A IAL PURE MATHS 3 PAST PAPERS

Chapter 6 [MS]

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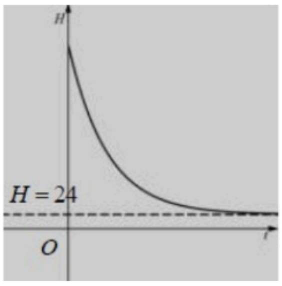
1. Jan 2025 [5]

5 (a)	$304(°\text{C})$	B1
		(1)
(b)	 Shape or Asymptote	M1
	Fully correct Shape and Asymptote	A1
		(2)
(c)	$144 = 280\mathrm{e}^{-0.05t} + 24 \Rightarrow 280\mathrm{e}^{-0.05t} = 120$	M1
	Correct use of lns $\Rightarrow -0.05t = \ln\dfrac{120}{280} \Rightarrow t = 16.95$	dM1 A1
		(3)
(d)	$\left(\dfrac{\mathrm{d}H}{\mathrm{d}t} =\right) -0.05\times 280\mathrm{e}^{-0.05t}$	M1
	$\dfrac{\mathrm{d}H}{\mathrm{d}t} = -0.05\times(H-24) \Rightarrow \dfrac{\mathrm{d}H}{\mathrm{d}t} = 1.2 - 0.05H$	M1, A1cso
		(3)
		(9 marks)

(a)

B1: 304

(b)

M1: Correct shape or correct asymptote. For the shape the curve must be at least in the upper half plane with negative gradient which levels out to 0, but which may be asymptotic to the x-axis. Be tolerant with curves that begin to turn back on themselves for this mark if the intent is clear.

For the asymptote it must be an asymptote for their curve (not just a dashed line with no curve approaching it), and may just be labelled 24 on the y-axis for this mark.

A1: Correct shape and asymptote. For the shape the curve must be in quadrant 1 only, starting on the H axis with negative gradient which levels to 0 at the asymptote which must be above the x-axis. It must not cross the asymptote or clearly turn back on itself, but condone touching the asymptote and slight "pen flicks" at the end.

The asymptote must be an equation, $H = 24$ but condone $y = 24$. Must be an equation, not just 24 labelled as the intercept of the asymptote.

(c)

M1: Substitutes $H = 144$ into the equation and proceeds at least as far as $Ae^{\pm 0.05t} = B$ condoning slips.

dM1 Uses correct ln work to find t.

Method 1: $Ae^{-0.05t} = B \to e^{\pm 0.05t} = k \to \pm 0.05t = \ln k \to t = ...$

Method 2: $Ae^{-0.05t} = B \to \ln A - 0.05t = \ln B \to t = ...$

May be implied by a correct f.t. value following reaching an equation $Ae^{\pm 0.05t} = B$

A1 Awrt 16.95. Condone awrt 16.95 minutes.

(d)

M1: Differentiates to a form $ke^{-0.05t}$ where $k \neq 1$ or 280. May be called $\frac{dy}{dx}$ or unlabelled as long as it is clearly an attempt at the derivative.

M1: Rearranges the equation for H to find $e^{-0.05t}$ or a multiple of $e^{-0.05t}$ which is then substituted into their $ke^{-0.05t}$ (not dependent, so their k may be 280 for this mark). May be implied as long as a derivative has been attempted.

A1: cso $\frac{dH}{dt} = 1.2 - 0.05H$ or with fractions e.g. $\frac{dH}{dt} = \frac{6}{5} - \frac{H}{20}$, which must follow correct work and notation. The $\frac{dH}{dt}$ must have been seen to score this mark.

Alt:

M1: Makes t the subject and differentiates: $H = 280e^{-0.05t} + 24 \Rightarrow t = -20\ln\frac{H-24}{280} \Rightarrow \left(\frac{dt}{dH} = \right)\frac{-20}{H-24}$

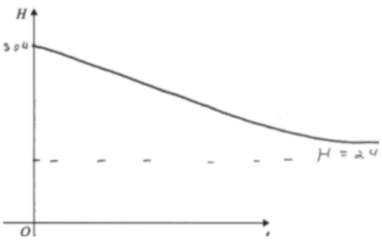
M1: Applies $\frac{dH}{dt} = 1 \div \frac{dt}{dH}$ with their attempt at $\frac{dt}{dH}$

A1: cso per main scheme.

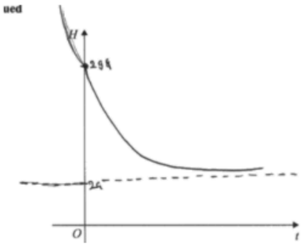
Alt 2: for Second M (first M and A as per scheme, allowing for a and b to be stated as values).

M1: Sets $\frac{dH}{dt}$ equal to $a + bH$, substitutes for H and compares coefficients to find values for a and b.

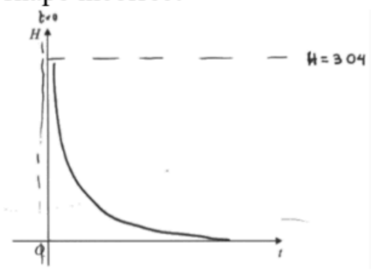
Some examples of scores for graphs:



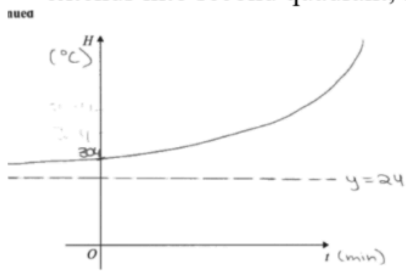
M1A0 Asymptote correct and labelled, shape incorrect



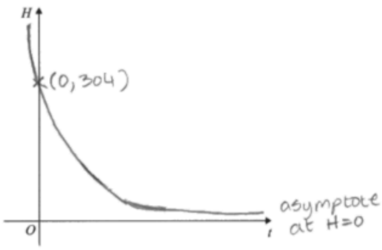
M1A0 Asymptote correct labelled but graph extends into second quadrant, and no equation.



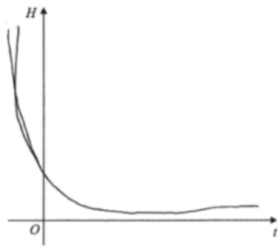
M1A0 Shape acceptable, but asymptote incorrect and does not start on H axis



M1A0 Asymptote correct and labelled (and is an asymptote to their graph), shape incorrect



M1A0 Shape acceptable, asymptote incorrect (and curve extends to Q2)



M0A0 Shape and asymptote both incorrect. Gradient increases above zero.

2. JAN 2025 [10]

10.(a)	$x = 3\cos 2y \Rightarrow \left(\frac{dx}{dy} =\right) - 6\sin 2y$	M1 A1
		(2)
(b)	E.g. $\frac{dx}{dy} = -6\sqrt{1-kx^2}$ or $\frac{dy}{dx} = \frac{1}{\text{their}\frac{dx}{dy}}$	M1
	$\frac{dy}{dx} = -\frac{1}{6\sqrt{1-\cos^2 2y}} = -\frac{1}{6\sqrt{1-\left(\frac{x}{3}\right)^2}} = -\frac{1}{2\sqrt{9-x^2}}$	dM1 A1
		(3)
(c)	Sets $\frac{dy}{dx} = -\frac{1}{2\sqrt{9-x^2}} = -\frac{1}{4}$	
	$x^2 = 5 \Rightarrow a = \sqrt{5}$	M1 A1
	$\sqrt{5} = 3\cos 2y \Rightarrow b = \frac{1}{2}\arccos\left(\frac{\sqrt{5}}{3}\right)$	dM1 A1
Alt (c)	Sets $\frac{dy}{dx} = -\frac{1}{6\sin 2y} = -\frac{1}{4}$	
	$\Rightarrow \sin 2y = \frac{2}{3} \Rightarrow y = \frac{1}{2}\arcsin\left(\frac{2}{3}\right)$	M1A1
	$x = 3\cos 2y = 3\sqrt{1-\sin^2\left(\arcsin\frac{2}{3}\right)} = \sqrt{5}$	dM1 A1
		(4)
		(9 marks)

(a)

M1: $\left(\frac{dx}{dy} =\right) k\sin 2y$ Alt: $x = 3\left(2\cos^2 y - 1\right) \Rightarrow \left(\frac{dx}{dy} =\right) k\cos y \sin y$ (oe with sine identity etc)

A1: $\left(\frac{dx}{dy} =\right) -6\sin 2y$ Alt: $\left(\frac{dx}{dy} =\right) -12\cos y \sin y$

(b)

M1: Either

- Uses $\sin^2 2y+\cos^2 2y=1$ with $x=3\cos 2y$ to find $\sin 2y$ in terms of x (condone slips with the 3 when rearranging)
- Or uses $\frac{\mathrm{d}y}{\mathrm{d}x}=1\div\frac{\mathrm{d}x}{\mathrm{d}y}$

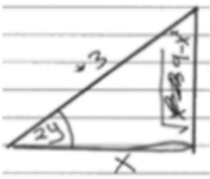
dM1: Uses both of the above to find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x.

A1: $\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{1}{2\sqrt{9-x^2}}$ including the $\frac{\mathrm{d}y}{\mathrm{d}x}$ seen at some stage associated with the answer, may have the $-1/2$ in the numerator seen and isw attempts to take the 2 into the numerator.

Alt for first bullet point: Longer ways may be possible, but look for correct formulae used, allowing algebraic slips simplifying, and must reach an expression for sin 2y in terms of x. E.g.

$$\sin 2y=2\sin y\cos y=2\sqrt{\frac{1}{2}\left(1-\frac{x}{3}\right)}\sqrt{\frac{1}{2}\left(1+\frac{x}{3}\right)}$$

Another alternative for finding sin 2y in terms of x is use of appropriate triangles, e.g.



To score the M they must proceed as far as identifying sin 2y in terms of x, here $\sin 2y=\frac{\sqrt{9-x^2}}{3}$. The trig ratios and application of Pythagoras theorem must be correct.

Notes: The question says "hence" and so answer via arcos formula are not being credited any marks.
You may allow recovery of marks for part (b) if the work is done in part (c) instead.

(c)

M1: Sets $\frac{dy}{dx} = -\frac{1}{4}$ and proceeds to $x^2 = p, p > 0$. Condone slips in rearranging.

A1: $a = \sqrt{5}$ from a correct answer to (b) Accept $x = \ldots$

dM1: Substitutes the value of x in $x = 3\cos 2y$ and uses arccos to find y (allowing decimal values – you may need to check).

A1: $b = \frac{1}{2}\arccos\left(\frac{\sqrt{5}}{3}\right)$ following a correct answer to (b). Accept $y = \ldots$ Accept alternative notation such as $\frac{1}{2}\cos^{-1}\left(\frac{\sqrt{5}}{3}\right)$ or even acos.

Alt method

M1: Sets $\frac{dy}{dx} = -\frac{1}{4}$ and proceeds to $\sin 2y = p, |p| < 1$. Condone slips in rearranging.

A1: $b = \frac{1}{2}\arcsin\frac{2}{3}$ Accept $y = \ldots$ Accept alternative notation such as $\frac{1}{2}\sin^{-1}\left(\frac{2}{3}\right)$ or even asin

dM1: Substitutes the value of y in $x = 3\cos 2y$ and uses Pythagorean identity to find exact value of x.

A1: $a = \sqrt{5}$ Accept $x = \ldots$

3. OCT 2024 [2]

2 (a)	$x = 2y^2 + 5y - 6$	
	$\frac{dx}{dy} = 4y + 5 \Rightarrow \frac{dy}{dx} = \frac{1}{4y+5}$	M1, A1
		(2)
(b)	Sets their $4y + 5 = 0 \Rightarrow y = -\frac{5}{4}$	M1
	Substitutes their $y = -\frac{5}{4}$ into $x = 2y^2 + 5y - 6$ to find x	dM1
	$\left(-\frac{73}{8}, -\frac{5}{4}\right)$	A1
		(3)
		Total 5

Mark as one complete question

Main method via differentiation in (a) and (b)

(a)

M1: Differentiates $2y^2+5y-6$ to a linear term in y AND attempts to take the reciprocal.

Condone for this mark attempts such as $\frac{dx}{dy}=4y+5 \Rightarrow \frac{dy}{dx}=\frac{1}{4y}+\frac{1}{5}$

We may see other methods including implicit differentiation from WMA14.

Look for $x=2y^2+5y-6 \Rightarrow 1=ay\frac{dy}{dx}+b\frac{dy}{dx}$ followed by an attempt to make $\frac{dy}{dx}$ the subject

A1: $\frac{dy}{dx}=\frac{1}{4y+5}$ ISW after a correct answer.

(b) **Marks in part (b) can only be awarded after sight of** $\frac{dx}{dy}=\frac{ay+b}{k}$ **or** $\frac{dy}{dx}=\frac{k}{ay+b}$

So full marks can only be awarded after sight of $\frac{dx}{dy}=4y+5$ **or** $\frac{dy}{dx}=\frac{1}{4y+5}$

M1: Sets $'ay+b'=0 \Rightarrow y=....$ following sight of $\frac{dx}{dy}=\frac{ay+b}{k}$ **or** $\frac{dy}{dx}=\frac{k}{ay+b}$

Ignore/condone incorrect statements such as $\frac{dy}{dx}=0$ here

This can be implied. E.g $\frac{dy}{dx}=\frac{1}{4y+5}$ followed by $y=-\frac{5}{4}$

dM1: Substitutes their solution of their $4y+5=0$ into $x=2y^2+5y-6$ to find x.

This is dependent upon the previous M

A1: $\left(-\frac{73}{8},-\frac{5}{4}\right)$ following $\frac{dx}{dy}=4y+5$ **or** $\frac{dy}{dx}=\frac{1}{4y+5}$

The answer may be given separately, for example, as $x=-9.125,\quad y=-1.25$

ISW after a correct answer.

Further guidance to highlighted sentence.

If (a) is incorrect or incomplete follow the following guidance.

Incorrect (a): If they incorrectly state $\frac{dy}{dx} = 4y + 5$ o.e. and go on to use $4y + 5 = 0 \Rightarrow y = ...$ and write down a correct set of coordinates for P they will score NO MARKS in (b)

Incomplete (a): If they state $\frac{dx}{dy} = 4y + 5$ in (a) and go on to use $4y + 5 = 0 \Rightarrow y = ...$ in (b) they will score no marks in (a) but could potentially score full marks in (b)

You can award all 3 marks for $\left(-\frac{73}{8}, -\frac{5}{4}\right)$ following either $\frac{dx}{dy} = 4y + 5$ or $\frac{dy}{dx} = \frac{1}{4y + 5}$

....

Alternative approach: Part (a) may well be blank or incorrect. Part(b) done via completion of square

It is possible to do part (b) by not doing any differentiating.

$$x = 2y^2 + 5y - 6 \Rightarrow x = 2\left(y + \frac{5}{4}\right)^2 - \frac{73}{8}$$

M1: An allowable attempt to complete the square followed by a valid attempt at either x or y from their attempt
A1: For a correct x or y
A1: For a correct x and y
....

4. JUNE 2024 [9]

9 (a)	$(k=)\ 4\sin^2\left(\frac{\pi}{3}\right)-1=2$ *	B1*
		(1)
(b)	(i) $x=4\sin^2 y-1 \Rightarrow \quad \frac{dx}{dy}=8\sin y\cos y$ o.e.	M1 A1
	(ii) Attempts either $\sin^2 y=\frac{\pm x\pm 1}{4}$ or $\cos^2 y=\pm 1\pm\frac{x+1}{4}$ (both for dM1)	M1 dM1
	$\frac{dy}{dx}=\frac{1}{8\sin y\cos y}=\frac{1}{8\times\sqrt{\frac{x+1}{4}}\times\sqrt{\pm 1\pm\frac{x+1}{4}}}=\frac{1}{2\sqrt{x+1}\sqrt{3-x}}$ *	ddM1 A1*
		(6)
(c)	At $x=2$, gradient of curve $=\frac{1}{2\sqrt{3}} \Rightarrow$ gradient of normal is $-2\sqrt{3}$	M1
	Point N (base length of triangle) is solution of	
	$\cancel{y}-\frac{\pi}{3}=-2\sqrt{3}(x-2) \Rightarrow x=2+\frac{\pi}{6\sqrt{3}}$ $(=2.3...)$	dM1
	Area $=\frac{1}{2}\times\left(2+\frac{\pi}{6\sqrt{3}}\right)\times\frac{\pi}{3}=\frac{\pi}{3}+\frac{\pi^2}{36\sqrt{3}}$	A1
		(3)
		(10 marks)

(a)

B1*: Verifies that $k=2$ with no errors seen. (Condone $x=2$) Do not allow this mark if rounded numbers are seen in their solution for e.g. $\frac{\pi}{3}$. They may work in degrees and substitute in $60°$ which is acceptable. Look for as a minimum e.g. $(k=)\ 4\sin^2\frac{\pi}{3}-1=2$ Do not allow $k=4\sin\left(\frac{\pi}{3}\right)^2-1=2$

If they rearrange the equation to e.g. $y=\sin^{-1}\left(\sqrt{\frac{x+1}{4}}\right)$ and substitute in $x=2$ proceeding to e.g. $\frac{\pi}{3}$ then they must conclude that $k=2$ (or a preamble followed by a minimal conclusion e.g. QED, tick)

(b) (i)

M1: Differentiates to a form $p\sin y\cos y$ or equivalent. Indices do not need to be processed for this mark. e.g. $...\sin^{2-1}y\cos y$

May use the identity $\cos 2y = \pm1\pm2\sin^2 y \Rightarrow\ x = 4\sin^2 y - 1 = 1 - 2\cos 2y \Rightarrow \frac{dx}{dy} = q\sin 2y$

A1: Correct lhs and rhs. For $\frac{dx}{dy} = 8\sin y\cos y$ or $4\sin 2y$. Note that $\frac{dy}{dx} = ...$ is A0. isw for this mark after a correct answer is seen. You may see $\frac{dx}{dy}$ (or allow x') on an earlier line which is fine for this mark. May be seen at the start of (b)(ii). Indices must be processed for this mark.

APPROVED) :PEN this is M1A1dM1A1* we are marking this as M1dM1ddM1A1*

Note this is a given answer so full working must be seen.

M1: Attempts to find either $\sin y$ (or $\sin^2 y$) **or** $\cos y$ (or $\cos^2 y$) in terms of x (or possibly multiples of these) Typically look for $\sin^2 y = \frac{\pm x\pm1}{4}$ or $\sin y = \sqrt{\frac{\pm x\pm1}{4}}$ Also allow e.g. $4\sin^2 y = \pm x\pm1$

May attempt to use $\pm\sin^2 y\pm\cos^2 y = \pm1 \Rightarrow \cos^2 y = \pm1\pm\sin^2 y \Rightarrow \cos^2 y = \pm1\pm\frac{x\pm1}{4}$ or

$\cos y = \sqrt{\pm1\pm\frac{x\pm1}{4}}$ Ignore $\pm\sqrt{...}$ for this mark. (may be seen within their expression for $\frac{dy}{dx}$ or $\frac{dx}{dy}$)

dM1: Attempts to find both $\sin y$ (or $\sin^2 y$) **and** $\cos y$ (or $\cos^2 y$) or possibly multiples of these in terms of x. (See previous method mark for guidance) It is dependent on the previous method mark.

ddM1: Full method to get $\frac{dy}{dx}$ of the form $\frac{1}{p\sin y\cos y}$ o.e. in terms of x e.g. $\frac{dy}{dx} = \frac{1}{"8"\sqrt{\frac{\pm x\pm1}{4}}\sqrt{\pm1\pm\frac{\pm x\pm1}{4}}}$

Condone square root notation which does not fully go over any fraction provided it is implied by further work which is not the given answer. Condone missing arguments for sin and cos provided the intention is clear or implied. Condone missing brackets to be recovered **before** achieving the given answer. If they have rearranged or simplified incorrectly for $\cos y$ before substituting, ddM1 can still be scored.

A1*: Achieves the given answer **provided all previous method marks have been scored in (b) and no errors.** Look for

- a correct expression for $\cos^2 y$ or $\cos y$ in x which has not already been simplified to $...\sqrt{3-x}$
- a correct expression for $\sin^2 y$ or $\sin y$ in x which has not already been simplified to $...\sqrt{x+1}$

$\frac{dy}{dx}$ (or y') may appear on an earlier line which is acceptable. Note that if they work backwards by substituting $x = 4\sin^2 y$ into the given answer there must be a conclusion that $\frac{dy}{dx} = \frac{1}{2\sqrt{x+1}\sqrt{3-x}}$ isw

(c)

M1: Full method for the gradient of the normal using the x or y value. Condone arithmetical slips but the gradient must be the negative reciprocal of the gradient of the tangent to the curve.

Note that they may use an earlier incorrect line in part (b) for $\frac{dy}{dx}$ (or even $\frac{dx}{dy}$).

Allow use of an earlier line provided it is an attempt at a correct method substituting in x or y and proceeding from e.g. $\frac{dx}{dy} = \alpha \Rightarrow$ gradient of the normal $= -\alpha$

May be implied by awrt -3.5 If no method is shown where the gradient of the tangent or normal has come from, then it must be correct.

dM1: Attempts to find the x coordinate of N. It is dependent on the previous method mark. Uses their gradient of the normal with $\left(2, \frac{\pi}{3}\right)$, sets $y=0$ in $y - y_1 = m(x - x_1)$ and proceeds to find x. May also use $y = mx + c$ to find the equation for the straight line and then sets $y = 0$ and rearranges to find x. Do not be concerned by the mechanics of the rearrangement for this mark.

A1: $\frac{\pi}{3} + \frac{\pi^2}{36\sqrt{3}}$ or exact equivalent such as $\frac{\pi}{3} + \frac{\pi^2\sqrt{3}}{108}$ which is in the form $a\pi + b\pi^2$

5.JAN 2023 [7]

5(a) Way One	$\cot^2 x - \tan^2 x \equiv \frac{\cos^2 x}{\sin^2 x} - \frac{\sin^2 x}{\cos^2 x} \equiv \frac{\cos^4 x - \sin^4 x}{\sin^2 x \cos^2 x}$	M1
	$\equiv \frac{(\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x)}{\sin^2 x \cos^2 x} \equiv \frac{\cos 2x}{\ldots}$ or $\frac{\ldots}{\left(\frac{1}{2}\sin 2x\right)^2}$	dM1
	$\equiv \frac{\cos 2x}{\left(\frac{1}{2}\sin 2x\right)^2}$	A1
	$\equiv 4\frac{\cos 2x}{\sin 2x \sin 2x} \equiv 4\cot 2x \operatorname{cosec} 2x$ *	A1*
		(4)
(b)	$4\cot 2\theta \operatorname{cosec} 2\theta = 2\tan^2\theta \Rightarrow \cot^2\theta - \tan^2\theta = 2\tan^2\theta \Rightarrow \cot^2\theta - 3\tan^2\theta = 0$	M1
	$\cot^2\theta - 3\tan^2\theta = 0 \Rightarrow \frac{1}{\tan^2\theta} - 3\tan^2\theta = 0 \Rightarrow \tan^4\theta = \frac{1}{3}$	A1
	$\tan^4\theta = "\frac{1}{3}" \Rightarrow \tan\theta = \pm\sqrt[4]{"\frac{1}{3}"} = "\pm 0.7598.." \Rightarrow \theta = \ldots$	M1
	$\theta =$ awrt $0.65,\ -0.65$	A1A1
		(5)
		Total 9

(a) Way One LHS to RHS
M1: Changes the LHS to $\sin x$ and $\cos x$ and attempts to make a single fraction using a correct common denominator. Condone errors/slips on the numerator
dM1: Attempts/ applies

- Either $\cos^4 x - \sin^4 x = \left(\cos^2 x - \sin^2 x\right)\left(\cos^2 x + \sin^2 x\right) = \cos^2 x - \sin^2 x = \cos 2x$ on the numerator
- Or $\sin 2x = 2\sin x \cos x$ to the denominator condoning bracketing slip

A1: Applies both of the above correctly to achieve a correct expression in terms of $\cos 2x$ and $\sin 2x$
A1*: Reaches the right hand side with sufficient working shown. Expect to see $\sin^2 2x$ split into $\sin 2x \sin 2x$
Penalise consistent (not once or twice) use of poor notation on this mark only.
Examples $\cos^2 x \leftrightarrow \cos x^2$, $\sin^2 \leftrightarrow \sin^2 x$, constantly switching between $x \leftrightarrow \theta$

(b) Allow use of $x \leftrightarrow \theta$
M1: Uses part (a) and attempt to collects terms. (See Appendix III for ways not using part (a))
See below for equations in $\sin\theta$ or $\cos\theta$ where this mark is awarded for an equation in just $\sin\theta$ or $\cos\theta$
A1: Reaches a correct equation in a single term, usually $\tan\theta$. Look for $\tan^4\theta = \frac{1}{3}$ o.e. such as $3\tan^4\theta = 1$
Other correct intermediate forms are $2\sin^4\theta + 2\sin^2\theta - 1 = 0$ and $2\cos^4\theta - 6\cos^2\theta + 3 = 0$
M1: Takes the 4th root of their $\frac{1}{3}$ (o.e) and uses $\tan^{-1}$ (you may need to check) to obtain at least one value for θ
For the other intermediate forms look for working such as $\sin^2\theta = \frac{\sqrt{3}-1}{2} \Rightarrow \sin\theta = \sqrt{\frac{\sqrt{3}-1}{2}} \Rightarrow \theta = \ldots$

A1: Either awrt 0.65 or awrt –0.65. Allow either answer in degrees, so awrt $\pm 37.2°$
A1: Both answers in radians, awrt ± 0.65, and no extras in range
There are many different ways to do part (a). Generally, this is how they will be marked.
Most cases can be aligned to one of the three cases.

RHS to LHS

(a) Way 2	$4\cot 2x \operatorname{cosec} 2x \equiv 4\frac{\cos 2x}{\sin 2x} \times \frac{1}{\sin 2x} \equiv \frac{4\left(\cos^2 x - \sin^2 x\right)}{\ldots}$ or $\frac{\ldots}{4\sin^2 x \cos^2 x}$	M1
	$\frac{\cos^2 x - \sin^2 x}{\sin^2 x \cos^2 x} \equiv \frac{1}{\sin^2 x} - \frac{1}{\cos^2 x} \equiv \operatorname{cosec}^2 x - \sec^2 x$	dM1A1
	$\equiv 1 + \cot^2 x - 1 - \tan^2 x \equiv \cot^2 x - \tan^2 x$ *	A1*

M1: Changes to $\sin 2x$ and $\cos 2x$ or $\tan 2x$ and $\sin 2x$ and attempts single angles in $\sin x$ and $\cos x$
dM1: Changes to single angles throughout and splits into 2 separate fractions (which don't need to be simplified)
A1: Correct expression in terms of the single angles cosec x and sec x
A1*: Reaches the left hand side with sufficient working shown

Working on both sides: One possible way

(a) Way 3	$\cot^2 x - \tan^2 x \equiv 4\cot 2x\,\text{cosec}2x$ $\dfrac{\cos^2 x}{\sin^2 x} - \dfrac{\sin^2 x}{\cos^2 x} \equiv 4 \times \dfrac{\cos 2x}{\sin 2x} \times \dfrac{1}{\sin 2x}$ $\cos^4 x - \sin^4 x \equiv 4\sin^2 x\cos^2 x\dfrac{\cos 2x}{\sin 2x} \times \dfrac{1}{\sin 2x}$	M1
	$\cos^4 x - \sin^4 x \equiv 4 \times \sin^2 x\cos^2 x\dfrac{\left(\cos^2 x - \sin^2 x\right)}{\left(2\sin x\cos x\right)^2}$	dM1A1
	$\left(\cos^4 x - \sin^4 x\right) \equiv \left(\cos^2 x - \sin^2 x\right)$ $\left(\cos^2 x - \sin^2 x\right)\underbrace{\cancel{\left(\cos^2 x + \sin^2 x\right)}}_{1} \equiv \left(\cos^2 x - \sin^2 x\right)$ Hence true	A1*

M1: Changes to sin x, cos x, sin $2x$ and cos2 x and attempts to cross multiply
dM1: Applies
- either $\cos^2 x - \sin^2 x = \cos 2x$ to the numerator
- or $\sin 2x = 2\sin x \cos x$ to the denominator

A1: Correct identity in terms of cos x and sin x
A1*: Reaches a point where both sides are equal and makes a minimal comment
Example of how you mark a "different" approach.

6. JAN 2023 [9]

9(a)	$y=\sqrt{3+4e^{x^2}}=\left(3+4e^{x^2}\right)^{\frac{1}{2}}\Rightarrow\frac{dy}{dx}=\frac{1}{2}\left(3+4e^{x^2}\right)^{-\frac{1}{2}}\times 8xe^{x^2}$	M1
	$=4xe^{x^2}\left(3+4e^{x^2}\right)^{-\frac{1}{2}}$	A1
		(2)
(b)	$\frac{\left(3+4e^{x^2}\right)^{\frac{1}{2}}}{x}=4xe^{x^2}\left(3+4e^{x^2}\right)^{-\frac{1}{2}}$	M1
	$\frac{\left(3+4e^{x^2}\right)}{x}=4xe^{x^2}$	dM1
	$4x^2e^{x^2}-4e^{x^2}-3=0$ *	A1*
		(3)
(c)	$f(x)=4x^2e^{x^2}-4e^{x^2}-3\Rightarrow f(1)=-3$ AND $f(2)=652\cdot\ldots$	M1
	Change of sign and f(x) is continuous hence root in (1, 2)	A1
		(2)
(d)	$4x^2e^{x^2}-4e^{x^2}-3=0\Rightarrow x^2=\frac{4e^{x^2}+3}{4e^{x^2}}=\frac{4+3e^{-x^2}}{4}\Rightarrow x=\frac{1}{2}\sqrt{4+3e^{-x^2}}$ *	B1*
		(1)
(e)(i)	$x_1=1\Rightarrow x_2=\frac{1}{2}\sqrt{4+3e^{-1}}$	M1
	$x_3=1.0997$	A1
(ii)	$\alpha=1.1051$	A1
		(3)
		Total 11

(a)

M1: Differentiates using the chain rule to obtain $kxe^{x^2}\left(3+4e^{x^2}\right)^{-\frac{1}{2}}$ OR $ke^{x^2}\left(3+4e^{x^2}\right)^{-\frac{1}{2}}$

If they are using implicit differentiation accept $\frac{kxe^{x^2}}{y}$ or $\frac{ke^{x^2}}{y}$

A1: Correct derivative in simplest form. Also award for $\frac{dy}{dx}=\frac{4xe^{x^2}}{y}$ o.e. simplified answer

(b)

M1: Sets their $\frac{dy}{dx}=\frac{\left(3+4e^{x^2}\right)^{\frac{1}{2}}}{x}$ or equivalent such as use of $y=mx$ with $m=$ their $\frac{dy}{dx}$ and $y=\left(3+4e^{x^2}\right)^{\frac{1}{2}}$.
Allow with another variable $x\to\alpha$
dM1: Multiplies up to eliminate the square root. Allow a consistent use of another variable $x\to\alpha$
Dependent upon the previous M1 and a suitable $\frac{dy}{dx}$, that is one of the form that scored an M1 in part (a)

A1*: Correct proof

An alternative method may be seen using the equation of the line with gradient $\frac{dy}{dx}$ and point $\left(\alpha, \sqrt{3+4e^{\alpha^2}}\right)$

M1: For $y-\sqrt{3+4e^{\alpha^2}} = "\left.\frac{dy}{dx}\right|_{x=\alpha}"(x-\alpha)$ and sets $(x, y)=(0,0)$

dM1: Proceeds as the main method with the same condition on the gradient

A1*: Correct proof. Condone with variable α

(c)

M1: Attempts both $f(1)$ and $f(2)$ (or tighter) and obtains at least one value correct to 1sf rounded or truncated.

Condone $f(2) = 12e^4 - 3$ If f or y is not stated assume that $f(x) = 4x^2e^{x^2} - 4e^{x^2} - 3$

A1: Requires

- both values either correct or rounded/ truncated with accuracy to at least 1 sf
- a reference to a sign change e.g. $f(1) = -3 < 0$, $f(2) = 652 > 0$ or $f(1) \times f(2) < 0$
- a mention of continuity
- a minimal conclusion, e.g. hence root

(d)

B1*: Correct proof.

Look for the following evidence

- the line $4x^2e^{x^2} - 4e^{x^2} - 3 = 0$ but may be implied by $4x^2e^{x^2} = 4e^{x^2} + 3$ o.e.
- the line $x^2 = \frac{4e^{x^2}+3}{4e^{x^2}}$ o.e $x^2 = 1 + \frac{3}{4e^{x^2}}$ or $4x^2 = \frac{4e^{x^2}+3}{e^{x^2}}$ o.e $4x^2 = 4 + \frac{3}{e^{x^2}}$
- correct square root work leading to the given answer

(e)

M1: Attempts to use the iteration formula.

This may be implied by awrt 1.13 for x_2 or awrt 1.10 for x_3 or sight of a correctly embedded value.

It cannot be awarded for just the value of α

A1: awrt 1.0997

A1: 1.1051 cao following the award M1

7. JUNE 2023 [10]

10 (a)	$x=\dfrac{2y^2+6}{3y-3}\Rightarrow\left(\dfrac{\mathrm{d}x}{\mathrm{d}y}=\right)\dfrac{4y(3y-3)-3(2y^2+6)}{(3y-3)^2}$	M1 A1
	$\dfrac{\mathrm{d}x}{\mathrm{d}y}=\dfrac{6y^2-12y-18}{9(y-1)^2}=\dfrac{2y^2-4y-6}{3(y-1)^2}$ o.e	dM1, A1
		(4)
(b)	P and Q are where $\dfrac{\mathrm{d}x}{\mathrm{d}y}=0$ or where $2y^2-4y-6=0$	B1
	Solves $2y^2-4y-6=0\Rightarrow 2(y-3)(y+1)=0\Rightarrow y=3,-1$	M1
	Subs $y=-1$ and 3 in $x=\dfrac{2y^2+6}{3y-3}\Rightarrow x=..$	dM1
	Achieves $x=-\dfrac{4}{3}$ and $x=4$	A1cso
		(4) **8 marks**

Notes
(a)
M1 Attempts the quotient rule. Condone slips on the coefficients - look for $\frac{Ay(3y-3)-B(2y^2+6)}{(3y-3)^2}$ $A,B>0$. Allow a product rule attempt: $x=(2y^2+6)(3y-3)^{-1} \Rightarrow \left(\frac{dx}{dy}=\right) Ay(3y-3)^{-1}+(2y^2+6)\times -B(3y-3)^{-2}$
A1 Correct differentiation which may be unsimplified. Allow if the $\frac{dx}{dy}$ is missing or called $\frac{dy}{dx}$ for this mark. By product rule $4y(3y-3)^{-1}+(2y^2+6)\times -3(3y-3)^{-2}$ Condone missing brackets if recovered.
dM1 Requires an attempt to get a single fraction with some attempt to simplify. For the quotient rule look for a simplification of the numerator with like terms collected giving a 3TQ. Attempts via the product rule will require a correct method to put as a single fraction.
A1 $\left(\frac{dx}{dy}=\right)\frac{2y^2-4y-6}{3y^2-6y+3}$ or exact simplified equivalent such as $\frac{2(y-3)(y+1)}{3(y-1)^2}$ isw after a correct simplified answer. Common factor 3 must have been cancelled. Must be seen in part (a). A0 if called $\frac{dy}{dx}$ but allow A1 if LHS is not stated.
Attempts at $\frac{dy}{dx}$ can score the first 3 marks if correct. Allow use of x in place of y for the Ms.
(b)
B1 Indicates P and Q are where $\frac{dx}{dy}=0$ or where their $2y^2-4y-6=0$ (which may be the denominator of $\frac{dy}{dx}$ if they found this instead).
M1 Solves their 3TQ from an attempt at $\frac{dx}{dy}=0$ (or denominator of their $\frac{dy}{dx}$=0), usual rules.

dM1 Substitutes both their solutions to $2y^2-4y-6=0$ into $x=\dfrac{2y^2+6}{3y-3}$. Condone slips if the attempt is clear. At least one should be correct if no method is shown.

A1cso Achieves $x=-\dfrac{4}{3}$ and $x=4$ only. Must be equations not just values but isw after correct equations seen as long as no contrary work is shown (such as giving horizontal lines). Accept equivalents. Must have come from a correct derivative - though allow from an isw form if a numerical factor was lost in the numerator. Must be exact.

Answers from no working score 0/4 as the question instructs use of part (a), so must see the attempt at setting $\dfrac{dx}{dy}=0$

Alt (a)	$x=\dfrac{2y^2+6}{3y-3}\Rightarrow 3xy-3x=2y^2+6\Rightarrow 3x+3y\dfrac{dx}{dy}-3\dfrac{dx}{dy}=4y$	M1 A1
	$\dfrac{dx}{dy}=\dfrac{4y-3x}{3(y-1)}$	dM1, A1
		(4)
(b) First 2 marks.	States that P and Q are where $\dfrac{dx}{dy}=0$ or where $4y-3x=0$	B1
	$\Rightarrow \dfrac{4}{3}y=\dfrac{2y^2+6}{3y-3}\Rightarrow 4y^2-4y=2y^2+6\Rightarrow$ as main scheme	M1
Alt II (a)	$x=\dfrac{2y^2+6}{3y-3}=\dfrac{2y}{3}+\dfrac{2}{3}+\dfrac{8}{3(y-1)}\Rightarrow\dfrac{dx}{dy}=\dfrac{2}{3}-\dfrac{8}{3(y-1)^2}$	M1 A1
	$\dfrac{dx}{dy}=\dfrac{2(y-1)^2-8}{3(y-1)^2}=\dfrac{2y^2-4y-6}{3(y-1)^2}$ oe	dM1, A1
		(4)

Notes

(a)

M1 Attempts long division or other method to achieve $Ay+B+\dfrac{C}{3y-3}$ oe and differentiates.

A1 Correct differentiation.

dM1 Attempts to get a single fraction and simplifies numerator to 3TQ or uses difference of squares to factorise.

A1 Correct answer.

8. OCT 2023 [5]

5(a)	$\dfrac{dy}{dx}=\dfrac{(x^2+k)\times\dfrac{2x}{x^2+k}-2x\ln(x^2+k)}{(x^2+k)^2}$	M1
	$\dfrac{dy}{dx}=\dfrac{2x-2x\ln(x^2+k)}{(x^2+k)^2}=\dfrac{2x(1-\ln(x^2+k))}{(x^2+k)^2}$	M1A1
		(3)
(b)	$x=0$	B1
	$"1"-\ln(x^2+k)=0\Rightarrow x^2=e^{"1"}\pm k$	M1
	$x=\pm\sqrt{e-k}$	A1ft
		(3)
(c)	Upper limit is e or $k<e$	B1ft
		(1)
		(7 marks)

Notes
(a) M1: Attempts to apply the quotient rule. Score for an expression of the form $\dfrac{dy}{dx}=\dfrac{(x^2+k)\times\dfrac{\dots}{x^2+k}-Qx\ln(x^2+k)}{(x^2+k)^2}$ oe (may be simplified) where … is P or Px and $P,Q>0$ May also attempt the product rule, in which case look for $-Px(x^2+k)^{-2}\ln(x^2+k)+(x^2+k)^{-1}\times\dfrac{\dots}{x^2+k}$ where … is Q or Qx and $P,Q>0$ M1: Cancels x^2+k and correctly takes a factor of (2)x from the numerator to achieve "$B-\ln(x^2+k)$" (may be done in one step). They must have a factor x in both terms. Note this may be scored following M0 e.g. if a numerator $uv'-vu'$ is used, or if the denominator misses the square. A1: $\left(\dfrac{dy}{dx}=\right)\dfrac{2x(1-\ln(x^2+k))}{(x^2+k)^2}$ from fully correct work. Both M's must have been scored. **(b)** B1: $x=0$ M1: Sets $"1"-\ln(x^2+k)=0$ and proceeds to $x^2=\dots$ with correct undoing of ln. A1ft: $x=\pm\sqrt{e-k}$ or follow through $x=\pm\sqrt{e^B-k}$ for a numerical B in their $B-\ln(x^2+k)$ factor **(c)** B1ft: Upper limit is e or $k<e$ stated. Follow on their (b) of form $x=(\pm)\sqrt{e^B\pm k}$ B0 if $k=e$ or $k\leqslant e$ is given as the answer without statement about upper limit being e.

9. OCT 2023 [10]

10(a)	$\frac{1}{4}=\sin^2 4y \Rightarrow y=\frac{\pi}{24}$	M1A1
		(2)
(b)	$\frac{dx}{dy}=8\sin 4y\cos 4y$	M1A1
		(2)
(c)	$\frac{dx}{dy}=8\sin 4y\cos 4y \rightarrow \frac{dy}{dx}=\frac{1}{8\sin 4y\cos 4y}$	M1
	$\frac{dy}{dx}=\frac{1}{8\sqrt{x(1-x)}}$	M1
	$\frac{dy}{dx}=\frac{1}{\sqrt{16-64\left(x-\frac{1}{2}\right)^2}}$	A1
		(3)
(d)(i)	$x=\frac{1}{2}$	B1ft
(ii)	$\frac{dy}{dx}=\frac{1}{4}$	B1ft
		(2)
		(9 marks)

Notes
(a)
M1: Attempts to substitute in $x=\frac{1}{4}$ and rearranges to find an exact value for *y*. Look for $y = M\arcsin\left(\pm\frac{1}{2}\right) \to k\pi$ or $k(30°)$ allowing errors in dividing by the 4, or equivalents criteria via a double angle identity, correct up to sign error.
A1: $\frac{\pi}{24}$ Ignore extra solutions outside the domain.
(b)
M1: Attempts to differentiate achieving a form $\left(\frac{dx}{dy}=\right)A\sin 4y\cos 4y$ or $A\sin 8y$ oe Accept alternative forms e.g. via implicit differentiation $1 = A\sin 4y\cos 4y\frac{dy}{dx}$. The $\frac{dx}{dy}$ may be missing, or labelled $\frac{dy}{dx}$ for this mark.
A1: $\frac{dx}{dy}=8\sin 4y\cos 4y$ oe eg $\frac{dx}{dy}=4\sin 8y$ Coefficients must be simplified. Must include the $\frac{dx}{dy}$
(c)
M1: Uses $\frac{dy}{dx}=1\div\frac{dx}{dy}$ (allow for the reciprocal of their answer to (b) if left hand side is omitted). Variables must be consistent.

M1: Attempts to use $\sin 4y = \pm\sqrt{x}$ and $\cos 4y = \pm\sqrt{1-x}$ to write $\frac{dy}{dx}$ or $\frac{dx}{dy}$ in terms of *x* only with no trig terms.
A1: $\frac{dy}{dx}=\frac{1}{\sqrt{16-64\left(x-\frac{1}{2}\right)^2}}$ and isw after correct answer. Allow with terms under the square root reversed.
(d) Note to score marks in (d) they must have a derivative which does have a minimum and is of a completed square form (the *r* may be inside the bracket) or in some other way clearly uses their (c) (e.g. minimising their quadratic).
(i) B1ft: $(x=)\frac{1}{2}$ ft their $-s$ provided their q>0, their r <0 and their x is in the range $0 \leqslant x \leqslant 1$
(ii) B1ft: $\left(\frac{dy}{dx}=\right)\frac{1}{4}$ ft their q provided it is positive and their r was negative.

(b) Alt	$x=\sin^2 4y \Rightarrow y=\frac{1}{4}\arcsin\sqrt{x} \Rightarrow \frac{dy}{dx}=\frac{1}{4}\frac{1}{\sqrt{1-(\sqrt{x})^2}}\times\frac{1}{2}x^{-\frac{1}{2}}$	M1
	$\frac{dx}{dy}=8\sqrt{x}\sqrt{1-x}$	A1
		(2)
(c)	$\frac{dx}{dy}=8\sin 4y\cos 4y \rightarrow \frac{dy}{dx}=\frac{1}{8\sin 4y\cos 4y}$	M1
	$\frac{dy}{dx}=\frac{1}{8\sqrt{x(1-x)}}$	M1
	$\frac{dy}{dx}=\frac{1}{\sqrt{16-64\left(x-\frac{1}{2}\right)^2}}$	A1
		(3)

Notes

Alt by finding *y* first. Mark (b) and (c) together via such approaches. But note that part (c) says "Hence" and the reciprocal law must have been used at some stage to score full marks.

(b)

M1: Makes y the subject and differentiates to reach form $\frac{dy}{dx}=A\frac{1}{\sqrt{1-(\sqrt{x})^2}}\times\ldots$ where … is a function of x

A1: For $\frac{dx}{dy}=8\sqrt{x}\sqrt{1-x}$ oe with square roots combined. Coefficients must have been gathered.

(c)

M1: For use of the reciprocal rule of derivatives evidenced in the working – allow for it being used in (b) to find $\frac{dx}{dy}$ from $\frac{dy}{dx}$.

M1: Award for the correct procedure of having made y the subject and differentiating to reach the correct form for the answer, ie $\frac{dy}{dx}=A\frac{1}{\sqrt{1-(\sqrt{x})^2}}\times Bx^{-\frac{1}{2}}$

A1: As main scheme.

10. JAN 2022 [1]

1	$y=(2x+5)e^{3x}$	
	$\frac{dy}{dx}=2e^{3x}+(2x+5)3e^{3x}$	M1 A1
	$\frac{dy}{dx}=0 \Rightarrow 2+3(2x+5)=0 \Rightarrow x=-\frac{17}{6}$	dM1A1
		(4) **(4 marks)**

M1: Attempts the product rule and achieves a form $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right) = Ae^{3x} + B(2x+5)e^{3x}$ where A and B are positive constants. Condone missing/poor bracketing

Note that this could be attempted from $y = 2xe^{3x} + 5e^{3x} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = Ae^{3x} + Bxe^{3x} + Ce^{3x}$

A1: Correct $\frac{\mathrm{d}y}{\mathrm{d}x} = 2e^{3x} + (2x+5)3e^{3x}$ o.e. There is no requirement to simplify this

dM1: Dependent upon the previous M, it is for using a correct strategy to find a value for x.

E.g. Sets or implies $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$, cancels or factorises out the e^{3x} term and solves a linear equation in x

In most cases where lns are used it will be M0

Example of M0

$e^{3x}(6x+17) = 0 \Rightarrow \ln\left(e^{3x}(6x+17)\right) = \ln(0) \Rightarrow 3x(6x+17) = ...$

Example of M1

$e^{3x}(6x+17) = 0 \Rightarrow (6x+17) = 0, \quad \left(e^{3x} = 0\right)$

$x = ...$

A1: $x = -\frac{17}{6}$ o.e. only. If an extra solution is given, e.g. from $e^{3x} = 0$, it is A0

Condone awrt -2.83 following a correct equation. Ignore any attempt to find y

11. JAN 2022 [10]

10 (a)	$x = y\mathrm{e}^{2y} \Rightarrow \dfrac{\mathrm{d}x}{\mathrm{d}y} = \mathrm{e}^{2y} + 2y\mathrm{e}^{2y}$	M1, A1
	$\Rightarrow \dfrac{\mathrm{d}x}{\mathrm{d}y} = \dfrac{x}{y} + 2x$	
	$\Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{1}{\dfrac{x}{y} + 2x}$	dM1
	$\Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{y}{x + 2xy} = \dfrac{y}{x(1+2y)}$ *	A1*
		(4)
(b)	Deduces $y = -\dfrac{1}{2}$	B1
	Substitutes $y = -\dfrac{1}{2} \Rightarrow x = -\dfrac{1}{2\mathrm{e}}$ Range for k $-\dfrac{1}{2\mathrm{e}} < k < 0$	M1, A1
		(3)
		(7 marks)

(a)

M1: For attempting to differentiate with respect to y.

Uses the product rule on $y\mathrm{e}^{2y} \Rightarrow \mathrm{e}^{2y} + ...y\mathrm{e}^{2y}$. The left hand side may be missing/incorrect

A1: Correct differentiation E.g. $\dfrac{\mathrm{d}x}{\mathrm{d}y} = \mathrm{e}^{2y} + 2y\mathrm{e}^{2y}$

dM1: Full method to get $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of just x and y.

This requires, in any order

- a correct attempt to invert, e.g. not inverting each term in a sum of terms
- e^{2y} being fully replaced. You should see e^{2y} being replaced by $\dfrac{x}{y}$ or equivalent and $y\mathrm{e}^{2y}$ being replaced by x

A1*: Correct proof. All relevant steps should be shown and there should be no errors.

If you feel that it hasn't been fully shown then please award M1 A1 dM1 A0

(b) Now being marked B1 M1 A1. On epen it is set up M1 A1 A1

B1: For deducing left hand end occurs when $y = -\frac{1}{2}$.

M1: For attempting to find x when $y = -\frac{1}{2} \Rightarrow x = ...$ This may be implied by k = awrt -0.183 or -0.184

A1: $-\frac{1}{2e} < k < 0$ or **exact** equivalent such as e.g. $-\frac{1}{2}e^{-1} < k < 0$, $k > -\frac{1}{2}e^{-1}$ **and** $k < 0$, $\left\{k : k > -\frac{1}{2e}\right\} \cap \{k : k < 0\}$

This must be correct so the candidate cannot have two separate inequalities with an "**or**" between
This must be the range for k not x

....

Alt via ln's

$x = ye^{2y} \Rightarrow \ln x = \ln y + 2y$ o.e.

M1: Attempts to differentiate wrt x. Consider just rhs $\ln y + 2y \to \frac{1}{y}\frac{dy}{dx} + ...\frac{dy}{dx}$

Alternatively attempts to differentiate wrt y. Consider both sides $...\frac{dx}{dy} = \frac{1}{y} + 2$

A1: Correct differentiation $\frac{1}{x} = \frac{1}{y}\frac{dy}{dx} + 2\frac{dy}{dx}$ or $\frac{1}{x}\frac{dx}{dy} = \frac{1}{y} + 2$

....

Alt via differentiating wrt x

$$x = y\mathrm{e}^{2y} \Rightarrow \quad 1 = \mathrm{e}^{2y}\frac{\mathrm{d}y}{\mathrm{d}x} + y \times 2\mathrm{e}^{2y}\frac{\mathrm{d}y}{\mathrm{d}x}$$

$$1 = \frac{x}{y}\frac{\mathrm{d}y}{\mathrm{d}x} + 2x\frac{\mathrm{d}y}{\mathrm{d}x}$$

$$1 = \left(\frac{x+2xy}{y}\right)\frac{\mathrm{d}y}{\mathrm{d}x}$$

$$\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{y}{x(1+2y)} \quad *$$

M1: For attempting to differentiate wrt x.

Uses the product rule on $y\mathrm{e}^{2y} \Rightarrow \quad \mathrm{e}^{2y}\frac{\mathrm{d}y}{\mathrm{d}x} + ...y\frac{\mathrm{d}y}{\mathrm{d}x}\mathrm{e}^{2y}$. The left hand side may be missing/incorrect

Uses the quotient rule on $y = \frac{x}{\mathrm{e}^{2y}} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{e}^{2y} - 2x\mathrm{e}^{2y} \times \frac{\mathrm{d}y}{\mathrm{d}x}}{\mathrm{e}^{4y}}$ condoning slips on the coefficient

A1: Correct differentiation including the lhs

dM1: Full method to get $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y.

12. JUNE 2022 [1]

1(a)	$\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right) 6\times 3(3x-2)^5 \quad (=18(3x-2)^5)$	M1A1
		(2)
(b)	$"18"(3\times\frac{1}{3}-2)^{"5"}=-18$	M1
	$-18\to\frac{1}{18}$	M1
	$y-1="\frac{1}{18}"\left(x-\frac{1}{3}\right)$	dM1
	$3x-54y+53=0$	A1
		(4)
		(6 marks)

(a)

M1 Applies the chain rule to achieve a form of $A(3x-2)^5$ which may be unsimplified.
The index does not need to be processed for this mark.

Alternatively, multiplies out the brackets to achieve an expression of the form
$Ax^6+Bx^5+Cx^4+Dx^3+Ex^2+Fx+G$ (where all coefficients do not need to be simplified) and reduces the power by 1 on at least one of the terms. You will need to check this carefully to make sure it is the relevant term where the power has decreased
eg $Bx^5\to\ldots\times Bx^4$
Condone slips in their expansion.

(The actual expansion is $64-576x+2160x^2-4320x^3+4860x^4-2916x^5+729x^6$)

A1 $\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)18(3x-2)^5$ or unsimplified equivalent eg $6\times 3(3x-2)^5$ but the index must be processed. If they have multiplied out then accept $-576+4320x-12960x^2+19440x^3-14580x^4+4374x^5$.
Accept x^1 for x

(b)

M1 Substitutes $x = \frac{1}{3}$ into their $\frac{\mathrm{d}y}{\mathrm{d}x}$ which must be of the form $A(3x-2)^5$ (or accept a polynomial of degree 5) and finds a value for $\frac{\mathrm{d}y}{\mathrm{d}x}$

M1 Finds the negative reciprocal of their gradient. May be implied by the equation of their line. It cannot be scored for setting $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$, solving to find x and then taking the negative reciprocal of this.

dM1 Attempts to find the equation of the normal. Look for $y - 1 = "\frac{1}{18}"\left(x - \frac{1}{3}\right)$ (must be a changed gradient from their tangent gradient). It is dependent on the first method mark. If they use $y = mx + c$ they must proceed as far as $c = \ldots$ $\left(= "\frac{53}{54}"\right)$

A1 $3x - 54y + 53 = 0$ or any integer multiple of this with all terms on one side of the equation eg $-6x + 108y - 106 = 0$ You must see the $= 0$
isw after a correct answer is seen. Eg if they proceed to state values for a, b and c which contradict their equation then take their equation as their final answer.

13.OCT 2022 [6]

6	$\dfrac{d}{dx}(\cos x+\sin x)=-\sin x+\cos x$	**B1**
	$\dfrac{dy}{dx}=\dfrac{(\cos x+\sin x)(3\cos x)-(-\sin x+\cos x)(2+3\sin x)}{(\cos x+\sin x)^2}$	**M1** **A1**
	$=\dfrac{3\cos^2 x+\cancel{3\sin x\cos x}+2\sin x+3\sin^2 x-2\cos x-\cancel{3\sin x\cos x}}{\cos^2 x+2\sin x\cos x+\sin^2 x}$ $=\dfrac{3+2\sin x-2\cos x}{1+2\sin x\cos x}$	**M1**
	$=\dfrac{3+2\sin x-2\cos x}{1+2\sin x\cos x}\times\dfrac{\sec x}{\sec x}=\dfrac{3\sec x+2\sec x\sin x-2}{\sec x+2\sin x}$	**M1**
	$=\dfrac{2\tan x+3\sec x-2}{\sec x+2\sin x}$	**A1**
		(6)
		(6 marks)

Notes:

B1: Correct differentiation of $\cos x+\sin x\to-\sin x+\cos x$ seen somewhere in the proof. This can be scored if seen in workings for quotient rule, or even in the denominator of an incorrect attempt at *u*'/*v*'.

M1: Differentiates using the quotient rule or product rule. For quotient rule look for

$$\frac{(\cos x+\sin x)(\pm a\cos x)-(\pm\sin x\pm\cos x)(2+3\sin x)}{(\cos x+\sin x)^2}$$

For product rule look for $(\cos x+\sin x)^{-1}(\pm a\cos x)\pm(\cos x+\sin x)^{-2}(\pm\cos x\pm\sin x)(2+3\sin x)$

A1: Fully correct derivative.

M1: Expands numerator or denominator and applies $\sin^2 x+\cos^2 x=1$ or other appropriate correct Pythagorean identity at least once in the proof.

M1: Attempts to multiply through by sec *x* in numerator and denominator (to achieve the 2sin *x* term). Not dependent and may be scored before the previous M.

A1: Correct answer. Terms may be in different order. Allow minor slips in notation (e.g. a single missing *x*) and recovery from missing brackets if the intent is clear, but A0 for persistent incorrect notation throughout (e.g no *x*'s in the trig terms).

14.JAN 2021[10]

10(a)	$\frac{dx}{dy}=12\sec^2 2y\tan 2y$	M1, A1
		(2)
(b)	$\frac{dx}{dy}=12\left(\frac{x}{3}\right)\sqrt{\sec^2 2y-1}\Rightarrow\frac{dx}{dy}=12\left(\frac{x}{3}\right)\sqrt{\frac{x}{3}-1}$	M1, A1ft
	$\frac{dy}{dx}=\frac{\sqrt{3}}{4x\sqrt{x-3}}$	A1
		(3)
(c)	$y=\frac{\pi}{12}\Rightarrow x=4$	B1
	$\frac{dx}{dy}=12\times\frac{4}{3}\times\frac{1}{\sqrt{3}}$ or $\frac{dy}{dx}=\frac{1}{12\left(\frac{4}{3}\right)\sqrt{\frac{4}{3}-1}}$	M1
	Correct $m_N=-\frac{16}{\sqrt{3}}$ o.e	A1
	$y-\frac{\pi}{12}=-\frac{16}{\sqrt{3}}(x-4)$	dM1
	$y=-\frac{16\sqrt{3}}{3}x+\frac{64\sqrt{3}}{3}+\frac{\pi}{12}$	A1
		(5)
		Total 10

(a)

M1: Differentiates to a form on the rhs of $\alpha\sec^2 2y\tan 2y$ which may be written $...\sec 2y\times...\sec 2y\tan 2y$

Note that the same scheme can also be applied to students who adapt $x=3\sec^2 2y$ to $x=\pm 3\tan^2 2y\pm 3$

A1: $\frac{dx}{dy}=12\sec^2 2y\tan 2y$. If the lhs is included it must be correct. So $\frac{dy}{dx}=12\sec^2 2y\tan 2y$ is M1 A0

Condone this to be unsimplified $\frac{dx}{dy}=6\sec 2y\times 2\sec 2y\tan 2y$ ISW after sight of correct answer

(b)

M1: For an attempt to

- replace $\sec^2 2y$ with αx
- use the identity $\pm1 \pm \tan^2 2y = \pm\sec^2 2y$ and replaces $\tan 2y = b\sqrt{\pm1 \pm d\,x}$

to obtain an expression for $\dfrac{dx}{dy}$ or $\dfrac{dy}{dx}$ in terms of x only.

The expression in part (a) must have had $\dfrac{dx}{dy}$ as a function of both $\sec 2y$ and $\tan 2y$ o.e.

A1ft: Requires a substitution of both $\sec^2 2y$ with $\dfrac{x}{3}$ and $\tan 2y = \sqrt{\dfrac{x}{3} - 1}$ to obtain a correct expression for $\dfrac{dx}{dy}$ or $\dfrac{dy}{dx}$ in terms of x. Follow through on their $\dfrac{dx}{dy} = \alpha \sec^2 2y \tan 2y$

APPROVED)

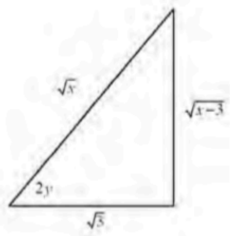
For a correct $\dfrac{dx}{dy}$ it is awarded for $\dfrac{dy}{dx} = \dfrac{1}{4x\sqrt{\frac{1}{3}x - 1}}$

A1: Correct answer in the required form.

Allow equivalents e.g. $\dfrac{3\sqrt{3}}{12x\sqrt{x-3}}$. Form required is $\dfrac{p}{qx\sqrt{x-3}}$ where p is irrational and q is an integer

Alt method for (a) and (b) which can be marked in a similar way

(a) M1 A1: $x = 3(\cos 2y)^{-2} \Rightarrow \dfrac{dx}{dy} = 12(\cos 2y)^{-3} \sin 2y$



(b) If $x = 3(\cos 2y)^{-2} \Rightarrow \cos 2y = \dfrac{\sqrt{3}}{\sqrt{x}}$

Score in a similar way to the main scheme

M1 A1: $\dfrac{dx}{dy} = \dfrac{12\sin 2y}{(\cos 2y)^3} = \dfrac{12\sqrt{\frac{x-3}{x}}}{\left(\sqrt{\frac{3}{x}}\right)^3}$

Alt (b) via arccos

$$y = \frac{1}{2}\arccos\sqrt{\frac{3}{x}} \Rightarrow \frac{dy}{dx} = \frac{1}{2}\frac{1}{\sqrt{1-\left(\sqrt{\frac{3}{x}}\right)^2}} \times \frac{\sqrt{3}}{2}x^{-\frac{3}{2}}$$

M1: For $\dfrac{dy}{dx} = \lambda \dfrac{1}{\sqrt{1-\left(\sqrt{\frac{\alpha}{x}}\right)^2}} \times -x^{-\frac{3}{2}}$ A1: Correct and unsimplied A1: Correct and in the required form

(c)
B1: Correct value for x

M1: Attempts to find the value of $\frac{dx}{dy}$ using their part (a) with $y=\frac{\pi}{12}$

the value of $\frac{dy}{dx}$ from an inverted $\frac{dx}{dy}$ using their part (a) with $y=\frac{\pi}{12}$

or the value of $\frac{dy}{dx}$ using their part (b) with their value of x found using $y=\frac{\pi}{12}$.

These may be called m or f ' and not identified as $\frac{dx}{dy}$ or $\frac{dy}{dx}$

A1: Correct normal gradient

dM1: Attempt at the equation of normal at $y=\frac{\pi}{12}$.

The gradient should be either an attempt at the value of $-\frac{dx}{dy}$ at $y=\frac{\pi}{12}$ for their $\frac{dx}{dy}$

or an attempt at the negative reciprocal of their $\frac{dy}{dx}$ at their "4" which must have been found from $y=\frac{\pi}{12}$

A1: Fully correct equation in the required form. ISW after a correct answer

Allow equivalent exact forms e.g. $y=-\frac{16}{\sqrt{3}}x+\frac{64}{\sqrt{3}}+\frac{\pi}{12}$, $y=-\frac{16}{\sqrt{3}}x+\frac{256\sqrt{3}+\pi}{12}$

15.JUNE 2021 [7]/ 16.JUNE 2021 [7]

7	$x = 6\sin^2 2y \Rightarrow \frac{dx}{dy} = 24\sin 2y\cos 2y$	M1 A1
	Attempts to use $\frac{dy}{dx} = 1 \div \frac{dx}{dy}$	M1
	Attempts to use both $\sin 2y = \sqrt{\frac{x}{6}}$ and $\cos 2y = \sqrt{1-\sin^2 2y} = \sqrt{1-\frac{x}{6}}$	M1
	$\frac{dy}{dx} = \frac{1}{24\sin 2y\cos 2y} = \frac{1}{24\times\sqrt{\frac{x}{6}}\times\sqrt{1-\frac{x}{6}}} = \frac{1}{4\sqrt{6x-x^2}}$	A1
		(5)
Way 2	$x = 6\sin^2 2y = 3-3\cos 4y \Rightarrow \frac{dx}{dy} = 12\sin 4y$	M1A1
	Attempts to use $\frac{dy}{dx} = 1 \div \frac{dx}{dy}$	M1
	Attempts to use $\sin 4y = \frac{\sqrt{6x-x^2}}{3}$	M1
	$\frac{dy}{dx} = \frac{1}{12\sin 4y} = \frac{1}{12\times\frac{\sqrt{6x-x^2}}{3}} = \frac{1}{4\sqrt{6x-x^2}}$	A1

Way 3	$x = 6\sin^2 2y = 24\sin^2 y\cos^2 y \Rightarrow \frac{dx}{dy} = 48\sin y\cos^3 y - 48\sin^3 y\cos y$	M1A1
	Attempts to use $\frac{dy}{dx} = 1 \div \frac{dx}{dy}$	M1
	$\frac{dy}{dx} = \frac{1}{48\sin y\cos^3 y - 48\sin^3 y\cos y} = \frac{1}{48\sin y\cos y\left(\cos^2 y - \sin^2 y\right)}$ $\frac{dy}{dx} = \frac{1}{24\sin 2y\cos 2y} = \frac{1}{24\times\sqrt{\frac{x}{6}}\times\sqrt{1-\frac{x}{6}}}$	M1
	$= \frac{1}{4\sqrt{6x-x^2}}$	A1
		(5 marks)

<u>In general, apply the following marking guidance for this question:</u>

M1: Attempts to differentiate to obtain $\frac{dx}{dy}$ in a correct form

A1: Correct derivative

M1: Attempts to use $\frac{dy}{dx} = 1 \div \frac{dx}{dy}$.

M1: Change fully to a function of x.

A1: All correct

E.g.

M1: Attempts to use the chain rule on the rhs to achieve $k\sin 2y\cos 2y$

A1: Fully correct derivative $\frac{dx}{dy}=24\sin 2y\cos 2y$

M1: Attempts to use $\frac{dy}{dx}=1\div\frac{dx}{dy}$.

Condone a slip on the coefficient. E.g. $\frac{dx}{dy}=24\sin 2y\cos 2y\Rightarrow\frac{dy}{dx}=\frac{24}{\sin 2y\cos 2y}$

Do not condone slips/errors on the variable. E.g. $\frac{dx}{dy}=24\sin 2y\cos 2y\Rightarrow\frac{dy}{dx}=\frac{1}{24\sin 2x\cos 2x}$

M1: Attempts to change both sin 2y and cos 2y to functions in x.

Expect to see $\sin 2y=p\sqrt{x}$ and $\cos 2y=\sqrt{1-qx}$

A1: CSO $\frac{dy}{dx}=\frac{1}{4\sqrt{6x-x^2}}$

Way 2

M1: Differentiates to obtain $k\sin 4y$

A1: Fully correct derivative $\frac{dx}{dy}=12\sin 4y$

M1: Attempts to use $\frac{dy}{dx}=1\div\frac{dx}{dy}$.

Condone a slip on the coefficient. E.g. $\frac{dx}{dy}=12\sin 4y\Rightarrow\frac{dy}{dx}=\frac{12}{\sin 4y}$

M1: Attempts to change both sin 4y to a function in x.

Expect to see $\sin 4y=p\sqrt{6x-x^2}$ or equivalent e.g. $\sin 4y=p\sqrt{9-(9-6x+x^2)}$

A1: CSO $\frac{dy}{dx}=\frac{1}{4\sqrt{6x-x^2}}$

Way 3

M1: Differentiates to obtain $A\sin y\cos^3 y + B\sin^3 y\cos y$

A1: Fully correct derivative $\frac{dx}{dy} = 48\sin y\cos^3 y - 48\sin^3 y\cos y$

M1: Attempts to use $\frac{dy}{dx} = 1 \div \frac{dx}{dy}$.

M1: Attempts to change both $\sin 2y$ and $\cos 2y$ to functions in x.

Expect to see $\sin 2y = p\sqrt{x}$ and $\cos 2y = \sqrt{1-qx}$

A1: CSO $\frac{dy}{dx} = \frac{1}{4\sqrt{6x-x^2}}$

A possible alternative:

$$x = 6\sin^2 2y \Rightarrow \sin 2y = \sqrt{\frac{x}{6}} \Rightarrow 2y = \sin^{-1}\sqrt{\frac{x}{6}} \Rightarrow y = \frac{1}{2}\sin^{-1}\sqrt{\frac{x}{6}}$$

M1: For a correct attempt to make y or $2y$ the subject

A1: Correct rearrangement

$$\frac{dy}{dx} = \frac{1}{2} \times \frac{1}{\sqrt{1-\frac{x}{6}}} \times \frac{1}{12}\left(\frac{x}{6}\right)^{-\frac{1}{2}}$$

M1: For $\frac{A}{\sqrt{1-\frac{x}{6}}} \times \ldots$ M1: For $\ldots \times B\left(\frac{x}{6}\right)^{-\frac{1}{2}}$

A1: CSO $\frac{dy}{dx} = \frac{1}{4\sqrt{6x-x^2}}$

17.OCT 2021 [6]

6 (i)	$\left(\frac{dy}{dx}=\right)\frac{6x}{x^2-5}-8x$ Stationary point when $\frac{6x}{x^2-5}-8x=0 \Rightarrow x^2=\frac{23}{4} \Rightarrow x=\frac{\sqrt{23}}{2}$ only	M1 A1 dM1 A1
		(4)
(ii)(a) **(b)**	$\left(\frac{dy}{dx}=\right)4-24\sin x\cos x$ $\left(\frac{dy}{dx}=\right)4-12\sin 2x$ Maximum gradient = 16	M1 dM1 A1 A1
		(4)
		(8 marks)

(i)

M1 For differentiating $3\ln\left(x^2-5\right)\to\dfrac{Ax}{x^2-5}$ or $\dfrac{A}{x^2-5}$

A1 $\left(\dfrac{dy}{dx}=\right)\dfrac{6x}{x^2-5}-8x$ seen or implied by eg $\dfrac{6x}{x^2-5}=8x$

dM1 For proceeding from $\left(\dfrac{dy}{dx}=\right)\dfrac{Ax}{x^2-5}-Bx=0$ to a quadratic or cubic equation. Eg

$6x-8x^3+40x=0$ oe

Do not be too concerned with the mechanics of their rearrangement.

A1 For $(x=)\dfrac{\sqrt{23}}{2}$ only provided dM1 has been scored. Solving the quadratic or cubic with a calculator is acceptable. Condone recovery of slips provided the method is correct.

$x=0$ and $x=-\dfrac{\sqrt{23}}{2}$ must be rejected if found or condone some minimal conclusion eg stating $p=23$

Mark (ii) (a) + (b) together

(ii) Way One

M1 For differentiating using the chain rule with $\sin^2 x\to\ldots\sin x\cos x$

dM1 .....and then using $\sin 2x=2\sin x\cos x$ to proceed to $\left(\dfrac{dy}{dx}=\right)A+B\sin 2x$. Condone the slip writing $A+B\sin x$ provided their coefficient of $\sin x\cos x$ has been halved or writing $2\sin\cos x$

A1 $\left(\dfrac{dy}{dx}=\right)4-12\sin 2x$ or $4+(-12)\sin 2x$ or $4-12\times\sin 2x$ (or states A and B following a correct method) Condone recovery of slips provided the method is correct.

A1 Maximum gradient = 16 (the previous A1 must have been scored)

(ii) Way Two

M1 Attempts to use $\cos 2x = \pm 1 \pm 2\sin^2 x$ in an attempt to write $y = 4x - 12\sin^2 x$ in terms of $\cos 2x$

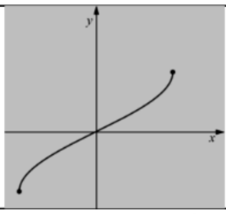
dM1 ...and then differentiates $\cos 2x \to k\sin 2x$ to proceed to $\left(\frac{\mathrm{d}y}{\mathrm{d}x} =\right) A + B\sin 2x$

A1 $\left(\frac{\mathrm{d}y}{\mathrm{d}x} =\right) 4 - 12\sin 2x$ or $4 + (-12)\sin 2x$ or $4 - 12 \times \sin 2x$ (or states A and B following a correct method) Condone recovery of slips provided the method is correct.

A1 Maximum gradient = 16 (the previous A1 must have been scored)

(Note some candidates may find the second derivative and set equal to 0 and then substitute in their angle to find the maximum gradient.)

18. OCT 2021 [8]

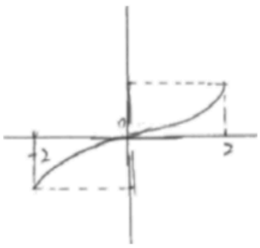
8 (a)	 Shape and position	B1 **(1)**
(b)	$x = 2\sin y \Rightarrow \left(\frac{dx}{dy} =\right) 2\cos y$ and attempts to use **both** $\frac{dy}{dx} = 1 \div \frac{dx}{dy}$ and $\cos y = \sqrt{1-\sin^2 y} = \sqrt{1-...x^2}$	M1
	$\left(\frac{dy}{dx} =\right) \frac{1}{2\sqrt{1-\frac{x^2}{4}}}$	A1
	$\left(\frac{dy}{dx} =\right) \frac{1}{\sqrt{4-x^2}}$	A1cso
		(3)
(c)	Substitutes $x = \sqrt{2}$ into their $\frac{dy}{dx} = \frac{1}{\sqrt{4-x^2}}$ $\left(= \frac{\sqrt{2}}{2}\right)$	M1
	Finds the equation of the tangent at P $y - \frac{\pi}{4} = "\frac{\sqrt{2}}{2}"\left(x - "\sqrt{2}"\right)$	dM1
	$y = \frac{\sqrt{2}}{2}x - 1 + \frac{\pi}{4}$	A1
		(3)
		(7 marks)

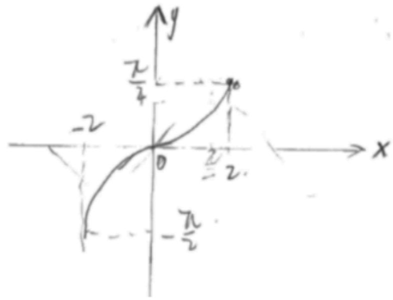
Alt(b)	$y = \arcsin\left(\frac{x}{2}\right) \Rightarrow \left(\frac{dy}{dx} =\right) \frac{1}{2} \times \frac{1}{\sqrt{1-\left(\frac{x}{2}\right)^2}}$	M1A1
	$\left(\frac{dy}{dx} =\right) \frac{1}{\sqrt{4-x^2}}$	A1

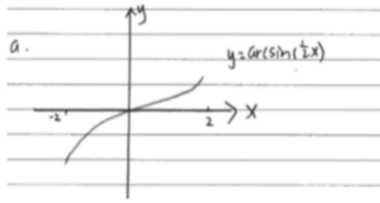
(a)

B1 Correct shape and position: Look for a curve in quadrants 1 and 3 with non zero gradient at the origin and gradient $\to \infty$ at both ends. Ignore values labelled on the axes.
If there is more than one attempt, mark the one which appears in the main body of the work as their fullest attempt. If there is more than one sketch on the same graph, then it must be clearly labelled which is the one to be marked.

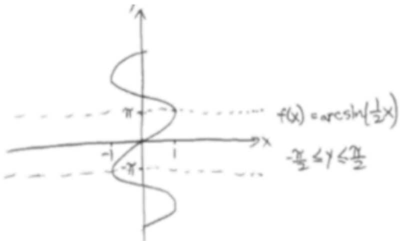
Examples of B1

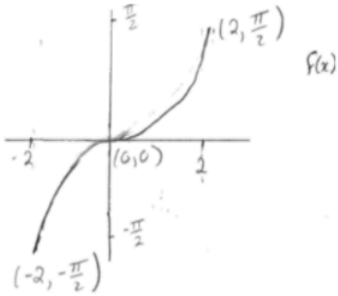






Examples of B0





(b) Note on EPEN it is M1M1A1 which is now marked as M1A1A1

M1 $x = 2\sin y \Rightarrow \frac{dx}{dy} = \pm k\cos y$ where k is non zero and attempts to use both $\frac{dy}{dx} = 1 \div \frac{dx}{dy}$ and $\sin^2 y + \cos^2 y = 1$ in order to obtain $\frac{dy}{dx}$ in terms of x. Allow slips with dealing with their "2"

Allow working leading to $\frac{...}{...\sqrt{1-...x^2}}$ to score M1.

A1 A correct expression for $\frac{dy}{dx}$ as a function of x which may be unsimplified.

eg $\left(\frac{dy}{dx}=\right)\frac{1}{2\sqrt{1-\frac{x^2}{4}}}$ or $\frac{1}{\sqrt{\frac{4}{4}-\frac{x^2}{4}}}$ or $\frac{1}{2\sqrt{\frac{4-x^2}{4}}}$

A1cso $\left(\frac{dy}{dx}=\right)\frac{1}{\sqrt{4-x^2}}$

Alt (b)

M1 $y = \arcsin\left(\frac{x}{2}\right) \Rightarrow \frac{dy}{dx} = k \times \frac{1}{\sqrt{1-\left(\frac{x}{2}\right)^2}}$ (where k can equal 1) then A1A1 as above

SC $y = \arcsin\left(\frac{x}{2}\right) \Rightarrow \frac{dy}{dx} = \frac{1}{\sqrt{4-x^2}}$ with no correct intermediate working score as 100.

(c)

M1 Attempts to find the value of $\frac{dy}{dx}$ or $\frac{dx}{dy}$ at P. See scheme but allow sub of $y = \frac{\pi}{4}$ into their $\frac{dx}{dy} = \pm k\cos y$

dM1 Full method to find the equation of the tangent at P. Their $\frac{dy}{dx}$ may not be exact. If they use $y = mx + c$ they must proceed as far as $c = ...$

A1 Correct equation and in the correct form. The coefficient of x and the constant must be exact. Isw