Relating the doubling period for cases to R_0

In the UK Government presentation on 2nd April 2020, the Health Secretary, Matt Hancock, said that the doubling period for the coronavirus outbreak was between 3 and 4 days. These two numbers lead to quite different outcomes numerically. Today (9th April 2020), The Chief Medical Officer, Chris Whitty, mentioned it again, saying it was as high as six days, which surprised me, although, afterwards Sir Patrick Vallance (Chief Scientist) seemed to suggest this might have been for ICU cases (and not for the overall case numbers). Not a clear situation.

Charts presented by Johns Hopkins University have usefully included guidelines for doubling periods of one, two, three and seven days (on their log charts) so that we can compare where different nations around the world are placed with their growth rates for the epidemic.

I was interested to relate this key number - the doubling period in days - to the R_0 Reproduction Number derived and explained in my previous paper at [SIR model equations and Ro](). I am indebted to Prof. Alex de Visscher for his help with reviewing this outline.

Recalling the differential equations modelling the outbreak, covered in that previous paper (equation numbering per that paper)

$$ds/dt = -\beta si \qquad (2)$$
$$di/dt = (\beta s - \nu)i \qquad (3)$$
$$dr/dt = \nu i, \qquad (4)$$

and adding (3) and (4) together, we see that:

$$d(i + r)/dt = \beta si. \qquad (12)$$

As before, take $s \approx 1$, and also $i \approx (i + r)$at the start of the epidemic (no recoveries yet), and call it j.

Then (12) becomes $dj/dt = \beta j$, and upon integration, we see that

$j = j_0 e^{\beta t}$, where $j_0$is the value of $j$at time $t = 0$.

β, the rate of infection, can now easily be expressed from the case doubling time T_D: if the growth of infected cases is showing an exponential growth to base 2, against time, then

$j/j_0 = e^{\beta t}$.

Taking $j/j_0 = 2$, i.e. the infections have doubled, say in time $t = T_D$, where T_D is the doubling time, we see that

$e^{\beta T_D} = 2$, and taking logs of both sides,

$\beta T_D = log_e 2$, or

$\beta = (log_e 2)/T_D$ (13)

Now, recalling (1), (7) and (8) from my previous paper at SIR model equations and Ro;

$R_0 = \tau.\bar{c}.d$, and (1)

$\beta = \tau\bar{c}$, (7) the effective contact rate,

and that the expected duration of infection d, the inverse of the recovery rate ν (recoveries per time interval):

$d = 1/\nu$. (8)

We have

$R_0 = \beta d$. Substituting from (13), we finally have

$R_0 = d(log_e 2)/T_D$ (14)

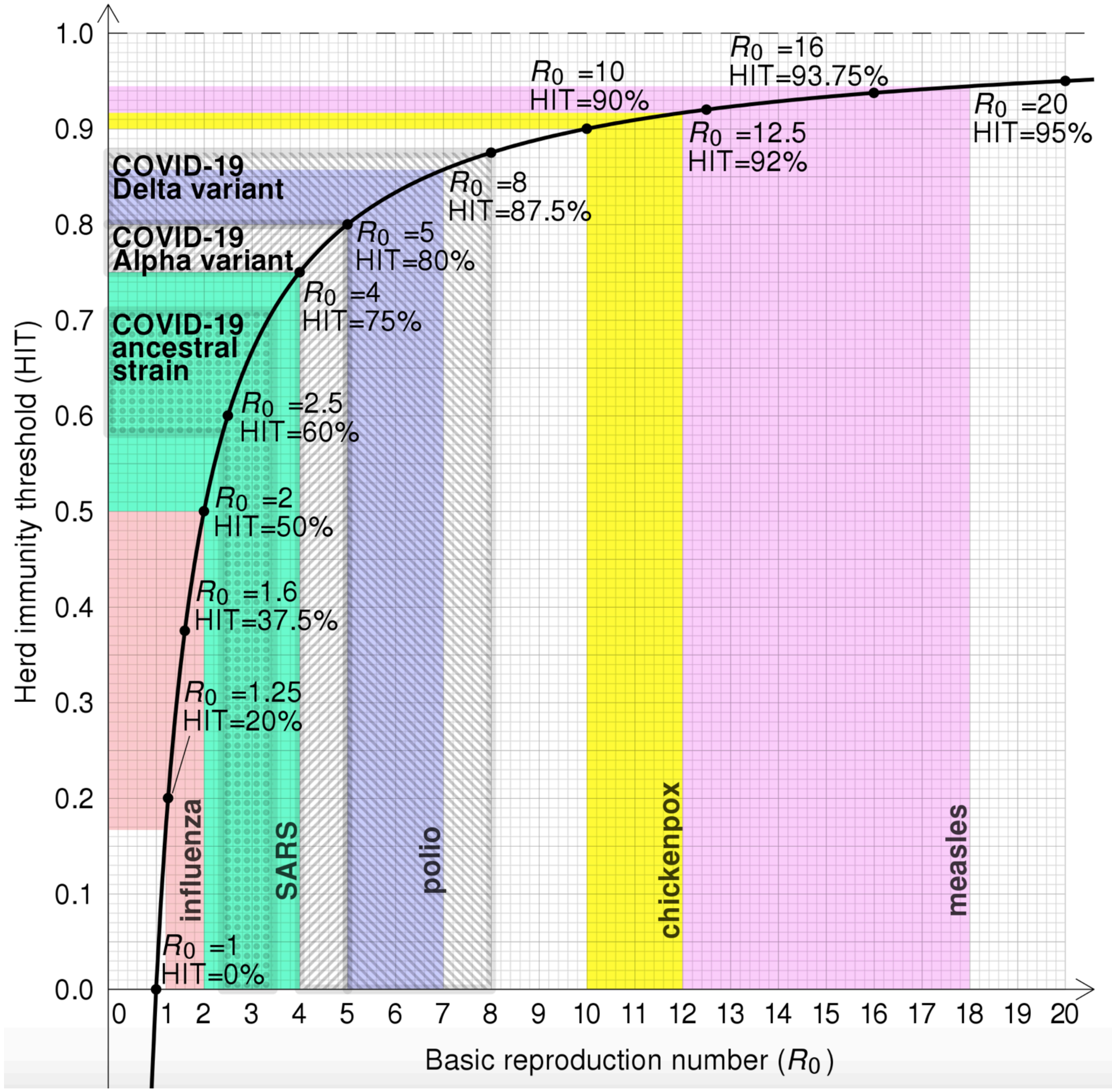
as our expression for R_0 in terms of the doubling period T_D, where d is the expected duration of infection, the inverse of the recovery rate ν.

So for a doubling period T_D of 3 days, say, and a disease duration d of 2 weeks, we would have

$R_0 = 14 \times 0.7/3 = 3.266$.

Assuming that the disease duration ***is*** 2 weeks, to achieve an $R_0 < 1$ we would need the doubling period to be more than 10 days or so. If the average disease duration were 1 week, then the doubling period could be as low as 5 days for $R_0 < 1$, and the reduction of the disease.

This all very much depends on the nature of the disease, because different infections have quite widely varying R_0 values. This useful chart from [Basic reproduction number] at Wikipedia shows some comparative examples of R_0 for influenza, SARS, polio, chicken pox and measles compared to the original Covid strain and its Alpha and Delta variants, with the related required Herd Immunity percentages HIT.



From (14), we see that HIT, given by (1 - 1/R_0)* is related directly to the doubling period T_D by

$$\text{HIT} = (1 - T_D / d(log_e 2)).$$

*You can see the derivation of (1 - 1/R_0) as the Herd Immunity requirement at my Google document [The Mathematics of Vaccination Efficacy].