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Learning Outcomes

- Simplify Nth roots.
- Write radicals as rational exponents.

Launch Desmos Calculator

Using Rational Roots

Although square roots are the most common rational roots, we can also find cube roots, 4th roots, 5th roots, and more. Just as the square root function is the inverse of the squaring function, these roots are the inverse of their respective power functions. These functions can be useful when we need to determine the number that, when raised to a certain power, gives a certain number.

Suppose we know that [latex]{a}^{3}=8[/latex]. We want to find what number raised to the 3rd power is equal to 8. Since [latex]{2}^{3}=8[/latex], we say that 2 is the cube root of 8.

The *n*th root of [latex]a[/latex] is a number that, when raised to the *n*th power, gives [latex]a[/latex]. For example, [latex]-3[/latex] is the 5th root of [latex]-243[/latex] because [latex]{\left(-3\right)}^{5}=-243[/latex]. If [latex]a[/latex] is a real number with at least one *n*th root, then the **principal *n*th root** of [latex]a[/latex] is the number with the same sign as [latex]a[/latex] that, when raised to the *n*th power, equals [latex]a[/latex].

The principal *n*th root of [latex]a[/latex] is written as [latex]\sqrt[n]{a}[/latex], where [latex]n[/latex] is a positive integer greater than or equal to 2. In the radical expression, [latex]n[/latex] is called the **index** of the radical.

A General Note: Principal ***n***th Root

If [latex]a[/latex] is a real number with at least one *n*th root, then the **principal *n*th root** of [latex]a[/latex], written as [latex]\sqrt[n]{a}[/latex], is the number with the same sign as [latex]a[/latex] that, when raised to the *n*th power, equals [latex]a[/latex]. The **index** of the radical is [latex]n[/latex].

Example: Simplifying ***n***th Roots

Simplify each of the following:

1. [latex]\sqrt[5]{-32}[/latex]
2. [latex]\sqrt[4]{4}\cdot \sqrt[4]{1,024}[/latex]
3. [latex]-\sqrt[3]{\dfrac{8{x}^{6}}{125}}[/latex]
4. [latex]8\sqrt[4]{3}-\sqrt[4]{48}[/latex]

Show Solution

1. [latex]\sqrt[5]{-32}=-2[/latex] because [latex]{\left(-2\right)}^{5}=-32 \\ \text{ }[/latex]
2. First, express the product as a single radical expression.
   [latex]\sqrt[4]{4\text{,}096}=8[/latex] because [latex]{8}^{4}=4,096[/latex]
3. [latex]\begin{align}\\ &\frac{-\sqrt[3]{8{x}^{6}}}{\sqrt[3]{125}} && \text{Write as quotient of two radical expressions}. \\ &\frac{-2{x}^{2}}{5} && \text{Simplify}. \\ \end{align}[/latex]
4. [latex]\begin{align}\\ &8\sqrt[4]{3}-2\sqrt[4]{3} && \text{Simplify to get equal radicands}. \\ &6\sqrt[4]{3} && \text{Add}. \end{align}[/latex]

Try It

Simplify.

1. [latex]\sqrt[3]{-216}[/latex]
2. [latex]\dfrac{3\sqrt[4]{80}}{\sqrt[4]{5}}[/latex]
3. [latex]6\sqrt[3]{9,000}+7\sqrt[3]{576}[/latex]

Show Solution

1. [latex]-6[/latex]
2. [latex]6[/latex]
3. [latex]88\sqrt[3]{9}[/latex]

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Using Rational Exponents

Radical expressions can also be written without using the radical symbol. We can use rational (fractional) exponents. The index must be a positive integer. If the index [latex]n[/latex] is even, then [latex]a[/latex] cannot be negative.

[latex]{a}^{\frac{1}{n}}=\sqrt[n]{a}[/latex]

We can also have rational exponents with numerators other than 1. In these cases, the exponent must be a fraction in lowest terms. We raise the base to a power and take an *n*th root. The numerator tells us the power and the denominator tells us the root.

[latex]{a}^{\frac{m}{n}}={\left(\sqrt[n]{a}\right)}^{m}=\sqrt[n]{{a}^{m}}[/latex]

All of the properties of exponents that we learned for integer exponents also hold for rational exponents.

Rational Exponents

Rational exponents are another way to express principal *n*th roots. The general form for converting between a radical expression with a radical symbol and one with a rational exponent is

[latex]\begin{align}{a}^{\frac{m}{n}}={\left(\sqrt[n]{a}\right)}^{m}=\sqrt[n]{{a}^{m}}\end{align}[/latex]

How To: Given an expression with a rational exponent, write the expression as a radical.

1. Determine the power by looking at the numerator of the exponent.
2. Determine the root by looking at the denominator of the exponent.
3. Using the base as the radicand, raise the radicand to the power and use the root as the index.

Example: Writing Rational Exponents as Radicals

Write [latex]{343}^{\frac{2}{3}}[/latex] as a radical. Simplify.

Show Solution

The 2 tells us the power and the 3 tells us the root.

[latex]{343}^{\frac{2}{3}}={\left(\sqrt[3]{343}\right)}^{2}=\sqrt[3]{{343}^{2}}[/latex]

We know that [latex]\sqrt[3]{343}=7[/latex] because [latex]{7}^{3}=343[/latex]. Because the cube root is easy to find, it is easiest to find the cube root before squaring for this problem. In general, it is easier to find the root first and then raise it to a power.

[latex]{343}^{\frac{2}{3}}={\left(\sqrt[3]{343}\right)}^{2}={7}^{2}=49[/latex]

Try It

Write [latex]{9}^{\frac{5}{2}}[/latex] as a radical. Simplify.

Show Solution

[latex]{\left(\sqrt{9}\right)}^{5}={3}^{5}=243[/latex]

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Example: Writing Radicals as Rational Exponents

Write [latex]\dfrac{4}{\sqrt[7]{{a}^{2}}}[/latex] using a rational exponent.

Show Solution

The power is 2 and the root is 7, so the rational exponent will be [latex]\dfrac{2}{7}[/latex]. We get [latex]\dfrac{4}{{a}^{\frac{2}{7}}}[/latex]. Using properties of exponents, we get [latex]\dfrac{4}{\sqrt[7]{{a}^{2}}}=4{a}^{\frac{-2}{7}}[/latex].

Try It

Write [latex]x\sqrt{{\left(5y\right)}^{9}}[/latex] using a rational exponent.

Show Solution

[latex]x{\left(5y\right)}^{\frac{9}{2}}[/latex]

Watch this video to see more examples of how to write a radical with a fractional exponent.

Video Link

Example: Simplifying Rational Exponents

Simplify:

1. [latex]5\left(2{x}^{\frac{3}{4}}\right)\left(3{x}^{\frac{1}{5}}\right)[/latex]
2. [latex]{\left(\dfrac{16}{9}\right)}^{-\frac{1}{2}}[/latex]

Show Solution

1.

[latex]\begin{align}&30{x}^{\frac{3}{4}}{x}^{\frac{1}{5}}&& \text{Multiply the coefficients}. \\ &30{x}^{\frac{3}{4}+\frac{1}{5}}&& \text{Use properties of exponents}. \\ &30{x}^{\frac{19}{20}}&& \text{Simplify}. \end{align}[/latex]

2.

[latex]\begin{align}&{\left(\frac{9}{16}\right)}^{\frac{1}{2}}&& \text{Use definition of negative exponents}. \\ &\sqrt{\frac{9}{16}}&& \text{Rewrite as a radical}. \\ &\frac{\sqrt{9}}{\sqrt{16}}&& \text{Use the quotient rule}. \\ &\frac{3}{4}&& \text{Simplify}. \end{align}[/latex]

Try It

Simplify [latex]{\left(8x\right)}^{\frac{1}{3}}\left(14{x}^{\frac{6}{5}}\right)[/latex].

Show Solution

[latex]28{x}^{\frac{23}{15}}[/latex]

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