Fundamentals of Physics II Review

PHY 1122 - Michael C. H. Wong - Winter 2023

Thermodynamics

1.0 FLUID MECHANICS

1.1 Density and Pressure

Density is the mass per volume

$\rho = m/V$ (density = mass / volume)

Pressure is the force that a fluid exerts on a surface within the fluid

$p = F/A$ (pressure = perpendicular force / area its applied to)

Pressure also changes depending on the depth within the liquid.

$p = p_0 + \rho g h$ (the pressure at the top plus the weight of the liquid (density gravity height))

Pascal's Law: If the pressure applied at the surface of the water changes then the pressure within the water also changes proportional to the change in area applied. This is the way hydraulic lifts work

$p = F_1/A_1 = F_2/A_2$

1.2 Buoyancy Force and Archimedes' Principle

Buoyancy is the force that the water exerts on an object that displaced the water.

The force of buoyancy is equal to the weight of the water displaced

$B = \rho V g$ (force of buoyancy is equal to the density of the liquid, times the volume displaced, times the gravity)

If the object is floating on the water then the buoyant force is equal to the weight of the object

1.3 Fluid Flow and Dynamics

If fluid flows through a changing cross sectional area then the flow rate will change proportionally to the change in area.

$A_1 v_1 = A_2 v_2$ (Cross sectional area at point 1 times speed at point 1 = at point 2)

Volume flow rate at a specific point = Av

Bernoulli's Equation: To consider more complicated fluid flow problems we use the height of the liquid, the cross sectional area and the speed.

$p + \rho g y + \frac{1}{2}\rho v^2 = \text{constant}$ (pressure + weight of water above + kinetic energy of water)

To consider the change in any of these when working with two points along the same pipe, simply make it $p_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$

2.0 TEMPERATURE AND HEAT

2.1 Kelvin Temperature Scale

There is a theoretical temperature where most gasses have 0 pressure, this temperature is $-273.15^{o}C$ or 0K.

2.2 Thermal Expansion

When materials are heated, they expand

Linearly they expand by an amount of $\Delta L = \alpha L_0 \Delta T$

Volume expands by an amount $\Delta V = \beta V_0 \Delta T$ where $\beta = 3\alpha$

The α and β values depend on the material

Thermal stress is the force that a restricted thermally expanding substance places on a nearby surface. $F/A = -Y\alpha\Delta T$ (Force / area equals young's modulus times the material's coefficient times change in temperature)

2.3 Calculations for Heat Flow, Temperature Change, and Phase Change

Temperature is the state of the material while heat is the movement of energy based on the difference of temperature.

Typically we want to calculate the quantity of heat required to raise a certain amount of a substance by a certain temperature

$Q=mc\Delta T$ (Quantity of heat (in calories or joules) = mass x heat capacity of material x temperature change)

The heat required to change the state of matter (between solid, liquid, and gas) is:

$Q = \pm mL$ (Heat required = (removed or added) mass x latent heat (depending on the phase change and depending on the material))

To calculate the overall heat required to change a temperature through a phase change you calculate the change in temperature between phase changes and then add it with the heat required to change phase.

Ex. To change from -4℃ to 102℃ of water: $Q=mc(4)+mL_f+mc(100)+mL_v+mc(2)$

2.4 Heat Transfer Mechanisms

Calorimetry: When one substance loses heat the other gains it (in an ideally closed system)

Conduction of Heat: Heat can be conducted through a surface, to find the rate in watts

$H=kA(T_H-T_C)/L$ (k is a constant, A is the cross sectional area, T_H is the hotter temperature, T_C is the colder temperature, L is the length between the two)

3.0 THERMAL PROPERTIES OF MATTER

3.1 Ideal Gas Law

For an ideal gas, the ideal gas equation can be used:

$pV=nRT$ (pressure x volume = number of moles x R constant x temperature)

The equation can be rearranged with the idea that R and n are the same for a single substance at two different points **$p_1V_1/T_1=p_2V_2/T_2$**

3.2 Kinetic Energy of Molecules

- The total translational kinetic energy of a molecule can be calculated with **$K_{tr}=3/2\ nRT$**
- The average kinetic energy of some single molecule only depends on the temperature
  $K_{tr}/N = \frac{1}{2}m(v^2)_{avg}=3/2\ kT$ (N is avogadro's number, k is boltzmann constant)
- The root-mean-square of a single molecule represents the average speed of a molecule

$$v_{rms}=\sqrt{(v^2)_{avg}} = \sqrt{\frac{3kT}{m}}$$

3.3 Heat Capacities

The capacity for a molecule of gas to retain heat is dependent on the number of atoms it has.

- Monatomic gasses like helium and neon have the heat capacity $C_v=3/2\ R$
- Diatomic gasses like O_2 and N_2 have the heat capacity $C_v=5/2\ R$
- Monatomic solids like lead or aluminum have the heat capacity of $C_v=3R$

4.0 FIRST LAW OF THERMODYNAMICS

4.1 Work Done by Volume Change

Thermodynamic systems are systems that involve an exchange of energy (heat Q or work W) with its surroundings

Signs: When heat is positive it flows into the system, when work is positive it is done by the system

Work Done with Volume Change: The work (w) done by the gas pressure (p) on a changing volume (v) can be calculated with $W = \int_{v_1}^{v_2} p \, dV$ or with constant pressure $W=p(v_2-v_1)$

Path: The work required to move between thermodynamic states changes depending on the path taken. Work is the area under the graph so changing the pressure does not involve any work.

- Changing the volume at high pressure will mean more work done by the system, changing the volume at low pressure will mean less work done by the system

4.2 First Law and Internal Energy of a System

Internal Energy: Generally internal energy is the summation of potential energy and kinetic energy, but this is difficult to measure. We can instead consider the energy added and taken away (in the form of work and heat) to determine any changes in internal energy

First Law of Thermodynamics: The change of internal energy is equal to heat added to the system - work done by the system ($\Delta U=Q-W$)

Cyclic Process: When the initial internal energy is the same as the final internal energy (consider a daily schedule where you eat a certain amount (increase Q) but also do the same amount of work (W) resulting in a weight) this is a cyclic process.

$U_2=U_1$ and $Q=W$

Isolated System: When there is no change in internal energy because there is no work done with the system and no heat added or removed.

$U_2=U_1$ and $Q=W=0$

4.3 Types of Thermodynamic Processes

Adiabatic: There is no heat transferred, $Q=0$ and $\Delta U = -W$

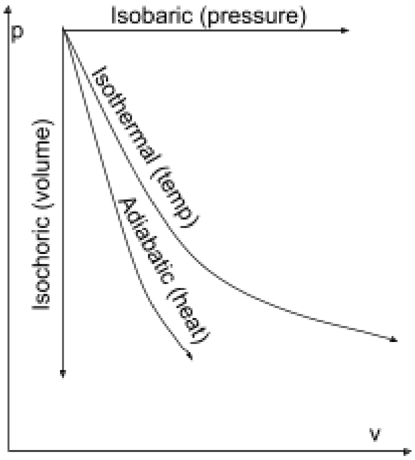
Ex: Cork pops, pressurized gas does work on cork but there is no heat exchanged

Isochoric: The volume remains constant, $W=0$ and $\Delta U = Q$

Ex: Most types of cooking, pot of boiling water has constant volume but changing heat

Isobaric: The pressure remains constant, $W=p(V_2-V_1)$

Isothermal: Temperature remains constant (no simple relationship between ΔU, Q and W)



4.4 Internal Energy of an Ideal Gas

For an ideal gas with constant volume, the change in temperature is linearly related to the change in internal energy: $\boldsymbol{\Delta U=nC_v\Delta T}$

4.5 Molar Heat Capacities

Heat capacity is the amount of heat required to raise the temperature of a substance.

If we add heat to an ideal gas with a **constant volume** (isochoric), all the heat energy will go directly to the substance and the temperature will increase.

Adding heat energy to a substance in a **constant pressure process** (isobaric) will mean the volume changes to accommodate for the warmer substance. This change in volume is work energy, meaning some of the heat energy that is put into the system gets converted to work.

This means that an **isobaric process can absorb more heat** and so has a higher heat capacity

The heat capacity of an isochoric process (C_v) and an isobaric process (C_p) can be related by

$C_p=C_v+R$

Ratio of Heat Capacities: There is a ratio for the heat capacities that is useful for simplifying equations: **$\gamma=C_p/C_v$**

4.6 Adiabatic Processes

When considering a temperature, pressure, or volume change of an ideal gas where no heat is added (adiabatic) then you can use the following equations

$T_1V_1^{\gamma-1}=T_2V_2^{\gamma-1}$ **$p_1V_1^{\gamma}=p_2V_2^{\gamma}$**

When considering the work done by an ideal gas in an adiabatic process we can use

$W=nC_v(T_1-T_2)$ **or** **$W=C_v(p_1V_1-p_2V_2) / R$**

5.0 SECOND LAW OF THERMODYNAMICS

5.1 Reversible and Irreversible Processes

There are certain processes that can be done forward but not in reverse (lava can vaporize water but can water vapor turn rock into lava?). Typically, thermodynamic processes work in one direction. Water won't just spontaneously freeze to make a glass warmer.

5.2 Internal Combustion Engines

Heat engines convert heat into work (aka mechanical energy). They work in a cyclic process meaning in the end the heat is equal to the work.

1. Heat is absorbed from a hot reservoir
2. Some of it is transformed into work while the other part is dumped into the cold reservoir

To calculate the work done by the heat engine, you can compare the heat leaving the hot reservoir and the heat entering the cold reservoir: **$e = 1 - |Q_c / Q_h|$**

e being the thermal efficiency of the heat engine

5.3 Other Thermodynamic Systems

Refrigerators take heat from a cold place and leave it in a warmer place. The amount of heat removed by the fridge is the sum of the work done and the heat taken from inside

K is the coefficient of performance: **$K = Q_C / W$**

5.4 Second Law of Thermodynamics and Limiting Efficiency

The second law states that there cannot be 100% efficient thermodynamic processes. (the fridge has to have work involved and the engine has to exert some heat)

Carnot Cycle: The carnot cycle is the theoretical most efficient cycle to convert heat to work. It works on the assumption that when heat is flowing, (Q_c and Q_h) both reservoirs do not change temperature.

The efficiency and coefficients of performance can be calculated as such

$$e_{carnot} = 1 - \frac{T_C}{T_h} \quad \text{and} \quad K_{carnot} = \frac{T_H}{T_H - T_C}$$

5.5 Entropy and What it Leads to

Entropy is a quantitative description for the randomness that results from any natural process

In reversible processes the change in entropy (ΔS) is just Q/T

The second law of thermodynamics can be restated as: there is no process where entropy decreases

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Electricity

6.0 ELECTRIC CHARGE AND FIELD

6.1 Electric Charge, Conservation of Charge

Electric charge is an invisible force that involves the movement of electrons. Objects can have positive, negative, or neutral charge depending on the excess of electrons in the system. Things with the same charge will repel each other and things with opposite charges will attract each other. Electric charge is not created, it has to come from somewhere else.

Materials: There are three categories for materials, conductors which move electrons with ease, insulators which inhibit the movement of electrons, and semiconductors which can act as either depending on conditions (temperature typically)

6.2 Coulomb's Law and Applications

Coulomb's Law defines the force that two charged particles will have on each other.

$F = k\frac{|q_1||q_2|}{r^2}$ where F is the force each particle applies to each other, k is a constant, the q s are the charges of each particle in coulombs, and r is the distance between them

6.3 Electric Field

To generalize the forces that any charged particle will feel near a charged object, we can use electric fields. Important notes:

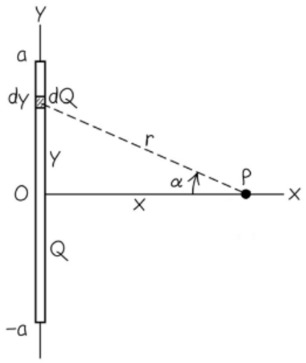
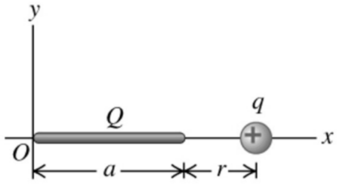
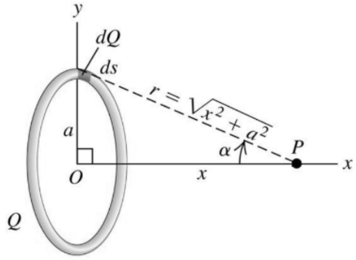
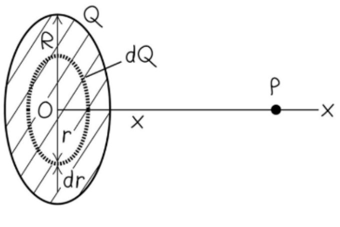
- Electric charge will go away from positive charges and toward negative charges.
- If there is a positive charge beside a negative charge, the electric field lines will leave the positive and bend towards the negative.
- A positively charged particle will experience force along the electric field lines whereas a negatively charged particle will experience force against the electric field line direction.

$E=kQ/r^2$: This is the equation for the electric field of some object. E is the electric field in N/C (leaves it generalized for any secondary charge), k is a constant, Q is the charge of the object (leaving it positive or negative) and r is the distance from the object that the electric field is being studied (electric field lessens with distance from charged object)

$E=F/q$: If you know the force that an object has on a charged particle with charge q then you can also determine the electric field that way

6.4 Continuous Charges

When dealing with a continuous charge, you need to break it into small segments and consider each of their effects on the particle. The electric field is the summation of all of these small segments.

Name	Drawing	Finite	Infinite
Field Perpendicular to a Vertical Line		$E_x = k\frac{Q}{x\sqrt{x^2+a^2}}$ **E**= electric field along x axis **k**= constant **Q**=charge of line **x**=horizontal distance from line **a**=height of line	$E_x = \frac{2k\lambda}{x}$ **E**= electric field along x axis **k**= constant **λ**=charge density in C/m **x**=horizontal distance from line
Field Inline with a Horizontal Line		$E_r = k\frac{Q}{r(r+a)}$ **E**= electric field along x axis **k**= constant **Q**=charge of line **r**=horizontal distance from line **a**=length of line	
Field from ring of charge for point along central axis		$E_x = k\frac{Qx}{(x^2+a^2)^{3/2}}$ **E**= electric field along x axis **k**= constant **Q**=charge of ring **x**=horizontal distance from center of ring **a**=radius of ring	
Field perpendicular to vertical disk		$E_x = \frac{\sigma}{2\epsilon_0}\left(1 - \frac{1}{\sqrt{(R^2/x^2)+1}}\right)$ **E**= electric field along x axis **ϵ₀**=1/4πk (constant) **σ**=charge density **x**=horizontal distance from center of ring **R**=radius of ring	$E_x = \frac{\sigma}{2\epsilon_0}$ **E**= electric field along x axis **ϵ₀**=1/4πk (constant) **σ**=charge density

7.0 ELECTRIC POTENTIAL

7.1 Electrical Potential Energy and Work

Electric potential energy works similarly to gravitational potential energy. It is the explanation for the source of energy used when work is done on a particle in an electric field.

Particle in Uniform Field: If there is a positive disk on top and a negative disk on the bottom and a positive particle starts beside the positive disk, this particle will have a lot of electric potential energy. As it moves down, along the electric field lines toward the negative plate, the field will do work on the particle and the electric potential will decrease.

For a uniform field E, the potential energy at height y for charge q is **$U=qEy$**
The work done to move from a to b is **$W=qE(y_a-y_b)$**

Work in a Nonuniform Field

For a point charge, the electric field will decrease with distance. Therefor the work required to move a particle q_p from distance r_a to r_b from a charged particle q is **$W=kqq_p(1/r_a-1/r_b)$**

- The electric potential energy in a nonuniform field from a particle with charge q on another particle with charge q_0 with distance r is **$U=kqq_0/r$**

Potential Energy from Multiple Charges: The potential energy required to assemble multiple charges into a specific arrangement is

$$U = k \sum_{i<j} \frac{q_i q_j}{r_{ij}}$$

7.2 Electric Potential

Similar to electric fields, this is a generalization of the electric potential energy that a charge causes for any charged particle within its range. Electric potential has a unit of voltage (V) which is 1 J/C.

Potential Difference: The work done by the electric force to move some particle from point a to b within an electric field. This is the potential difference. In the context of circuits and uniform electric fields, this is called voltage. The voltage of a battery is the difference between the electric potential energy for a generic particle between the two ends.

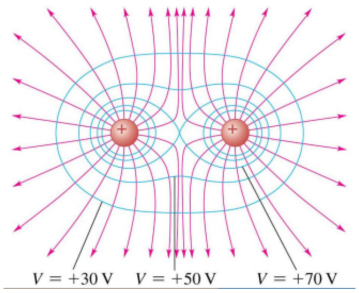
Voltage from a Point Charge: The voltage created by a point charge q is **$V=kq/r$**. If there are multiple charges it is simply the summation of both voltages **$V=k\Sigma q_i/r_i$**

Electric Potential from Nonuniformfield: If there is a known electric field you can calculate the potential difference with that: **$\Delta V=\Delta U/q= -\int_a^b E\,dr$**

Uniform Field: Much easier, **$\Delta V= -Ed$**

7.3 Equipotential Surfaces

Similar to the way contour maps work for hiking, we can use lines to represent where the electric potential is the same.



7.4 Calculating Electric Field from Electric Potential

If we have a formula to represent the electric potential, we could use it to determine the electric field by deriving it.

$\mathbf{E_x = -\partial V/\partial x}$ $\mathbf{E_y = -\partial V/\partial y}$ $\mathbf{E_z = -\partial V/\partial z}$

8.0 CAPACITANCE AND DIELECTRICS

8.1 Capacitor Introduction

Objects of opposite charge want to move toward each other, and so when they are held at a certain distance apart they will build up electrical potential energy.

Point Charges: The capacitance C of two point charges with charges ±Q charged by a battery with voltage V is $\mathbf{C=Q/V}$ with units 1F=1 C/V

Parallel Plate Capacitors: If there are two parallel plates with charges ±Q with area A and separated by distance d then $\mathbf{C=\epsilon_0 A/d}$ where $\epsilon_0=1/4\pi k$

Spherical Capacitors: There is a positively charged inner ball with radius r_a and a negatively charged outer ball with radius r_b then $\mathbf{C=4\pi\epsilon_0 r_a r_b / (r_b-r_a)}$

Cylindrical Coordinates: There's a positively charged smaller cylinder with radius r_a and larger negatively charged outer cylinder with radius r_b and the whole thing has a length L then $\mathbf{C=2\pi\epsilon_0 L / \ln(r_b/r_a)}$

8.2 Capacitors in a Network

Series: When capacitors are connected in series, an equivalent capacitor would be $C=\Sigma 1/C_i$

Parallel: When capacitors are connected in parallel, an equivalent capacitor would be $C=\Sigma C_i$

8.3 Calculating Energy in a Capacitor

The energy stored in a capacitor is equal to the work required to charge it which is:

$$W = U = \frac{1}{2}CV^2 = \frac{Q^2}{2C} = \frac{1}{2}QV$$

The energy within a parallel plate capacitor specifically is $\mathbf{U=\frac{1}{2}\epsilon_0 AdE^2}$

The energy density within a capacitor is $\mathbf{u=\frac{1}{2}\epsilon_0 E^2}$

8.4 Dielectrics

Dielectrics are sheets of insulative material that goes between the plates of a capacitor. It keeps the plates mechanically separated, increases the charge that can be held in the capacitor before charge starts jumping between sheets, and it increases the capacitance.

Dielectric Constant: A unitless constant based on the material between the plates. It is the ratio of the new capacitance over the old one.

Changes:

- **Electric Field**: The induced charge from the capacitor plates to the dielectrics decrease the electric field by $\mathbf{E=E_o/K}$
- **Permittivity** constant of a dielectric is given by $\mathbf{\epsilon=K\epsilon_0}$
- **Capacitance** increases to $\mathbf{C=KC_0}$
- **Voltage** decreases to $\mathbf{V=V_0/K}$

Power: The power involved in a capacitors circuit is the rate that energy is delivered to the capacitor: $\mathbf{P=\epsilon i=i^2R+iq/C}$

Power equals the energy the battery delivers to the circuit (ϵi) which is equal to the the energy dissipated by the resistor i^2R plus the energy stored in the capacitor iq/C

9.0 CURRENT, RESISTANCE AND EMF

9.1 Electric Current

Current is the transfer of charge from one region to another, it is defined by the amount of charge that passes through an area in a time. Its units are ampere (1A=1C/s)

- Charge carriers are charged particles that contribute to this movement of charge
- In a normal electric circuit, electrons move opposite to the current
- Geometrically, I can also be defined as $I=nqv_dA$ where I is the current, n is the number of charge carriers in the region, q is the charge, v_d is the velocity, and A is the cross sectional area
- Current density is the current per unit area and includes the direction: $j=nqv_d$

9.2 Calculating Resistance

For a theoretical material, we can imagine the current density depends directly with the electric field, for these materials their conductivity (σ) or resistivity (ρ) would linearly affect the current density: **$J=\sigma E$ or $E=\rho J$**

Temperature: Although most materials don't actually work like that. The resistance of a material can also be affected by the temperature: **$\rho(T)=\rho_0(1+\alpha(T-T_0))$** where ρ_0 and T_0 are the resistivity and temperature initially and α is the temperature coefficient. As the temperature increases the resistivity increases (less likely for charge carriers to touch)

Exceptions: Semiconductors become more conductive with increased temperature, superconductors and they act like normal until they get cold enough and then they have no resistivity.

Ohm's Law: If there is a conductor with length L, resistivity ρ and area A the resistance can be calculated as **$R=\rho L/A$**, this has units ohms (Ω). Where **$R=V / I$**

9.3 EMF

Electromotive force (EMF) is a general term for something that moves current and has the symbol ϵ. Typically this is a battery or some other power supply.

Internal Resistance: Realistically, power supplies have internal resistance, so when considering the voltage of a system we are subtracting the internal resistance from its emf. A 12V battery might only exert 8V due to its internal resistance.

9.4 Energy and Power

Power is defined as the rate of change in energy transfer, its units are watts (1W=1J/s)

$P=VI=I^2R=V^2/R$

Something can be considered more powerful if it is quicker at transferring energy

10.0 DC CIRCUITS

10.1 Resistors in Series and Parallel

Series: When resistors are connected in series, an equivalent resistor would be the **sum** of the other resistors. **$R_{eq} = R_1+R_2+R_3\ldots$**

- Current is the same throughout the system
- Voltage changes depending on the resistance (V=IR)

Parallel: An equivalent resistor to multiple resistors connected in parallel is equal to the **sum of the inverse. $R_{eq}= 1/R_1 + 1/R_2 + \ldots$**

- Voltage stays the same throughout the system
- Current changes depending on the resistance, the lower the resistance the higher the current (path of least resistance)

10.2 Kirchhoff's Rules for Circuit Analysis

Junction: When there is a junction with 3 or more wires, the sum of the entering current has to equal the sum of the exiting current ($\Sigma I_{in} = \Sigma I_{out}$)

Loop: For any closed loop, the summation of voltages will be 0. A battery will increase the voltage and any device (resistor or light etc) will decrease voltage.

Sign Convention: For emf sources (batteries) going from - to + has +ϵ, + to - is -ϵ

For resistors, moving against current makes +V, moving with makes -V

10.3 Measuring Current, Voltage, and Resistance

Ammeter: Measures the current passing through it, must be connected in series with circuit element

Voltmeter: Measures the potential difference between two points, needs to be connected in parallel with circuit element

Ohmmeter: Measures the resistance, must be connected in parallel with element(s)

10.4 Analyzing RC Current

For a circuit with a capacitor and a resistor and a battery this is how the current changes with time

1. The circuit starts with the capacitor holding no charge and the circuit performing as normal. I=ϵ/R
2. As time passes, the capacitor charge increases and current decreases.
3. Eventually the capacitor charge is maxed out and the current is zero
4. The battery is then removed and the capacitor starts discharging
5. Eventually the resistor swallows all the remaining charge and the current goes to 0 again.

$q_c(t) = Q_f\left(1 - e^{-t/RC}\right)$ **q_c(t)**=function of increasing charge in capacitor as time progresses **Q_f**=final charge in capacitor	$I_c(t) = \frac{\epsilon}{R}e^{-t/RC}$ **I_c(t)**=function of current of a charging capacitor circuit **ϵ**=charge of battery	$q_d(t) = Q_i e^{-t/RC}$ **q_d(t)**=function of decreasing charge in capacitor **Q_i**=fully charged capacitor	$I_d(t) = -I_i e^{-t/RC} = -\frac{Q_i}{RC}e^{-t/RC}$ **I_d(t)**=function of current in circuit with discharging capacitor **I_i**=current at beginning of discharge **Q_i**=fully charged capacitor
R= Resistance, **C**=capacity of capacitor, **t**=time			

Time Constant: This is the amount of time that the current takes to reduce to 36.8% of its original charge: τ=RC

Optics

11.0 THE NATURE OF LIGHT

11.1 Light Rays and Wave Fronts

Light can be considered either a wave or a particle called a photon. Some of its characteristics are consistent with wave behaviour and some of them are consistent with particle behaviour.

Wave Fronts: Emanate from the light source and are similar to the ripples in a pond, they represent the crest of the waves.

Rays: The rays are imaginary but convenient for analysis. They run perpendicular to the wave fronts. The branch of optics that can use rays to describe light sufficiently is called geometrical optics

The further from the source you get, the wave fronts and rays become parallel. It's called a **plane wave** when all the rays are parallel

11.2 Reflection and Refraction of Light, Total Internal Reflection

When light hits a smooth interface (glass or water), some of it is refracted (bends and goes into the surface) and some of it is reflected (bounces back)

Definitions:

Normal Line: This is perpendicular to the surface

Incident Ray: This is the incoming ray

Angle of Incident is the angle between the normal and the incident ray

Reflected Ray: This is the ray that bounces off of the surface

Angle of Reflection: Equal to the angle of incident for a smooth surface, this is the angle between the reflected ray and the normal

Refracted Ray: This is the ray that goes into the surface after bending from the change in medium.

Angle of Refraction: Depends on the speed of light in both materials, if the light is going into a slower speed medium then it will bend closer to the normal (like from air to glass)

Index of Refraction: A unitless value that is used to calculate the angle of refraction.

$n=c/v$ where c is the speed of light in a vacuum and v is the speed of light in that medium

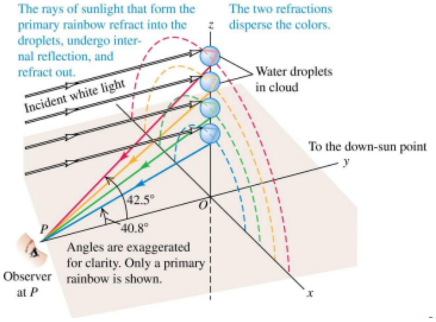
Snell's Law: To calculate the angle of refraction the formula is **$n_1\sin\theta_1=n_2\sin\theta_2$** Where the n values are the different indexes of refraction and the theta values are the angles from the normal to the ray

Total Internal Reflection: This only happens when the light is going from a slower to a faster medium (ex glass to air) and when the angle of refraction is greater than 90°.

$\sin\theta_{crit}=n_b/n_a$ where θ_{crit} is the initial angle of light, n_a is the initial substance and n_b is the secondary substance.

This is used in fiber optic cables by shooting a light ray at an angle greater than θ_{crit} the light does not refract out of the cable and simply reflects all the way to the end

Dispersion: For most substances, n decreases for longer wavelengths. Within the visible light spectrum the later colours in the rainbow have a shorter wavelength and so when passing through the same substance as a red light, the purple light will refract more and the red light will stay truer to the original ray. This difference in refraction is what causes rainbows, light hits a raindrop in the air, refracts into it at various angles then reflects back to the observer.



11.3 Polarization of Light

When treating light like a wave, light has many different waves at many different angles. One wave could be horizontal and one wave could be vertical or anything in between. Typically light is **unpolarized** and there are a bunch of waves going in all yz directions.

Polarizing Filter: These filters only allow light in direction producing linearly polarized light in the direction of the polarization axis. The intensity of the polarized light is exactly ½ of the original light

Malus' Law: When there are two polarizers at different angles, the intensity at the end can be calculated with $\mathbf{I=I_0\cos^2\phi}$ where I is the final intensity, I_0 is the initial intensity before the polarization, and ϕ is the difference between the axis of polarization of the the filters.

11.4 Scattering of Light

When light hits our atmosphere it is absorbed then rescattered in a variety of directions depending on the energy that the light has. Blue light has more energy due to its shorter wavelength so it will scatter more while red light has less energy and mostly continues on its original path.

During the day the sky appears blue because of the high energy rescattered light that hit our atmosphere and bounced down to us while the red light continues past us. When the sun is low, the light hits our atmosphere first, the blue light goes flying and the red light continues on its original path and hits us.

11.5 Huygens' Principle (Reflection and Refraction)

This is how the wave model works with reflection and refraction. It's a principle that states that any point along a wave can create its own secondary wavelet that goes in all directions. When the wave front gets disturbed by a changing substance, it creates a new wave at each of the points along that substance. These new waves go in all directions resulting in refraction and reflection.

12.0 GEOMETRIC OPTICS

12.1 Image Formation

Two different light rays are needed to determine the apparent point of an image from a mirror. For flat mirrors the rays do not actually intersect at a real point so the image is considered virtual.

The distance is also equal for a flat mirror, between the source of light and its reflected image from the mirror

12.2 Spherical Mirrors

Concave: Mirrors that bend inward create real images (the reflected light rays interact)

Definitions:

- **Radius**: This is the radius of the mirror (R)
- **Center of Curvature**: If the mirror was a full circle, this would be the center
- **Vertex**: This is the point where a horizontal light ray will reflect directly back onto itself (aka the far most point)
- **Distance**: The distance from object to the vertex is s, the distance from the real image to the vertex is s'. These distances can be calculated with

$$\mathbf{1/s + 1/s' = 2/R}$$

Focal Point: If there are numerous parallel rays coming at a concave mirror, they will all pass through a certain focal point before continuing. If there is a source of light at the focal point, it will reflect off the mirror and produce parallel rays. **2/R = 1/f**

Magnification: An extended object placed beyond the focal point will create an image that is upside down and smaller **m = y'/y = -s'/s** m is the magnification, y is the height of the object while y' is the height of the image, s is the distance from the object to the vertex (the negative indicates an inverted image)

Placements: Different placements of the object will determine different types of images

Beyond the Focal Point: Inverted, real and smaller
Placed at Center of Curvature: Inverted, real, and equal size
Placed at Focal Point: Does not create an image
Placed in Front of Focal Point: Virtual, erect and larger

Convex: Mirrors that bend outwards like a shiny ball create virtual images.

The main difference is that the radius R and the focal point F are negative meaning they are behind the mirror. s' is also negative (behind the mirror) and the image is upright.

Focal Point: F is the virtual focal point, if rays are hitting the mirror in parallel, they will all diverge from a virtual focal point behind the mirror. Likewise if there are multiple rays aiming for the virtual focal point, they will reflect back parallel.

12.3 Lenses

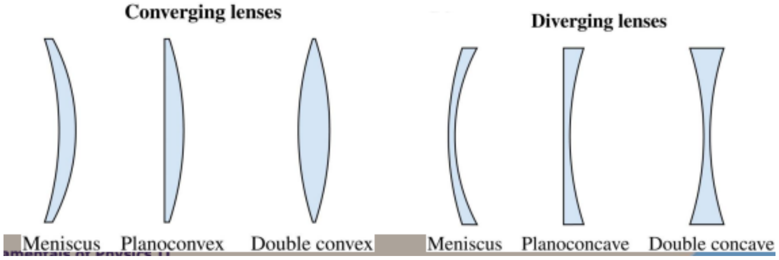
Lenses use the diffraction of light through various surfaces to alter images. Each side will have its own radius and the whole lens will have one focal point. The lensmaker's equation is

$$\frac{1}{f} = (n - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

If the center of curvature is behind the lens, then R is positive. If its in front of the lens, the R is negative. There are two focal points with an equal distance f from the center of the lens

Thin Converging Lens: These act similar to a concave mirror. Parallel rays will converge on the opposite side of the lens at the focal point while rays coming from the focal point will pass parallel on the other side. They are thicker in the middle and focus light.

Thin Diverging Lens: Similar to a convex mirror, parallel rays will diverge on the other side seemingly from the first focal point. Rays coming in aiming at the other side's focal point will continue parallel on the other side of the lens. They are thinner at the center.



Sign Convention: The front side is for the object's positive distance (+s) and the back side is the images positive distance (+s'). Converging lenses have a +f and diverging lenses have a -f. Real images are behind the lens while virtual images are in front of the lens.

Double Lens: When there are two lenses, first find the image created by the first lens and then treat it as the object for the second image. The magnification of the whole system together is the magnification of each lens multiplied, or: M= (- s_a'/s_a)(- s_b'/s_b)

13.0 INTERFERENCE

13.1 Interference of Two Waves

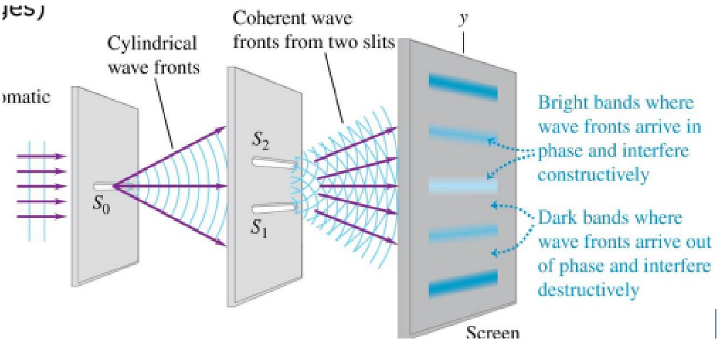
Light and sound produce waves that go in every direction. Whenever two waves intersect, the height of their waves add together. If one is at its lowest point and another is at its highest, they cancel out and it's called destructive interference. When both are at their highest when they intersect, they double in height and it's called constructive interference. If we know the wavelength and distance that two sources are producing at, we can determine their affects when they intersect.

Constructive Interference: If the difference between the distances of the sources and the point if an intersection is an integer multiple of the wavelength then the interference at that point is constructive. **$(r_2-r_1)=l\lambda$**

Destructive Interference: If the difference is halfway between an integer multiple of the wavelength then it is destructive **$(r_2-r_1)=(l+\frac{1}{2})\lambda$**

13.2 Intensity Distribution of an Interference Pattern

Double Slit Experiment: When light is shown through two slits, it causes the waves to start again at each of the two slit locations. These sources will then interfere with each other as though they are their own unique sources of light with the exact same wavelength. This creates a striped pattern on the next surface that shows where the waves are constructive and destructive.



Geometry: To consider the layout geometrically, we can use **$d\sin\theta=m\lambda$** where d is the distance between the slits, m refers to an integer that's counted away from the central fringe, and θ is the angle made with the center of the two slits and the location of the fringe

Calculating Height: For a small angle, the height of a fringe can be calculated with $y_m = R\frac{m\lambda}{d}$ where **y** is the height from the center, **R** is the distance between the double slit board and the screen, **m** is an integer to represent which fringe we're talking about (0 is the middle one and you count up or down from there) and **d** is the distance between the slits.

13.3 Electric Field in Interference

The amplitude of a light wave is determined by the electric field. This means we can represent two light waves with **$E_1(t)=E\cos(\omega t)$** and **$E_2(t)=E\cos(\omega t+\phi)$**. The phase difference between these will be **ϕ**. The electric field of the combined wavepoint will then be **$E_{tog}=2E|\cos(\frac{1}{2}\phi)|$**

E is the maximum amplitude of the light wave, ω is the angular frequency and is measured in rads/sec, it represents the frequency and so the type of light. **ϕ** is the phase difference. If ϕ is an even multiple of pi then its constructive, an odd multiple of pi is destructive.

Intensity: The intensity is proportional to the square of the electric field, so **$I=I_0\cos^2(\frac{1}{2}\phi)$** is the intensity of light at some point after a double slit where the phase difference is ϕ, the original light has intensity I_0

Phase Difference: The phase difference can be calculated by the geometry of the double slit experiment: **$\phi=2\pi(d\sin\theta)/\lambda$**

14.0 DIFFRACTION

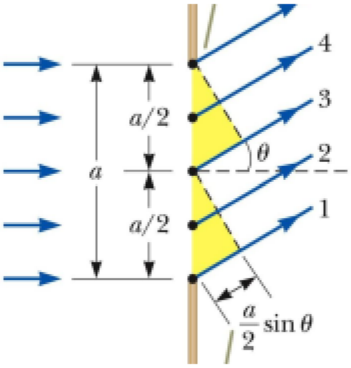
14.1 Fresnel and Fraunhofer Diffraction

If light is blocked by an opaque object, we would assume there would be a harsh line where that object is blocking, but there is actually a striped pattern where the light bends around the object.

Huygens' Principle states that any point along a wave front can be treated as its own new wavelet source that can interact with one another.

- When the light goes through the single slit it creates all new wavelets that are able to interfere with each other.
- The main bright fringe passes through as normal to the screen but the edges have the striped fringe pattern due to the interference of the wavelets.

Locating Dark Fringes: Dark fringes appear when there is a path difference of λ/2. For a slit with width a we can consider the wave that starts at the bottom and one that starts at the middle, their path difference will be ½asinθ (because triangles). We can set this to be λ/2 and then the angle that dark fringes appear will be **sinθ=mλ/a**. Where m is a positive or negative integer aside from 0.



14.2 Intensity Distribution

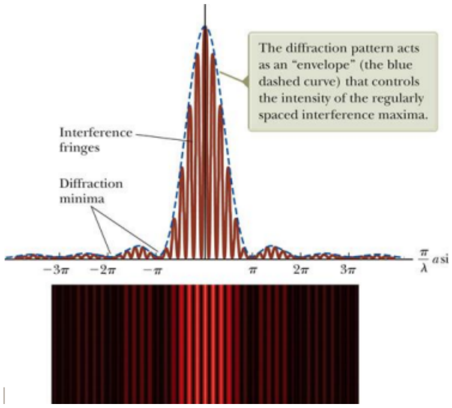
The intensity of a single slit experiment is calculated with

$$I = I_0\left[\frac{\sin(\pi a \sin\theta / \lambda)}{(\pi a \sin\theta / \lambda)}\right]^2$$

Where a is the width of the slit, θ is the angle from the center of the slit to the fringe location **in rads** and I_0 is the intensity before entering the slit.

To simplify this expression, if the angle is small then we can sub sinθ with y/R where **y** is the height from the center, **R** is the distance between the slit board and the screen.

Revisiting Double Slit: The interference of a double slit pattern is still roughly the same as discussed in the interference chapter, but there is a little more to it when considering the diffraction occurring within each slit. In reality, the double slit experiment will have many small teeth in each light fringe


The diffraction pattern acts as an "envelope" (the blue dashed curve) that controls the intensity of the regularly spaced interference maxima.

14.3 Diffraction Grating

A diffraction grating is essentially a double slit scenario but with many many closely placed together slits. It results in very nicely spaced out bright fringes that are at $d\sin\theta_{bright}=m\lambda$. These angles will no longer be small angles so we need to use trig. Diffraction gratings will create a rainbow effect for the fringes outside of the main central fringe. This is because different colours have different wavelengths and because the angles are so separate, the separate colours are more distinct