Practice – Explicit and Recursive Sequences

Part 1: Explicit Sequences

*** Key Problems:** Solve problems 1. d-g, 5, 7, and 8 first. Do more for more practice.
*** Answers given at the end of this document.**

Practice 1

1 Find the first five terms of each sequence.

a $u_n = 4n - 1$ for $n \geq 1$

b $u_n = 6n^2 + 2$ for $n \geq 1$

c $u_n = 10 - \frac{1}{4}n^2$ for $n \geq 1$

d $u_n = 2^n - 2$ for $n \geq 1$

e $u_n = (n + 4)(n - 3)$ for $n \geq 1$

f $u_n = 4n - 5$ for $n \geq 0$

g $u_n = \frac{25}{24}n^4 - \frac{155}{12}n^3 + \frac{1307}{24}n^2 - \frac{1051}{12}n + 45$ for $n \geq 1$

2 Find the tenth term of the sequence given by $u_n = 5n - 3$ for $n \geq 1$.

3 Determine if 54 is a term in the sequence $u_n = 4n - 1$ for $n \geq 1$.

4 Determine which term of the sequence $u_n = 3n - 5$ has value 61.

5 Find the fifteenth term of the sequence given by $u_n = 10 - \frac{10}{n}$ for $n \geq 1$.

6 Find the value of the fourth term of the sequence given by $u_n = 2n + 12$ for $n \geq 0$.

7 **a** Find the term of the sequence $u_n = 2n^2 + 4$ that has value 246.

b Show that 396 is a term in this sequence.

8 Find the eleventh term of the sequence given by $u_n = 100 - \frac{n^2}{5}$ for $n \geq 0$.

Problem solving

9 Identify which explicit formula, **a** to **f**, corresponds to each sequence, **i** to **vi**.

a $u_n = 4n + 1$ for $n \geq 0$

b $u_n = 2n + 3$ for $n \geq 1$

c $u_n = 5n - 4$ for $n \geq 1$

d $u_n = 3n$ for $n \geq 1$

e $u_n = 3n - 2$ for $n \geq 0$

f $u_n = 2n + 1$ for $n \geq 0$

i 3, 6, 9, 12, 15, …

ii 1, 3, 5, 7, 9, …

iii –2, 1, 4, 7, 10, …

iv 1, 5, 9, 13, 17, …

v 5, 7, 9, 11, 13, …

vi 1, 6, 11, 16, 21, …

Tip

The command term **identify** requires you to state briefly how you have made your decisions.

Recursive Sequences on back □

Part 2: Recursive Sequences

*** Key Problems:** Solve problems 1. a, e-g, 3, and 4 first. Do more for more practice.

Practice 2

1 Find the first five terms of each sequence.

a $u_{n+1} = 4u_n - 1, u_1 = 1$ for $n \geq 1$

b $u_{n+1} = 2u_n + 1, u_1 = 2$ for $n \geq 1$

c $u_{n+1} = u_n + 7, u_1 = -4$ for $n \geq 1$

d $u_{n+1} = \frac{1}{2}u_n + 1, u_1 = 3$ for $n \geq 1$

e $u_{n+1} = 2u_n(1 - u_n), u_1 = 0.8$ for $n \geq 1$

f $u_{n+1} = 4u_n(1 - u_n), u_1 = 0.8$ for $n \geq 1$

g $u_n = u_{n-1} + 3, u_1 = 4$ for $n \geq 2$

Tip

Look for patterns when you work with sequences. Do the terms of the sequence seem to increase, decrease, or bounce around chaotically?

2 A sequence is given by $u_{n+1} = 3 - u_n$ and $u_0 = -3$.
Find the first four terms of the sequence.

3 A sequence is given by $u_{n+1} = \frac{u_n - 1}{u_n}, u_1 = 2$ for $n \geq 1$.

a Find the first six terms of the sequence.

b Describe any patterns you notice.

c Predict the next few terms of the sequence.

Tip

It can be useful to list a few terms to explore an unfamiliar sequence.

Problem solving

4 A sequence is given by $u_{n+1} = 2u_n - 1, u_1 = 2$ for $n \geq 1$.
Find the term of the sequence that has value 257.

5 A sequence is given by $u_{n+1} = 3u_n - 2, u_1 = 2$ for $n \geq 1$.
Find the value of the largest term in the sequence that is less than 10 000.

ANSWERS

Explicit Sequences

1 **a** 3, 7, 11, 15, 19
b 8, 26, 56, 98, 152
c 9.75, 9, 7.75, 6, 3.75
d 0, 2, 6, 14, 30
e −10, −6, 0, 8, 18
f −5, −1, 3, 7, 11
g 0, 1, 8, 6, 5

2 47

3 It is not; all terms of the sequence are odd.

4 The 22nd term

5 $9\frac{1}{3}$

6 18 (Note: the first term of the sequence is u_0.)

7 **a** The 11th term
b The 14th term $= 2 \times 14^2 + 4 = 396$

8 80

9 **a, iv** **b, v** **c, vi** **d, i** **e, iii** **f, ii**

Recursive Sequences

1 **a** 1, 3, 11, 43, 171
b 2, 5, 11, 23, 47
c −4, 3, 10, 17, 24
d 3, 2.5, 2.25, 2.125, 2.0625
e 0.8. 0.32, 0.4352, 0.491602, 0.499859
f 0.8, 0.64, 0.9216, 0.28901376, 0.821939226
g 4, 7, 10, 13, 16

2 −3, 6, −3, 6

3 **a** 2, 0.5, −1, 2, 0.5, −1
b The numbers 2, 0.5, −1 keep repeating.
c 2, 0.5, −1, 2, 0.5, −1

4 The 9th term

5 6562