of square 1-4 equal to 1.

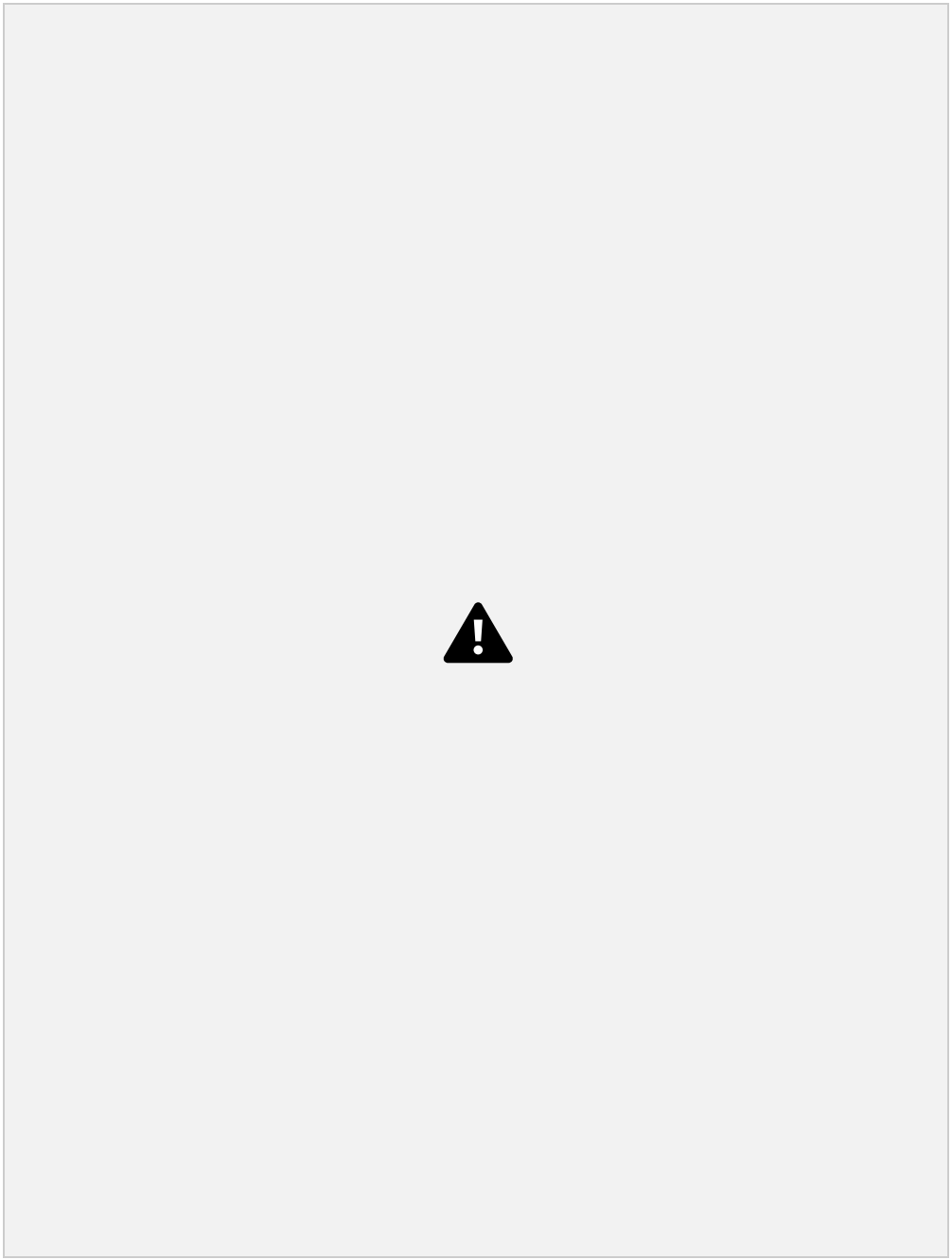
Step 3: In order to find the area of the shape I need to do area under a curve. I needed the height of the smaller length in order to do this, so I graphed a circle with desmos with a diameter of 4, and found that the that smaller length is about .732. Now I need to find the area, which requires me to use multiple smaller rectangles in between, one set are lower bound and one upper in order to find the area. The green rectangles are the lower bound rectangles, which have the width of .2, the horizontal measurement, and a height of where the circle touches the left corner. This also pertains to the upper bound rectangles except with the right corner of the rectangle and not the left. The table in red in the picture below depicts the rectangle's height and width, which I will add together and divide by 5 in order to get and average height, which can be seen with the lower bound rectangles on the far left and the upper bound on the far right





Then I multiplied the lower and upper bound average lengths by the width, which is 1 so that means that the area is the same as the length. Then I added the two lengths together,

and divide by 2. This gives me the approximate area of shapes 5-12, which is .9295.



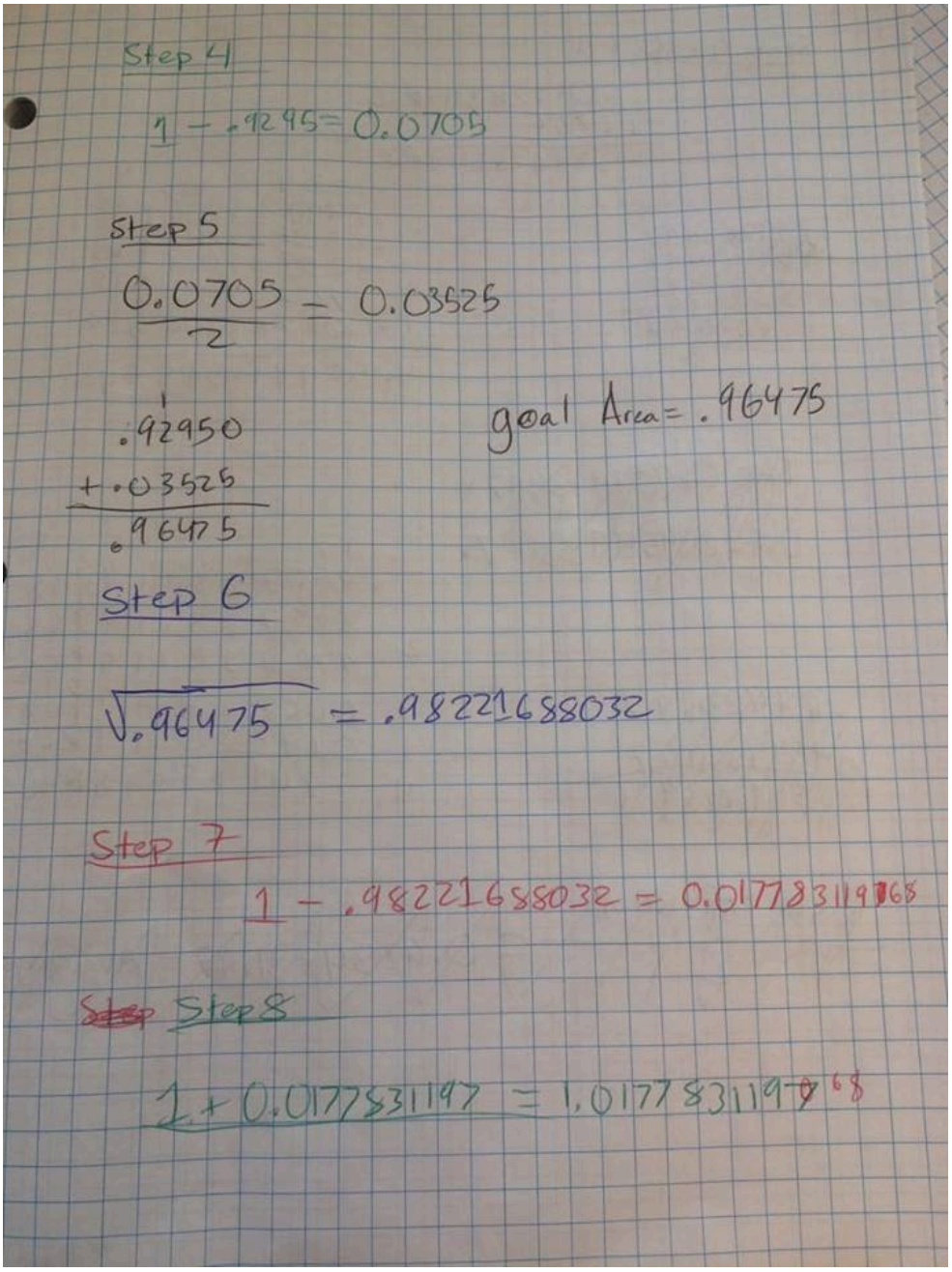
Step 4: I found the difference between the two areas to get a grasp of how far apart they were and how far I need to get to to have them be equal. The area of shapes 1-4 was

simply 1 and the areas of shapes 5-12 was .9295. So if you subtract .9295 from 1 you get 0.0705. The two areas were very slightly off in this particular setting. With these decimals we can approximate that the circle cuts off about 7% of the area if the shape were a square.

Step 5: In this step I half the difference between the areas and I add it to the smaller area. This will make pieces 5-12 larger while making pieces 1-4 smaller. I add it to the smaller area to find out what the new area I will propose will be. The new area ends up being .96475

Step 6: I square root the new area to find the length that square 1-4 will be. Since I am making the changes to the 4 cuts at halfway through the radius, it will not change the shape itself, just the dimensions. So the length and width of square 1-4 is going to be

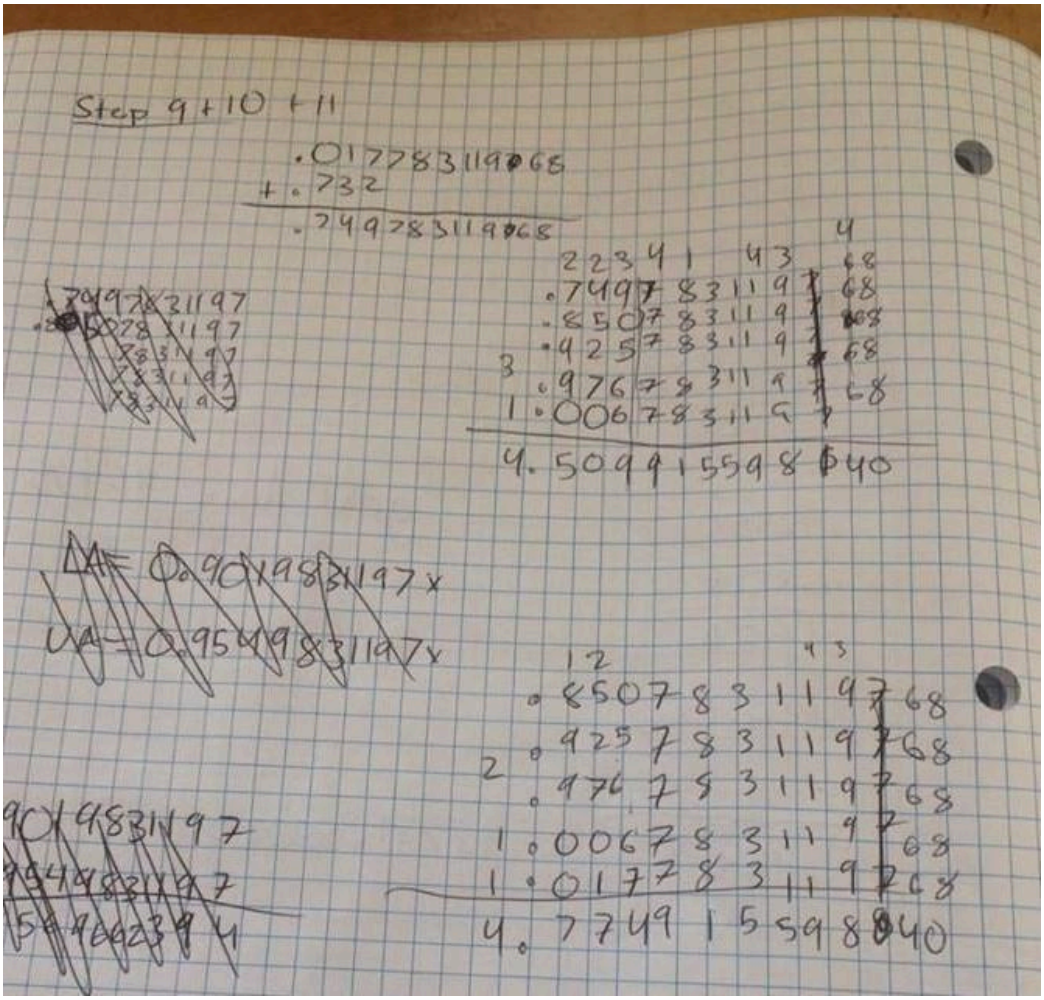
.98221688032. This will also be the size of the width of shapes 5-12.



Step 7: Now I subtract the new length of squares 1-4 from the old length,

1-.98221688032 and this will show me far the new cut will be from the old one. This new

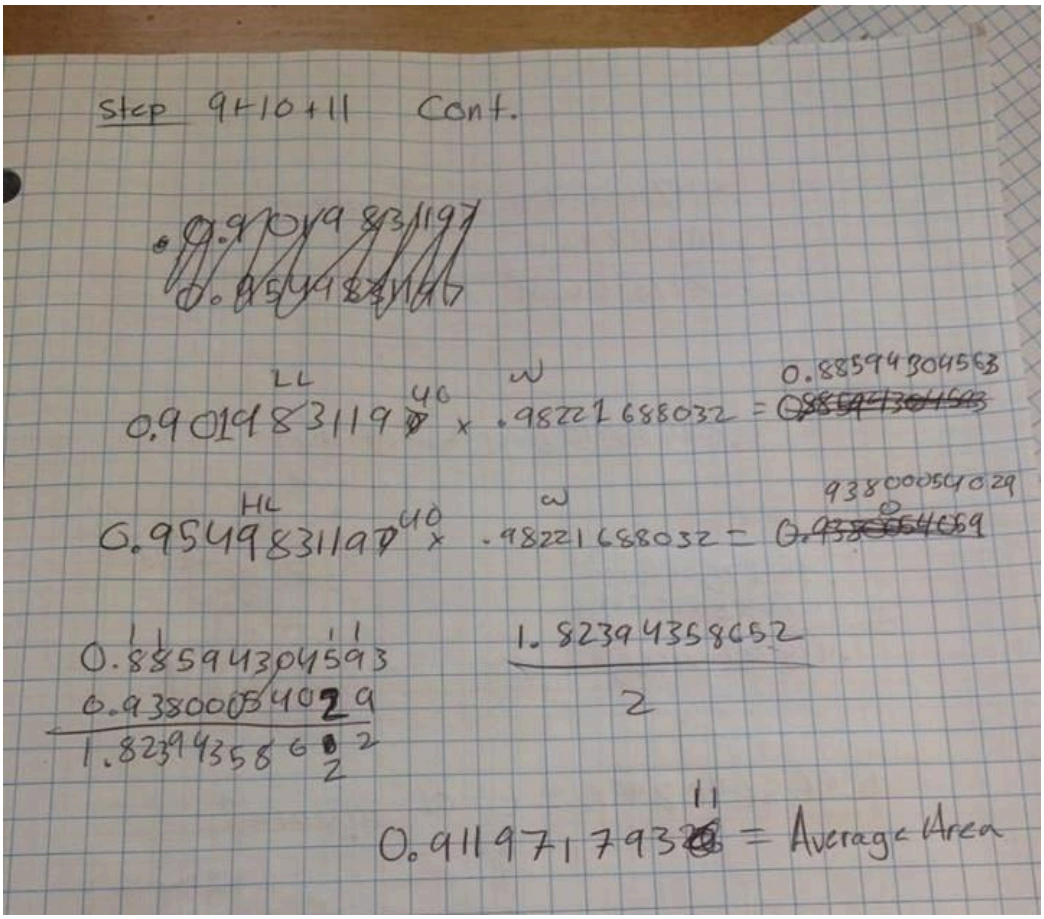
cut will be 0.01778211968 closer to the diameter.



Step 8: I find the larger length of shapes 5-12 by adding the number we got from step seven to it.

Step 9: I find the smaller length by adding 0.01778211968 (the distance from the old cut and new one) to .732 (the smaller length). I do this for every value step 3. This gives me the total length upper and lower bound lengths, but I need to divide them by five in order

to get the average. The total lower bound length is 4.50991559840, which when divided by five is 0.90198311940. The total upper bound length is 4.77491559840, and when you divide that by five you get 0.95498311940. This gives us the upper bound and lower bound lengths.

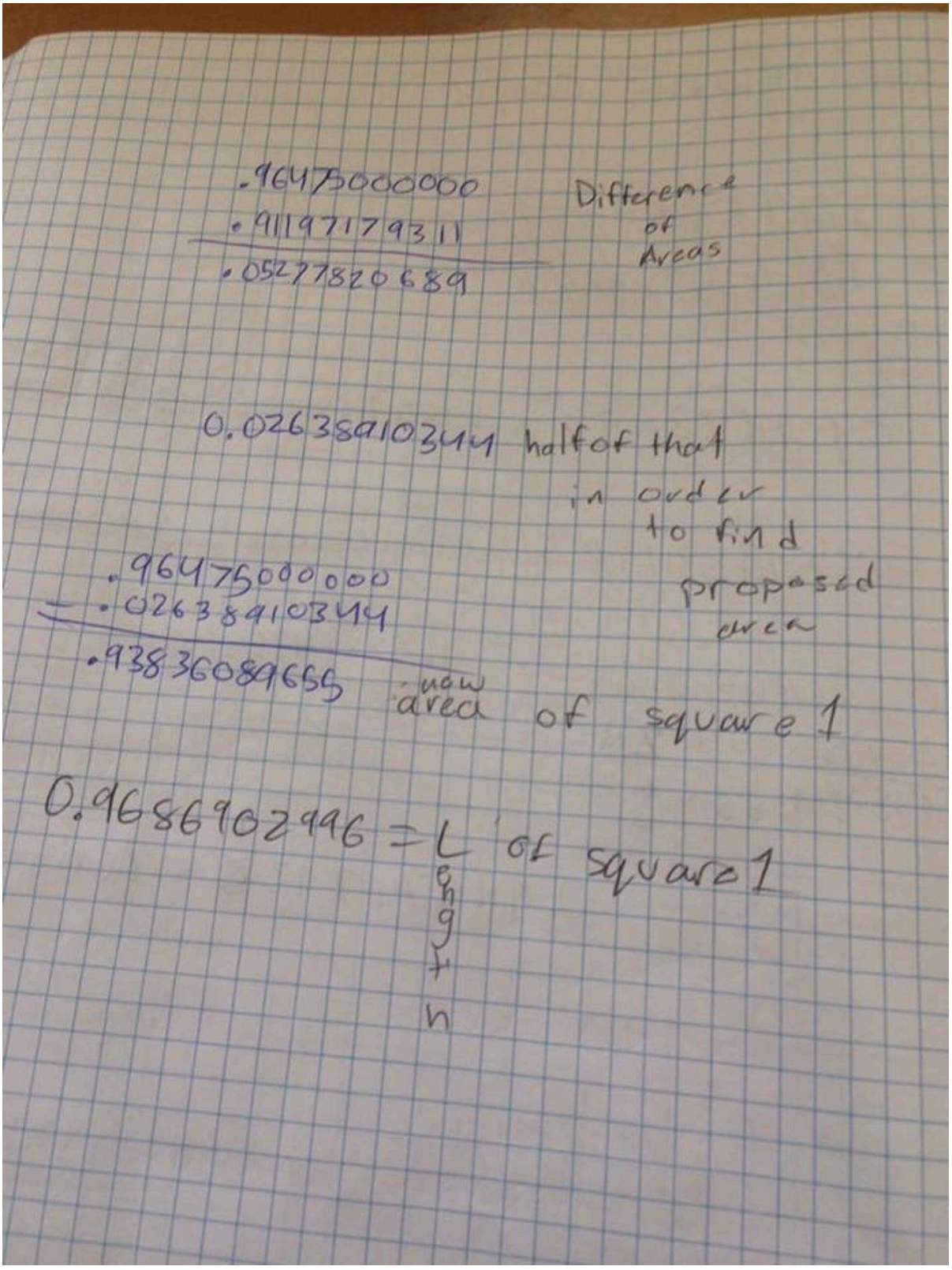


Step 10: I take these lengths, 0.90198311940 (lower) and 0.95498311940 (upper), and I multiply them by the width (.98221688032). This gives me the upper and lower bound

areas. (0.88594304563 is the lower and 0.93800054029 is the upper) I then add these together and divide by two to get the exact area, 0.91197179311.

The two new areas don't match, so I decided to try my strategy again on the new areas and lengths. So because I have already solved for the areas and lengths, I only needed to do steps 4-10. I took the difference between the two areas, which ended up being

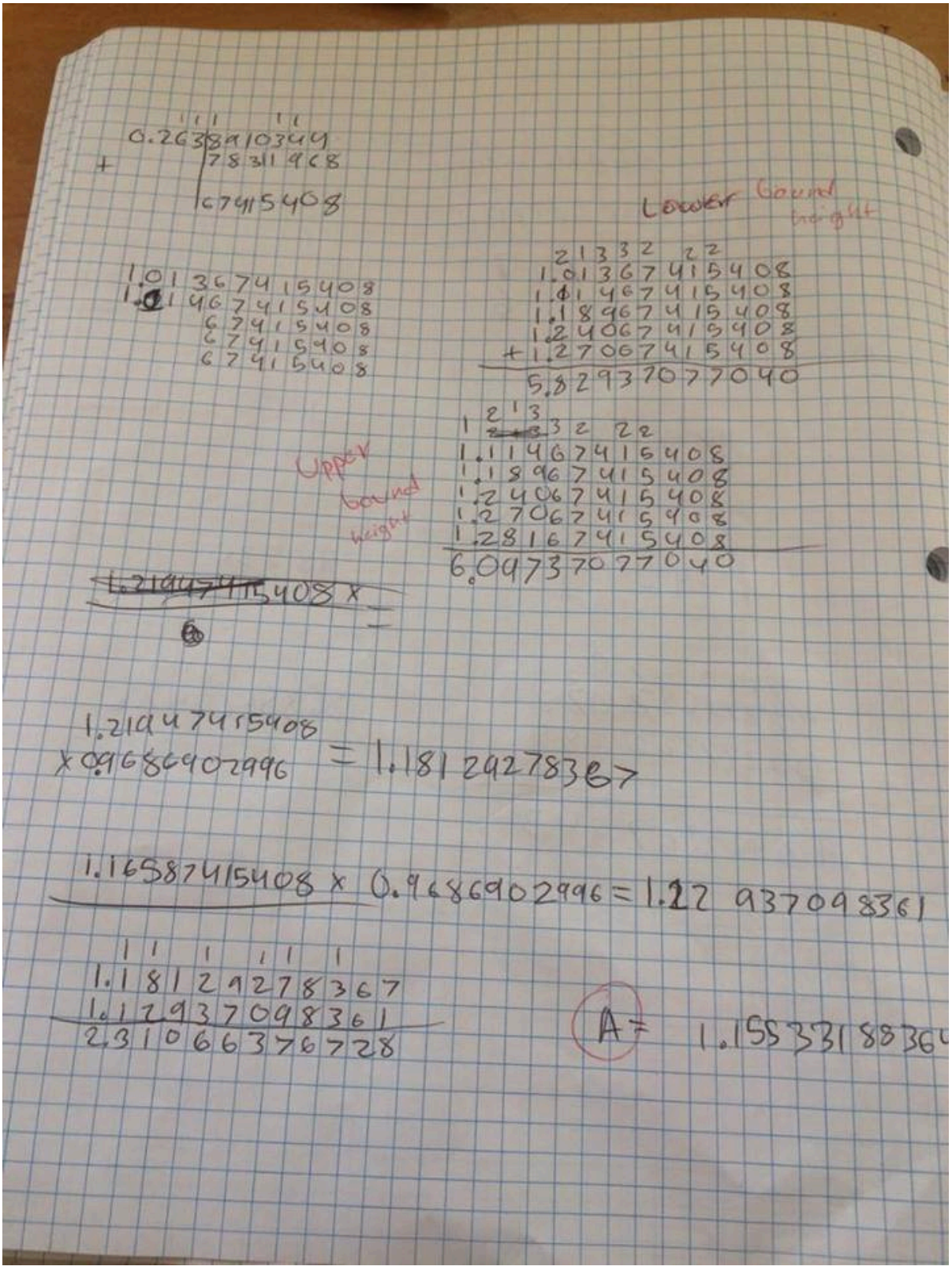
0.5277820639 and then halved it to find the new area of squares 1-4.


.9647500000
.9119717931
.0527782069
Difference of Areas
0.026384103444 half of that in order to find
.9647500000
-.0263841034 44
.9383658965 5 new area of square 1
0.9686902496 = L ength of square 1

When I subtracted it from square 1-4's area, .96475, I got .93836089655. This will also be the width of squares 5-12.

Then I add the distance of the new cut (half the difference of the old areas) to the rectangle lengths of squares 5-12. (see image on next page) This gets me the total lengths of the upper bound rectangles and the lower bound rectangles. The upper was 6.09737077040 and the lower was 5.82937077040. I divided both of these lengths by five to get the average length, lower being 1.16587415408 and upper being 1.21947415408. I then multiply these by the width, 0.9686902996. Upper ends up being 1.18129278367 and lower is 1.12937098361. Then I add these two areas together and divide by two,

getting the total area of 1.15533188364.


Lower bound height
Upper bound height
1.214474154088 x
0.9686902996 = 1.181292783867
1.165874154088 x 0.9686902996 = 1.22 937098361
A = 1.155331888364

As you can see, once again the areas do not match. In fact, shapes 5-12 have now become larger than shapes 1-4. The area's overall went like this

	Shapes 1-4 Area	Shapes 5-12 area
Initial (cut halfway between diameter and edge)	1	.9295
1st time using method (1.7% closer cut than initial to the diameter)	.96475	0.91197179311
2nd time using method (3% close cut than initial to the diameter.	.93836089655	1.15533188364

With these findings I see that as I used the method more and more, shapes 5-12 went a little bit smaller to a bit bigger and shapes 1-4 got smaller and smaller by little bits as well. Because the numbers that are changing are so small and there is such a wide variance in where you can cut, it would take many tries to figure out exactly where to cut to make shapes 5-12 and shapes 1-4 equivalent. I have concluded that if you cut somewhere between 1.7% closer to the diameter and 3% closer to the diameter you will get the perfect cut. Once I hit three percent closer the to areas got further apart from each other than 1.7%, which was the closest they got. This problem is interesting because it starts off very close to the answer already, so you leap too far very fast. I think that the answer is somewhere between 1.7% closer to the diameter and 2% closer, as three was too far and I think going closer is the correct answer, as it is the only way to make shapes 5-12 larger and shapes 1-4 smaller.