The North Carolina High School Collaborative Instructional Framework

NC Math 3
Unit 7: Introduction to Trigonometric Functions

10 Days Block Schedule | **September 2017 Update** | 20 Days Traditional Schedule

RESEARCH BRIEF: Trigonometric Functions

Essential Questions:

- How can the coordinates of any point on a circle be determined from the radius and angle of rotation?
- How can the trigonometric ratio be represented as a function of an angle measure?
- How do dependent variables change as independent variables increase for sine and cosine ratios represented on a circle and trigonometric curve?
- How can cosine and sine functions be used to model real world situations?

Learning Outcomes	Student Objectives
• Students should be able to convert between radian and degree measurements. • Students should explain the relationship between the domain and range represented as angle measurements and ratios respectively, of sine and cosine functions. • Students will understand the relationship between the cosine and sine values and the horizontal and vertical components of position. • Given a trigonometric function, students will determine key features of a graph, table, or context. • Students should be able to compare features of two functions in different representations. • Students will use prior knowledge of function transformations to build new sine and cosine functions.	• I will **calculate** an equivalent angle measurement given radians or degrees. • I will **interpret** the relationship between input and output of sine and cosine functions. • I will **evaluate** the value of sine and cosine functions given an angle measurement. • I will **determine** the coordinates of any point on a circle centered at the origin given the radius and angle measurement. • I can **sketch** and **recognize** angles in standard position to find coterminal and reference angles. • I will **compare** features of two functions in different representations.

- I will **interpret** parts of a function and their relationship with the graph, table, and context.
- I will be able to read a word problem or analyze a graph and **create** an equation or inequality.
- I will **describe** the transformations that have been applied to the sine and cosine functions given an equation or graph.

Standards Addressed in this Unit

Overarching Standards

- NC.M3.A-SSE.1b: Interpret the structure of expressions. Interpret expressions that represent a quantity in terms of its context. b. Interpret expressions composed of multiple parts by viewing one or more of their parts as a single entity to give meaning in terms of a context.

Understand that trigonometric functions can be represented on the coordinate plane in different ways based on the quantities used for the axises. Understand the relationships between different representations of trigonometric functions on the coordinate plane. Identify and interpret key features of trigonometric functions.

- **NC.M3.F-IF.1:** Extend the concept of a function by recognizing that trigonometric ratios are functions of angle measure.
- **NC.M1.F-IF.4:** Interpret functions that arise in applications in terms of the context. Interpret key features of graphs, tables, and verbal descriptions in context to describe functions that arise in applications relating two quantities, including: intercepts; intervals where the function is increasing, decreasing, positive, or negative; and maximums and minimums.
- NC.M3.F-IF.7: Analyze functions using different representations. Analyze piecewise, absolute value, polynomials, exponential, rational, and trigonometric functions (sine and cosine) using different representations to show key features of the graph, by hand in simple cases and using technology for more complicated cases, including: domain and range; intercepts; intervals where the function is increasing, decreasing, positive, or negative; rate of change; relative maximums and minimums; symmetries; end behavior; period; and discontinuities.
- NC.M3.F-IF.9: Analyze functions using different representations. Compare key features of two functions using different representations by comparing properties of two different functions, each with a different representation (symbolically, graphically, numerically in tables, or by verbal descriptions).
- NC.M3.F-BF.3: Build new functions from existing functions. Extend an understanding of the effects on the graphical and tabular representations of a

function when replacing $f(x)$ with $k \cdot f(x)$, $f(x) + k$, $f(x + k)$ to include $f(k \cdot x)$ for specific values of k (both positive and negative).

- **NC.M3.F-TF.1:** Understand radian measure of an angle as:
  - The ratio of the length of an arc on a circle subtended by the angle to its radius.
  - A dimensionless measure of length defined by the quotient of arc length and radius that is a real number.
  - The domain for trigonometric functions.

- **NC.M3.F-TF.2a:** Build an understanding of trigonometric functions by using tables, graphs and technology to represent the cosine and sine functions.
  a. Interpret the sine function as the relationship between the radian measure of an angle formed by the horizontal axis and a terminal ray on the unit circle and its y coordinate.

- **NC.M3.F-TF.2b:** Build an understanding of trigonometric functions by using tables, graphs and technology to represent the cosine and sine functions.
  b. Interpret the cosine function as the relationship between the radian measure of an angle formed by the horizontal axis and a terminal ray on the unit circle and its x coordinate.

- **NC.M3.F-TF.5:** Use technology to investigate the parameters, a, b, and h of a sine function, $f(x) = a \cdot sin(b \cdot x) + h$, to represent periodic phenomena and interpret key features in terms of a context.

Implementing the Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Aligned Resources for this Unit

-

The Math Resource for Instruction - Customized for the Content of this Unit

NC.M3.A-SSE.1b

Interpret the structure of expressions.

Interpret expressions that represent a quantity in terms of its context.

b. Interpret expressions composed of multiple parts by viewing one or more of their parts as a single entity to give meaning in terms of a context.

Concepts and Skills

Pre-requisite

- Interpret parts of a function as a single entity (NC.M2.A-SSE.1b)
- Interpret parts of an expression in context (NC.M3.A-SSE.1a)

Connections

- Create and graph equations (NC.M3.A-CED.1, NC.M3.A-CED.2, NC.M3.A-CED.3)
- Interpret one variable rational equations (NC.M3.A-REI.2)
- Interpret statements written in function notation (NC.M3.F-IF.2)
- Analyze and compare functions for key features (NC.M3.F-IF.4, NC.M3.F-IF.7, NC.M3.F-IF.9)
- Understand the effects on transformations on functions (NC.M3.F-BF.3)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

1 – Make sense of problems and persevere in solving them

4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

New Vocabulary:

Mastering the Standard

Comprehending the Standard

Students must be able to take the multi-part expressions we engage with in Math 3 and see the different parts and what they mean to the expression in context. Students have worked with this standard in Math 1 and Math 2, so the new step is applying it to our Math 3 functions.

As we add piecewise functions and expressions in Math 3, breaking down these expressions and functions into their parts are essential to ensure understanding.

For Example: Explain what operations are performed on the inputs -2, 0, and 2 for the following expression:

$f(x) = \{3x, \ for\ x < 0 \ \frac{1}{x}, \ for\ 0 \leq x < 2 \ x^3, \ for\ x \geq 2$

Which input is not in the domain? Why not?

Assessing for Understanding

Students must be able to demonstrate that they can understand, analyze, and interpret the information that an expression gives in context. The two most important parts are determining what a certain situation asks for, and then how the information can be determined from the expression.

Example: Given the expression: $a \sin(bx) + c$

a) What do a, b, c, and x represent?

b) How would increasing each variable by a factor of 2 change the value of the expression?

Note: This example could also fit NC.M3.F-TF.5. For this standards, students must recognize that changing b and x have different impacts then a or c because they are "inputs" of a sine function. Teachers can give values for the variables to help students interpret.

	Students should notice the similarity of this expression as with function transformations (e.g., $a \cdot f(b \cdot x) + c$).

NC.M3.A-SSE.2

Interpret the structure of expressions.

Use the structure of an expression to identify ways to write equivalent expressions.

Concepts and Skills

Pre-requisite

- Justifying a solution method (NC.M2.A-REI.1)

Connections

- Write an equivalent form of an exponential expression (NC.M3.A-SSE.3c)
- Create and graph equations (NC.M3.A-CED.1, NC.M3.A-CED.2, NC.M3.A-CED.3)
- Justify a solution method (NC.M3.A-REI.1)
- Solve one variable rational equations (NC.M3.A-REI.2)
- Analyze and compare functions for key features (NC.M3.F-IF.7, NC.M3.F-IF.9)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

7 – Look for and make use of structure

8 – Look for and express regularity in repeated reasoning

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Mastering the Standard

Comprehending the Standard

In Math 1 and 2, students factored quadratics. In Math 3, extend factoring to include strategies for rewriting more complicated expressions. Factoring a sum or difference of cubes, factoring a GCF out of a polynomial, and finding missing coefficients for expressions based on the factors can all be included.

For Example: When factoring a difference of cubes, is the trinomial factor always, sometimes or never factorable? How do you know?

Assessing for Understanding

This standard can be assessed mainly by performing the algebraic manipulation. Problems could include:

Example: Prove that $\sin(x - \pi/2)$ is the same as cos (x). Use the triangles below if you need.

NC.M3.F-IF.1

Understand the concept of a function and use function notation.

Extend the concept of a function by recognizing that trigonometric ratios are functions of angle measure.

Concepts and Skills

Pre-requisite

- Define a function (NC.M1.F-IF.1)
- Verify experimentally that the side ratios in similar triangles are properties of the angle measures in the triangle (NC.M2.G-SRT.6)
- Understand radian measure of an angle (NC.M3.F-TF.1)

Connections

- Analyze and compare functions in various representations (NC.M3.F-IF.4, NC.M3.F-IF.7, NC.M3.F-IF.9)
- Build an understanding of trig functions in relation to its radian measure (NC.M3.F-TF.2a, NC.M3.F-TF.2b)
- Investigate the parameters of the sine function (NC.M3.F-TF.5)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to discuss the output of trig functions as unit rates.

Mastering the Standard

Comprehending the Standard

This is an extension of previous learning. Students should already understand function notation, the correspondence of inputs and outputs, and evaluating functions. In Math 3, students should build an understanding of the unique relationship between the measure of the angle and the value of the particular trig ratio.

Also in Math 3, students build an understanding of radian measure.
See NC.M3.F-TF.1 for more information.

Students should also begin to see the graphical representations of trig functions, both on a unit circle and on a graph in which the domain is the measure of the angle and the range is the value of the associated trig ratio.
On the unit circle, the input is the measure of the angle and the output of the sine function is the *y*-coordinate of the vertex of the formed triangle and the output of the cosine function is the *x*-coordinate of the vertex of the formed triangle.
See NC.M3.F-TF.2a and NC.M3.F-TF.2b for more information.

Assessing for Understanding

Students should be able to create trig functions in various representations, recognizing that the domain of a trig function is the measure of the angle.

Example: Complete the function table for $f(\theta) = sin\,\theta$ and $f(\theta) = cos\,\theta$ and complete the following.

θ	$sin\,\theta$	$cos\,\theta$	θ	$sin\,\theta$	$cos\,\theta$
0			π		
$\frac{\pi}{6}$			$\frac{7\pi}{6}$		
$\frac{\pi}{4}$			$\frac{5\pi}{4}$		
$\frac{\pi}{3}$			$\frac{4\pi}{3}$		
$\frac{\pi}{2}$			$\frac{3\pi}{2}$		
$\frac{2\pi}{3}$			$\frac{5\pi}{3}$		
$\frac{3\pi}{4}$			$\frac{7\pi}{4}$		
$\frac{5\pi}{6}$			$\frac{11\pi}{6}$		

Based on the table:

a) Describe in your own words the relationship you see between the measure of the angle and the sine function.
b) If you were to graph $f(\theta) = sin\,\theta$, what would it look like? What would be some of the key features?
c) Describe in your own words the relationship between the measure of the angle and the cosine function.
d) If you were to graph $f(\theta) = cos\,\theta$, what would it look like? What would be some of the key feature?
e) How does $sin\,\theta$ and $cos\,\theta$ relate to each other?

NC.M3.F-IF.4

Interpret functions that arise in applications in terms of the context.

Interpret key features of graphs, tables, and verbal descriptions in context to describe functions that arise in applications relating two quantities to include periodicity and discontinuities.

Concepts and Skills

Pre-requisite

- Interpret key features from graph, tables, and descriptions (NC.M2.F-IF.4)
- Interpret parts of an expression in context (NC.M3.A-SSE.1a, NC.M3.A-SSE.1b)
- Recognize that trig ratios are functions of angle measure (NC.M3.F-IF.1)

Connections

- Analyze and compare functions (NC.M3.F-IF.7, NC.M3.F-IF.9)
- Build functions given a graph, description or ordered pair. (NC.M3.F-BF.1a)
- Use tables and graphs to understand relationships in trig functions (NC.M3.F-TF.2a, NC.M3.F-TF.2b, NC.M3.F-TF.5)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to justify their identified key features with mathematical reasoning.

New Vocabulary: periodicity, discontinuity, amplitude, period, radian.

Mastering the Standard

Comprehending the Standard

This standard is included in Math 1, 2 and 3. Throughout all three courses, students interpret the key features of graphs and tables for a variety of different functions. In Math 3, extend to more complex functions represented by graphs and tables and focus on interpreting key features of all function types. Also, include periodicity as motion that is repeated in equal intervals of time and discontinuity as values that are not in the domain of a function, either as asymptotes or "holes" in the graph.

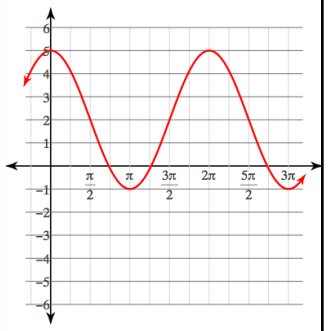
No limitations are listed with this standard. This means that all function types, even those found in more advanced courses. Students do not have to be able to algebraically manipulate a function in order to identify the key features found in graphs, tables, and verbal descriptions.

This is in contrast to NC.M3.F-IF.7, in which the specific function types are included. Students can work algebraically with those listed types and can

Assessing for Understanding

This standard must be assessed using three important forms of displaying our functions: graphs, tables, and verbal descriptions/word problems. Students must be able to interpret each and how they apply to the key input-output values.

Example*:* Jumper horses on carousels move up and down as the carousel spins. Suppose that the back hooves of such a horse are six inches above the floor at their lowest point and two-and-one-half feet above the floor at their highest point. Draw a graph that could represent the height of the back hooves of this carousel horse during a half-minute portion of a carousel ride.



Example: For the function to the right, label and describe the key features. Include intercepts, relative max/min, amplitude, period, midline, and frequency.

Example*:* Over a year, the length of the day (the number of hours from sunrise to sunset) changes every day. The table below shows the length of day every 30 days from 12/31/97 to 3/26/99 for Boston Massachusetts. During what part of the year do the days get longer?

Data on length of day

Date	12/31	1/30	3/1	3/31	4/30	5/30	6/29	7/29	8/28	9/27	10/27	11/26	12/26	1/25	2/24	3/26
Day Number	0	30	60	90	120	150	180	210	240	270	300	330	360	390	420	450
Length (hours)	9.1	9.9	11.2	12.7	14.0	15.0	15.3	14.6	13.3	11.9	10.6	9.5	9.1	9.7	11.0	12.4

analyze those functions in greater detail.	Support your claim using information provided from the table.

NC.M3.F-IF.9

Analyze functions using different representations.

Compare key features of two functions using different representations by comparing properties of two different functions, each with a different representation (symbolically, graphically, numerically in tables, or by verbal descriptions).

Concepts and Skills

Pre-requisite

- Analyze the key features of functions for tables, graphs, descriptions and symbolic form (NC.M3.F-IF.4, NC.M3.F-IF.7)

Connections

- Analyze and compare functions (NC.M3.F-IF.7, NC.M3.F-IF.9)
- Build functions given a graph, description or ordered pair. (NC.M3.F-BF.1a)
- Use tables and graphs to understand relationships in trig functions (NC.M3.F-TF.2a, NC.M3.F-TF.2b, NC.M3.F-TF.5)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should discuss how the comparison of a functions leads to a mathematical understanding, such as with transformations and choosing better models.

New Vocabulary: periodicity, discontinuity

Mastering the Standard

Comprehending the Standard

This standard is included in Math 1, 2 and 3. Throughout all three courses, students compare properties of two functions. The representations of the functions should vary: table, graph, algebraically, or verbal description.

In Math 3, this standard can include two functions of any type students have learned in high school math in any representation. Comparing the key features should be the focus of the teaching for this standard, so the actual functions involved are not as important.

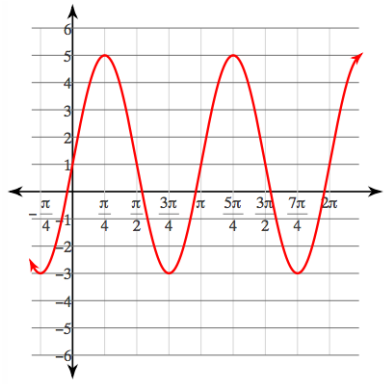
Students are expected to use and interpret compound inequalities using inequality and interval notation to describe key features when appropriate.

Assessing for Understanding

In assessing this standard, students must demonstrate that they can not only identify, but compare, the key features of two different functions. Appropriate question stems could include: Which is less/greater; Which will have a greater value at x = __; Which function has the higher maximum/lower minimum; etc.

Examples: If $f(x) = -2sin(x) - 3$ and $g(x)$ is represented on the graph.

a) Which function has the greatest amplitude? Explain your reasoning.
b) Which has the largest relative maximum?
c) Describe each function's period. Why are they different? What can be said about each function?



NC.M3.F-IF.7

Analyze functions using different representations.

Analyze piecewise, absolute value, polynomials, exponential, rational, and trigonometric functions (sine and cosine) using different representations to show key features of the graph, by hand in simple cases and using technology for more complicated cases, including: domain and range; intercepts; intervals where the function is increasing, decreasing, positive, or negative; rate of change; relative maximums and minimums; symmetries; end behavior; period; and discontinuities.

Concepts and Skills

Pre-requisite

- Analyze functions using different representations to show key features (NC.M2.F-IF.7)
- Interpret parts of an expression in context (NC.M3.A-SSE.1a, NC.M3.A-SSE.1b)
- Use the structure of an expression to identify ways to write equivalent expressions (NC.M3.A-SSE.2)
- Recognize that trig ratios are functions of angle measure (NC.M3.F-IF.1)
- Use function notation to evaluate piecewise functions (NC.M3.F-IF.2)

Connections

- Create and graph equations in two variables (NC.M3.A-CED.2)
- Analyze graphs and tables and compare functions (NC.M3.F-IF.4, NC.M3.F-IF.9)
- Build functions (NC.M3.F-BF.1a, NC.M3.F-BF.1b)
- Understand the effects of transformations on functions (NC.M3.F-BF.3)
- Use tables and graphs to understand relationships in trig functions (NC.M3.F-TF.2a, NC.M3.F-TF.2b, NC.M3.F-TF.5)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

4 – Model with mathematics

6 – Attend to precision

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should discuss which representation best shows each of the key features.

New Vocabulary: periodicity, discontinuity

Mastering the Standard

Comprehending the Standard

In previous math courses, students have identified the characteristic of graphs of other functions, including linear, quadratic, exponential, radical, and inverse variation functions. They should be familiar with the concept of intercepts, domain, range, intervals increasing/decreasing, relative maximum/minimum, and end behavior.

In Math 3, these concepts are extended to piecewise, absolute value, polynomials, exponential, rational, and sine and cosine functions. Discontinuity (asymptotes/holes) and periodicity are new features of functions that must be introduced. The intent of this standard is for students to find discontinuities in tables and graphs and to recognize their relationship to functions. Students are not expected to find an asymptote from a function. (This could be an extension topic.)

Assessing for Understanding

In assessing this standard, students must demonstrate their ability to represent and determine the key features from algebraic and graphical representations of the functions.

Example: Graph $y = 3\,sin\,(x) - 5$ and answer the following questions:

a) What is the period?

b) For the domain of $-2\pi < x < 2\pi$, identify

This standard will likely span multiple units, as most Math 3 courses teach polynomial, exponential, rational, and trigonometric functions in different units. These function characteristics will be repeated and reinforced throughout the course. Students should be able to use interval notation as appropriate.	any relative maxima and minima, intervals of increasing and decreasing, and lines of symmetry.

NC.M3.F-BF.3

Build new functions from existing functions.

Extend an understanding of the effects on the graphical and tabular representations of a function when replacing $f(x)$ with $k{\cdot}f(x)$, $f(x) + k$, $f(x + k)$ to include $f(k{\cdot}x)$ for specific values of k (both positive and negative).

Concepts and Skills

Pre-requisite

- Understand the effects of transformations on functions (NC.M2.F-BF.3)
- Interpret parts of an expression in context (NC.M3.A-SSE.1a, NC.M3.A-SSE.1b)

Connections

- Analyze and compare the key features of functions for tables, graphs, descriptions and symbolic form (NC.M3.F-IF.4, NC.M3.F-IF.7, NC.M3.F-IF.9)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

3 – Construct a viable argument and critique the reasoning of others

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to explain why $f(x + k)$ moves the graph of the function left or right depending on the value of k.

Mastering the Standard

Comprehending the Standard

Students learned the translation and dilation rules in Math 2 with regard to linear, quadratic, square root, and inverse variation functions. In Math 3, we apply these rules to functions in general.

Students should conceptually understand the transformations of functions and refrain from blindly memorizing patterns of functions. Students should be able to explain why $f(x + k)$ moves the graph of the function left or right depending on the value of k.

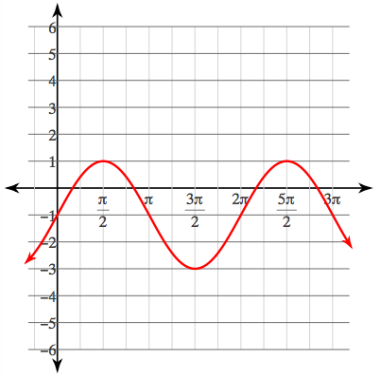
• Note: Phase shifts and transformations of trigonometric functions are NOT required in Math 3. Those will be covered in the fourth math course.

Assessing for Understanding

In demonstrating their understanding, students must be able to relate the algebraic equations, graphs, and tabular representations (ordered pairs) as functions are transformed. Appropriate questions will ask students to identify and explain these transformations.

Example: If $f(x) = sin(x)$and the given graph is $g(x)$, write the equation for $g(x)$.

Example: Explain how the graph of $f(x) = -1/2sin(3x) + 2$is transformed from the parent sine function.



NC.M3.F-TF.1

Extend the domain of trigonometric functions using the unit circle.

Understand radian measure of an angle as:

- The ratio of the length of an arc on a circle subtended by the angle to its radius.
- A dimensionless measure of length defined by the quotient of arc length and radius that is a real number.
- The domain for trigonometric functions.

Concepts and Skills
Pre-requisite
• Recognize that trig ratios are functions of angle measure (NC.M3.F-IF.1)
Connections
• Recognize that trig ratios are functions of angle measure (NC.M3.F-IF.1) • Define radian measure (NC.M3.G-C.5)

The Standards for Mathematical Practices
Connections
Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.
Disciplinary Literacy
As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication. Students should be able to discuss the relationship between degrees and radians. New Vocabulary: arc length

Mastering the Standard

Comprehending the Standard

To build the understanding of radian measure, students should first become familiar with degree measure.

In ancient times, when discussing angle measure, it was realized that the best way to describe angle measure was through a ratio. It was decided based on a different numbering system that they would divide a circle into 360 sectors and each of the sectors would measure 1 degree. The division of the circle into 360 sectors not only divided the angle, but also divided the arc of the circle as well. (Hence the measure of the central angle is the same as the measure of the intercepted arc.)

This means that a measure of $42°$ is $42(\frac{1}{360})$ of a circle or 42 divisions of the 360 divisions.

In modern times, as science and mathematics knowledge increased, the decision to divide a circle into 360 parts is arbitrary and less precise. This lead to the development of radian measures.

In this process, a ratio is still used, however the circle is not divided into parts but is described in the ratio of the circumference to the radius.

Here is a good resource to understand radian measure: Find radian measure by dividing arc length by radius (Learn Zillion)

By discovery (using string, rolling a can, etc.), students can determine that it takes just over 6 radii to create the circumference of a circle, and the teacher can relate that to 2π.

Assessing for Understanding

In mastering this standard, students will need to demonstrate an understanding of radian angle measure and applying the arc length formula (Arc Length = Radius • Radian Measure) to solve for any missing measure, both using basic measures and in the context of word problems. The following examples are from NC.M3.G-C.5 but require the understanding of this standard.

Example: An angle with a measure of 4 radians intercepts an arc with a length of 18 ft. What is the length of the radius of the circle?

Example: The minute hand on the clock at the City Hall clock in Stratford measures 2.2 meters from the tip to the axle.

a) Through what radian angle measure does the minute hand pass between 7:07 a.m. and 7:43 a.m.?

b) What distance does the tip of the minute hand travel during this period?

NC.M3.F-TF.2a

Extend the domain of trigonometric functions using the unit circle.

Build an understanding of trigonometric functions by using tables, graphs and technology to represent the cosine and sine functions.

a. Interpret the sine function as the relationship between the radian measure of an angle formed by the horizontal axis and a terminal ray on the unit circle and its *y* coordinate.

Concepts and Skills

Pre-requisite

- Recognize that trig ratios are functions of angle measure (NC.M3.F-IF.1)
- Understand radian measure (NC.M3.F-TF.1)

Connections

- Analyze and compare the key features of functions for tables, graphs, descriptions and symbolic form (NC.M3.F-IF.4, NC.M3.F-IF.7, NC.M3.F-IF.9)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

2 – Reason abstractly and quantitatively

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should describe the relationship between sine represented on a unit circle and graphical representation of the sine function.

Mastering the Standard

Comprehending the Standard

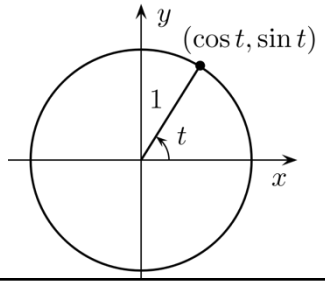
Students will be introduced to the unit circle and angle measures on the coordinate plane in Math 3 as a way to relate the sine and cosine ratios to the coordinates and the plane.

A unit circle is used to develop the concepts of this standard to simplify the picture for students. In Math 3, students are only introduced to the trigonometric functions.

This standard builds upon previous understanding of the trig ratios in right triangles.

$Sin\ \theta$ is the unit rate produced by the ratio of the length of the opposite side to the length of the hypotenuse.

$$sin\ \theta = \frac{length\ of\ opposide\ side}{length\ of\ hypotenuse}$$

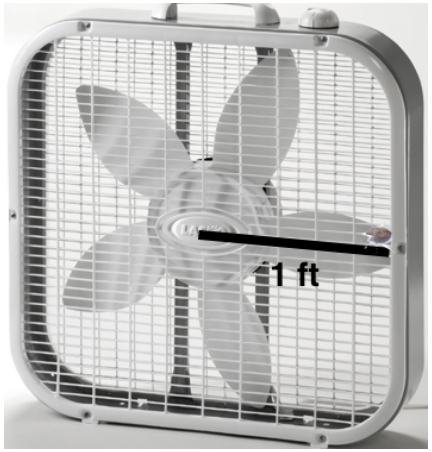


Since we are working within a unit circle, and the hypotenuse is the radius of the unit circle, so the length of the hypotenuse is 1 unit. This means that $sin\ \theta = \frac{length\ of\ opposide\ side}{1}$, so with the unit circle, $sin\ \theta$ is the length of the opposite side.

Assessing for Understanding

Students apply reasoning to their knowledge of the relationship between angles and the sides of right triangles.

Example: A stink bug has crawled into a box fan and sits on the tip of the blade of the fan as seen below. The fan starts to turn slowly due to a breeze in the room.



a) Create a function and a graph that describes its change in height from its original position based on the angle of the blade from its original position.

b) What is the height of the stink bug when the blade has rotated 2 radians? $\frac{11\pi}{6}$ radians?

c) How much has the blade rotated when the stink bug's height is $-\frac{3}{4}$ feet? Can there be more than one answer?

This means that the height of the triangle, which is the y-coordinate of the vertex on the circle, is $sin\ \theta$. The focus of this standard is on the relationship between the changing angle of the sine function and the value of the sine ratio. This should allow students to move from the unit circle to graphing the relationship on a coordinate plane in which the independent variable is the angle measure and the dependent variable is the value of the sine ratio (the y-coordinate from the unit circle). This is a strong connection to NC.M3.F-IF.1. In general, from the unit circle, students should see that as the angle is near zero, the ratio of the length of the opposite side to the length of the hypotenuse is also near zero. As the angle starts to increase and approaches $90°$ or $\frac{\pi}{2}$, the value of the sine ratio approaches 1. This pattern continues around the unit circle and eventually demonstrates the periodicity of the sine function. An in depth teaching of the unit circle, tangent and reciprocal ratios, coterminal angles, specific coordinates and the Pythagorean Identity are NOT appropriate for Math 3, as they will be covered in depth in the fourth math course. Students should understand these relationships in degree and radian angle measure.	

NC.M3.F-TF.2b

Extend the domain of trigonometric functions using the unit circle.

Build an understanding of trigonometric functions by using tables, graphs and technology to represent the cosine and sine functions.

b. Interpret the cosine function as the relationship between the radian measure of an angle formed by the horizontal axis and a terminal ray on the unit circle and its *x* coordinate.

Concepts and Skills

Pre-requisite

- Recognize that trig ratios are functions of angle measure (NC.M3.F-IF.1)
- Understand radian measure (NC.M3.F-TF.1)

Connections

- Analyze and compare the key features of functions for tables, graphs, descriptions and symbolic form (NC.M3.F-IF.4, NC.M3.F-IF.7, NC.M3.F-IF.9)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

2 – Reason abstractly and quantitatively

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should describe the relationship between cosine represented on a unit circle and graphical representation of the cosine function.

Mastering the Standard

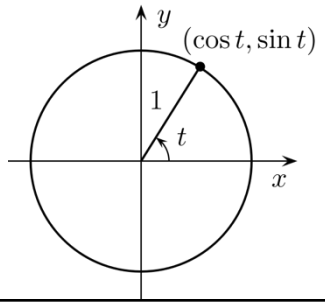
Comprehending the Standard

Students will be introduced to the unit circle and angle measures on the coordinate plane in Math 3 as a way to relate the sine and cosine ratios to the coordinates and the plane.

A unit circle is used to develop the concepts of this standard to simplify the picture for students. In Math 3, students are only introduced to the trigonometric functions.

This standard builds upon previous understanding of the trig relationship in right triangle. $Cos\ \theta$ is the unit rate produced by the ratio of the length of the adjacent side to the length of the hypotenuse.

$$cos\ \theta = \frac{length\ of\ adjacent\ side}{length\ of\ hypotenuse}$$



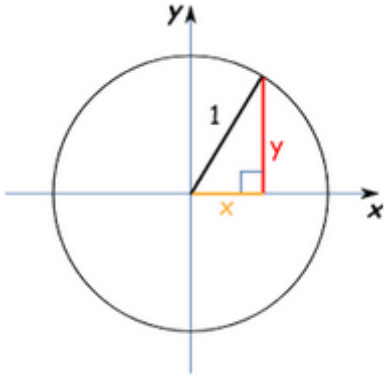
Since we are working within a unit circle, and the hypotenuse is the radius of the unit circle, so the length of the hypotenuse is 1 unit. This means that $cos\ \theta = \frac{length\ of\ adjacent\ side}{1}$, so with the unit circle, $cos\ \theta$ is the length of the adjacent side. This means that the base of the triangle, which is the *x*-coordinate of the vertex on the circle, is $cos\ \theta$.

Assessing for Understanding

Students apply reasoning to their knowledge of the relationship between angles and the sides of right triangles.

Example: Using the unit circle and segments below:

a) Why is the cosine value of the reference angle θ equal to x?
b) For $90^0 < \theta < 270^0$, why is the cosine value negative?
c) Why is the range of the cosine function $-1 \leq y \leq 1$?
d) Will the cosine value ever equal the sine value? Why or why not?



The focus of this standard is on the relationship between the changing angle of the cosine function and the value of the cosine ratio. This should allow students to move from the unit circle to graphing the relationship on a coordinate plane in which the independent variable is the angle measure and the dependent variable is the value of the cosine ratio (the *x*-coordinate from the unit circle). This is a strong connection to NC.M3.F-IF.1. From the unit circle, students should see that as the angle is near zero, the ratio of the length of the opposite side to the length of the hypotenuse is also near 1. As the angle starts to increase and approaches $90°$ or $\frac{\pi}{2}$, the value of the cosine ratio approaches 0. This pattern continues around the unit circle and eventually demonstrates the periodicity of the cosine function. As the angle changes, sine represents the change in the y-coordinate (height of the triangle) on the unit circle, cosine represents the change in the x-coordinate (length of the base of the unit circle). Students should be able to not only see the relationship between the functions represented on a unit circle and the graphical representation on the coordinate plane, but should understand the relationship between the sine and cosine functions. An in depth teaching of the unit circle, tangent and reciprocal ratios, coterminal angles, specific coordinates and the Pythagorean Identity are NOT appropriate for Math 3, as they will be covered in depth in the fourth math course. Students should understand these relationships in degree and radian angle measure.	

NC.M3.F-TF.5

Model periodic phenomena with trigonometric functions.

Use technology to investigate the parameters, a, b, and h of a sine function, $f(x) = a \cdot sin(b \cdot x) + h$, to represent periodic phenomena and interpret key features in terms of a context.

Concepts and Skills

Pre-requisite

- Interpret parts of an expression in context (NC.M3.A-SSE.1a)
- Recognize that trig ratios are functions of angle measure (NC.M3.F-IF.1)
- Understand radian measure (NC.M3.F-TF.1)
- Build an understanding of trig functions (NC.M3.F-TF.2a, NC.M3.F-TF.2b)

Connections

- Analyze and compare the key features of functions for tables, graphs, descriptions and symbolic form (NC.M3.F-IF.4, NC.M3.F-IF.7, NC.M3.F-IF.9)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

3 – Construct viable arguments and critique the reasoning of others

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to discuss how changing the parameters effects the different representations.

New Vocabulary: period, amplitude

Mastering the Standard

Comprehending the Standard

It is important to not overreach with this standard. In Math 3, students are just being introduced to the concepts of the sine function and the effects of the various representations by changing parameters. As the phrase at the beginning of the standards states, students should use technology to investigate these changes.

There are several excellent online resources to investigate the change in parameters of trig functions. For some of these resources, you may need to create an account. Some of these resources are listed below. Some of the resources explore horizontal phase shift, which is not part of this standard.

Phase shifts and complicated trigonometric functions are not part of the standards for Math 3, as they will be covered in depth in the fourth math course. This is an introduction to the concept of a periodic graph through learning the sine function.

Assessing for Understanding

Students should be able to explain how the change in parameters affects the various representations and interpret them in a context.

Example: The following function describes the stock price for Facebook where m stands for the number of months since May 2012. Use technology to graph and create tables as needed.

$$f(m) = -11 \cdot sin\left(\frac{2\pi}{4}m\right) + 38$$

a) Interpret the 38 in the context of the problem.
b) What does -11 mean in context of the problem and what is the significance of 11 being negative?
c) How long does it take for the pattern to start repeating?
d) During which months would you want to buy and sell stock in Facebook?