Physics Math Review pg 833-836

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Appendix A: Math Handbook

I. Symbols

Δ	change in quantity	$a \times b$	
$\pm$	plus or minus a quantity	ab	*a* multiplied by *b*
$\propto$	is proportional to	$a(b)$	
$=$	is equal to	$a \div b$	
$\approx$	is approximately equal to	a/b	*a* divided by *b*
$\cong$	is approximately equal to	$\frac{a}{b}$	
$\leq$	is less than or equal to	$\sqrt{a}$	square root of *a*
$\geq$	is greater than or equal to	$\lvert a \rvert$	absolute value of *a*
$<<$	is much less than	$\log_b x$	log to the base, *b*, of *x*
$\equiv$	is defined as		

II. Measurements and Significant Digits

Connecting Math to Physics Math is the language of physics. Using math, physicists are able to describe relationships among the measurements that they make using equations. Each measurement is associated with a symbol that is used in physics equations. The symbols are called variables.

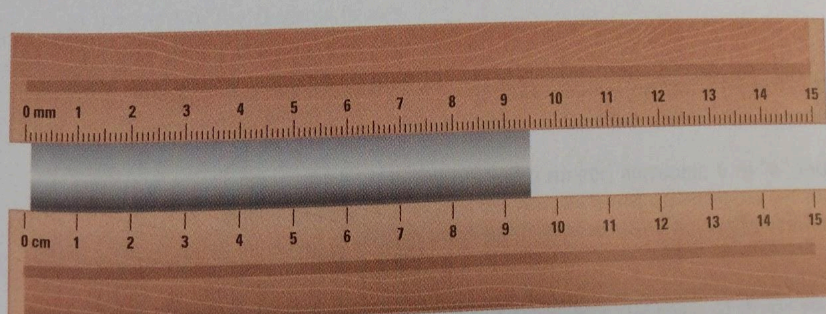
Significant Digits

All measured quantities are approximated and have significant digits. The number of significant digits indicates the precision of the measurement. Precision is a measure of exactness. The number of significant digits in a measurement depends on the smallest unit on the measuring tool. The digit farthest to the right in a measurement is estimated.

Example: What is the estimated digit for each of the measuring sticks in the figure below used to measure the length of the rod?

Using the lower measuring tool, the length is between 9 and 10 cm. The measurement would be estimated to the nearest tenth of a centimeter. If the length was exactly on the 9-cm or 10-cm mark, record it as 9.0 cm or 10.0 cm.

Using the upper measuring tool, the length is between 9.5 and 9.6 cm. The measurement would be estimated to the nearest hundredth of a centimeter. If the length was exactly on the 9.5-cm or 9.6-cm mark, record it as 9.50 cm or 9.60 cm.



All nonzero digits in a measurement are significant digits. Some zeros are significant and some are not. All digits between and including the first nonzero digit from the left through the last digit on the right are significant. Use the following rules when determining the number of significant digits.

1. Nonzero digits are significant.
2. Final zeros after a decimal point are significant.
3. Zeros between two significant digits are significant.
4. Zeros used only as placeholders are not significant.

Example: State the number of significant digits in each measurement.

5.0 g has two significant digits.	**Using rules 1 and 2**
14.90 g has four significant digits.	**Using rules 1 and 2**
0.0 has one significant digit.	**Using rules 2 and 4**
300.00 mm has five significant digits.	**Using rules 1, 2, and 3**
5.06 s has three significant digits.	**Using rules 1 and 3**
304 s has three significant digits.	**Using rules 1 and 3**
0.0060 mm has two significant digits (6 and the last 0).	**Using rules 1, 2, and 4**
140 mm has two significant digits (just 1 and 4).	**Using rules 1 and 4**

▶ PRACTICE Problems

1. State the number of significant digits in each measurement.
   - **a.** 1405 m
   - **b.** 2.50 km
   - **c.** 0.0034 m
   - **d.** 12.007 kg
   - **e.** 5.8×10^{6} kg
   - **f.** 3.03×10^{-5} mL

There are two cases in which numbers are considered exact, and thus, have an infinite number of significant digits.

1. Counting numbers have an infinite number of significant digits.
2. Conversion factors have an infinite number of significant digits.

Examples:

The factor "2" in 2*mg* has an infinite number.
The number 2 is a counting number. It is an exact integer.

The number "4" in 4 electrons has an infinite number.
Because you cannot have a partial electron, the number 4, a counting number, is considered to have an infinite number of significant digits.

60 s/1 min has an infinite number.
There are exactly 60 seconds in 1 minute, thus there are an infinite number of significant digits in the conversion factor.

Rounding

A number can be rounded to a specific place value (like hundreds or tenths) or to a specific number of significant digits. To do this, determine the place being rounded, and then use the following rules.

1. When the leftmost digit to be dropped is less than 5, that digit and any digits that follow are dropped. Then the last digit in the rounded number remains unchanged.
2. When the leftmost digit to be dropped is greater than 5, that digit and any digits that follow are dropped, and the last digit in the rounded number is increased by one.
3. When the leftmost digit to be dropped is 5 followed by a nonzero number, that digit and any digits that follow are dropped. The last digit in the rounded number increases by one.
4. If the digit to the right of the last significant digit is equal to 5, and 5 is followed by a zero or no other digits, look at the last significant digit. If it is odd, increase it by one; if it is even, do not round up.

Examples: Round the following numbers to the stated number of significant digits.

8.7645 rounded to 3 significant digits is 8.76.	**Using rule 1**
8.7676 rounded to 3 significant digits is 8.77.	**Using rule 2**
8.7519 rounded to 2 significant digits is 8.8.	**Using rule 3**
92.350 rounded to 3 significant digits is 92.4.	**Using rule 4**
92.25 rounded to 3 significant digits is 92.2.	**Using rule 4**

▶ PRACTICE **Problems**

2. Round each number to the number of significant digits shown in parentheses.

a. 1405 m (2)

b. 2.50 km (2)

c. 0.0034 m (1)

d. 12.007 kg (3)

Operations with Significant Digits

When using a calculator, do all of the operations with as much precision as the calculator allows, and then round the result to the correct number of significant digits. The correct number of significant digits in the result depends on the measurements and on the operation.

Addition and subtraction Look at the digits to the right of the decimal point. Round the result to the least precise value among the measurements—the smallest number of digits to the right of the decimal points.

Example: Add 1.456 m, 4.1 m, and 20.3 m.

The least precise values are 4.1 m and 20.3 m because they have only one digit to the right of the decimal points.

$$\begin{array}{r} 1.456\ \text{m} \\ 4.1\ \text{m} \\ +\ 20.3\ \text{m} \\ \hline 25.856\ \text{m} \end{array}$$

Add the numbers

The sum is only as precise as the least precise number being added.

25.9 m **Round the result to the estimated place with the largest place value**

Multiplication and division Look at the number of significant digits in each measurement. Perform the calculation. Round the result so that it has the same number of significant digits as the measurement with the least number of significant digits.

Example: Multiply 20.1 m by 3.6 m.

$(20.1\ \text{m})(3.6\ \text{m}) = 72.36\ \text{m}^2$

The least precise value is 3.6 m with two significant digits. The product can only have as many digits as the least precise of the multiplied numbers.

72 m — **Round the result to two significant digits**

PRACTICE Problems

3. Simplify the following expressions using the correct number of significant digits.

a. 5.012 km + 3.4 km + 2.33 km

b. 45 g − 8.3 g

c. 3.40 cm × 7.125 cm

d. 54 m ÷ 6.5 s

Combination When doing a calculation that requires a combination of addition/subtraction and multiplication/division, use the multiplication/division rule.

Examples:

$$d = 19\ \text{m} + (25.0\ \text{m/s})(2.50\ \text{s}) + \frac{1}{2}(-10.0\ \text{m/s}^2)(2.50\ \text{s})^2$$
$$= 5.0\times10^1\ \text{m}$$

19 m only has two significant digits, so the answer should only have two significant digits

$$\text{slope} = \frac{70.0\ \text{m} - 10.0\ \text{m}}{29\ \text{s} - 11\ \text{s}}$$
$$= 3.3\ \text{m/s}$$

26 s and 11 s only have two significant digits each, so the answer should only have two significant digits

Multistep calculations Do not round to significant digits in the middle of a multistep calculation. Instead, round to a reasonable number of decimal places that will not cause you to lose significance in your answer. When you get to your final step where you are solving for the answer asked for in the question, you should then round to the correct number of significant digits.

Example:

$$F = \sqrt{(24\ \text{N})^2 + (36\ \text{N})^2}$$

$$= \sqrt{576\ \text{N}^2 + 1296\ \text{N}^2}$$ **Do not round to 580 N² and 1300 N²**

$$= \sqrt{1872\ \text{N}^2}$$ **Do not round to 1800 N²**

$$= 43\ \text{N}$$ **Final answer, so it should be rounded to two significant digits**