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B.Sc. (Hons. Math) (Semester – 1st)

Subject Name: Calculus-I

Subject Code: BMATS1101

Paper ID: [19131201]

Time: 03 Hours **Maximum Marks: 60**

Instruction for candidates:

1. Section A is compulsory. It consists of 10 parts of two marks each.
2. Section B consist of 5 questions of 5 marks each. The student has to attempt any 4 questions out of it.
3. Section C consist of 3 questions of 10 marks each. The student has to attempt any 2 questions.

Section – A **(2 marks each)**

Q1. Attempt the following:

a) If $y = \sin sin\,(m\,x)$, prove that $\left(1 - x^2\right) y_2 - x\,y_1 + m^2\,y = 0.$

b) If $y = a^{2x} + \frac{x}{2x-1}$, find y_n.

c) If there is a possible error of 0.02 cm in the measurement of the diameter of a sphere, then find the possible percentage error in its volume, when the radius is 10 cm.

d) Prove that the curve $y = \log log\,x$ is everywhere concave downwards for $x > 0$.

e) Examine the nature of origin for the curve $y^2 = 2\,x^2\,y + x^4\,y - 2\,x^4$.

f) Find the asymptotes parallel to the axes of the curve $x^2\,y^2 + y^2 = 1$.

g) If $u(x, y) = x^2\,(y/x) - y^2\,(x/y), x > 0,\ y > 0$, then evaluate

$$x^2\,\frac{\partial^2 u}{\partial x^2} + 2\,xy\,\frac{\partial^2 u}{\partial x\,\partial y} + y^2\,\frac{\partial^2 u}{\partial y^2}.$$

h) Show that the variables $u = x - y + z,\ v = x + y - z,\ w = x^2 + xz - xy$, are functionally related. Find the relationship between them.

i) Show that $dV = d\vec{r}\,.\,\nabla V$

j) Find the normal to the surface $2\,x\,z^2 - 3\,x\,y - 4\,x = 7$ at $(1, -\,1,\ 2)$.

Section – B **(5 marks each)**

Q2. If $y = (x)^{\,2}$, find $y_n(0)$.

Q3. Find the intervals in which the curve $y = (\cos cos\,x + \sin sin\,x)\,e^x$ is concave upward or downwards in $(0,\ 2\pi)$. Find also the points of inflexion.

Q4. Prove that the radius of curvature at any point $P(x, y)$ on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $\frac{a^2 b^2}{p^3}$, where p is the length of perpendicular from the centre of ellipse on the tangent at P.

Q5. If $z = f(u, v)$ where $u = e^x\,\cos cos\,y\,,\ v = e^x\,\sin sin\,y\,,$ show that

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = \left(u^2 + v^2\right)\left(\frac{\partial^2 z}{\partial u^2} + \frac{\partial^2 z}{\partial v^2}\right)$$

Q6. Show that of all triangles inscribed in a circle, the one with maximum area is equilateral.

Section – C **(10 marks each)**

Q7. (a). Find asymptotes of the curve $x^2\,y - x\,y^2 + x\,y + y^2 + x - y = 0$.

(b). Find the *n*th derivative of $\sqrt{a\,x + b}$.

Q8. (a). In the curve $y = a\,\log log\,\sec sec\left(\frac{x}{a}\right)$, prove that the chord of curvature parallel to the axis of *y* is of constant length.

(b). Let $f(x, y) = (\sin sin\, x\,,\ \cos cos\, y\,)$ and $g(x, y) = \left(x^2,\ y^2\right)$. Let $F = fog$. Evaluate $J_F(x, y)$ and verify the result by direct differentiation.

Q9. (a). If $\vec{F}$ is a solenoidal vector, show that:

$$curl\ curl\ curl\ curl\ \vec{F} = \nabla^2\,\nabla^2\,\vec{F} = \nabla^4\,\vec{F}.$$

(b). Find the maximum and minimum value of $x^2 + y^2$ subject to the condition:

$$3\,x^2 + 4\,x\,y + 6\,y^2 = 140.$$