THESIS INFORMATION

Thesis title: ***Investigation of some boundary problems for nonlinear pseudoparabolic equations***.
Speciality: Mathematical Analysis
Code: **62460102**
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1. SUMMARY

This thesis focuses on studying the solvability and the properties of solutions of the boundary problems for nonlinear pseudo-parabolic equations with/without viscoelastic term. Main results are presented in four chapters as follows:

Chapter 1. Using the Faedo-Galerkin method and the weak compact method, the thesis proves the existence and uniqueness of weak solutions of the nonhomogeneous Dirichlet problem for a nonlinear pseudo-parabolic equation of the form

$$\begin{cases} u_t - \left(\mu(t) + \alpha(t)\dfrac{\partial}{\partial t}\right)\left(\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\gamma}{x}\dfrac{\partial u}{\partial x}\right) + f(u) = f_1(x,t),\ 1 < x < R,\ 0 < t < T, & (1) \\ u(1,t) = g_1(t),\ u(\mathrm{R},t) = g_R(t), & (2) \\ u(x,0) = \tilde{u}_0(x), & (3) \end{cases}$$

where $R > 1,\ \ \gamma > 0$ are given constants, and μ, α, $\tilde{u}_0$, f, f_1, g_1, g_R are given functions. After studying the existence of solutions of a stationary problem, the thesis proves that the weak solution of the problem (1)-(3) converges to the unique solution of the stationary problem as $t \to +\infty$. Moreover, by replacing the initial condition (3) by the “(η, T)-period” condition

$$u(x,0) = \eta u(x,T),\ 0 < |\eta| \le 1, \tag{4}$$

and by using the Faedo-Galerkin method combined with the operator method of Poincaré type, the thesis also obtains the existence and uniqueness of weak solutions of the problem (1), (2), (4).

Chapter 2. The thesis considers the problem (1) with nonhomogeneous Robin-Dirichlet

$$u_x(1,t) = h_1 u(1,t) + g_1(t),\ u(R,t) = g_R(t). \tag{5}$$

By applying the same methods used in Chapter 1, the thesis proves the existence, uniqueness and asymptotic behavior as $t \to +\infty$ of weak solutions of the problem (1), (3), (5). Moreover, by replacing the initial condition (3) by “$(N+1)$-points condition in time"

$$u(x,0) = \sum_{i=1}^{N} \eta_i u(x,T_i), \tag{6}$$

where $(\eta_i, T_i),\ i=\overline{1,N}$ satisfy $0<T_1<T_2<\cdots<T_N=T, \sum_{i=1}^{N}|\eta_i|\le 1,$ the thesis also proves the existence of weak solutions of the problem (1), (5), (6). In particular, when N = 1, the weak solution is unique.

Chapter 3. By applying the linear approximation method, the Faedo-Galerkin method and the weak compact method, the thesis proves the existence and uniqueness of weak solutions of the Neumann-Dirichlet problem for a nonlinear pseudo-parabolic equation with a viscoleastic term as follows

$$\begin{cases} u_t - \left(1+\frac{\partial}{\partial t}\right)\left(\frac{\partial^2 u}{\partial x^2}+\frac{1}{x}\frac{\partial u}{\partial x}\right)+\int_0^t g(t-s)\left(\frac{\partial^2 u}{\partial x^2}(s)+\frac{1}{x}\frac{\partial u}{\partial x}(s)\right)ds \\ \qquad = f(x,t,u,u_x,u_t,u_{xt}),\ 1<x<R,\ 0<t<T, & (7) \\ u_x(1,t)=u(R,t)=0, & (8) \\ u(x,0)=\tilde{u}_0(x), & (9) \end{cases}$$

where $R>1$ is given constant, and $\tilde{u}_0$, f, g are given functions. In the special case $f=F(u)+f_1(x,t)$, by constructing the Lyapunov functional and under the suitable assumptions, the thesis proves that the weak solution of the problem (7)-(9) is global and exponentially decay as $t\to+\infty$.

Chapter 4. The thesis considers the Robin-Dirichlet problem for a nonlinear viscoleastic pseudo-parabolic equation of the form

$$\begin{cases} u_t - \left(\mu+\alpha\frac{\partial}{\partial t}\right)\left(\frac{\partial^2 u}{\partial x^2}+\frac{1}{x}\frac{\partial u}{\partial x}\right)+\int_0^t g(t-s)\left(\frac{\partial^2 u}{\partial x^2}(s)+\frac{1}{x}\frac{\partial u}{\partial x}(s)\right)ds \\ \qquad = f(x,t,u),\ 1<x<R,\ 0<t<T, \\ u_x(1,t)-h_1u(1,t)=u(R,t)=0, \\ u(x,0)=\tilde{u}_0(x), \end{cases} \tag{10}$$

where $R>1,\ h_1>0$, $\mu>0$, $\alpha>0$ are given constants, and $\tilde{u}_0$, f, g are given functions. The existence and uniqueness of solutions of the problem (10) are given by a high-order iterative scheme, in which the existence of a nonlinear recurrent sequence is established and its convergence to the unique solution of the problem (10) is also proved.

2. NOVELTY OF THESIS

- The existence and uniqueness of weak solutions to the problem (1)-(3), the problem (1), (2), (4), the problem (1), (3), (5), the problem (1), (5), (6), the problem (7-(9).
- The asymptotic behavior of weak solutions as $t\to+\infty$ to the problem (1)-(3), the problem (1), (3), (5).
- The exponential decay of weak solutions as $t\to+\infty$ to the problem (7)-(9) with $f=F(u)+f_1(x,t)$.
- The high-order iterative scheme to the problem (10).

3. OPEN PROBLEMS

Uniqueness of a weak solution to the problem (1), (5), (6) in the case $N\ge 2$.

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