20-3 Trigonometry Notes

Topic 1.1: Refreshing the Pythagorean Theorem

Trigonometry is an ancient field of study that utilizes an understanding of geometry to extrapolate all kinds of information. The first introduction into trig happens in Jr. High with the idea of pythagorean theorem.

The pythagorean theorem states that in any given **right angle** triangle the squares of the legs will be equal to the square of the hypotenuse.

Definitions:

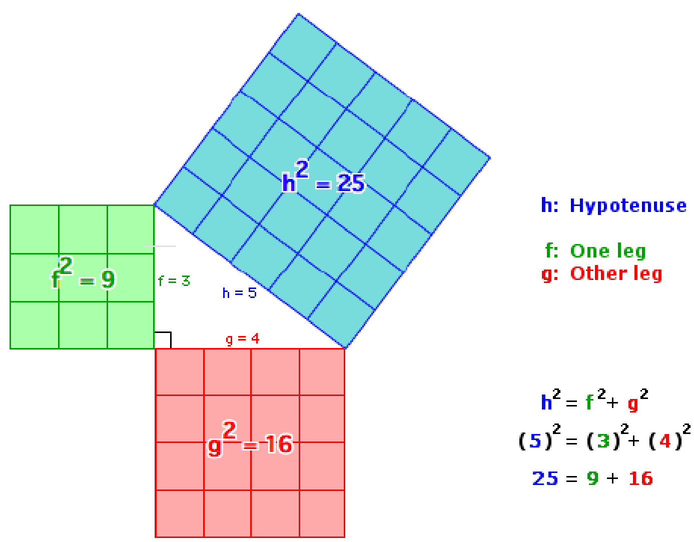
Legs: ____

____

Hypotenuse: ____

____

Pythagorean theorem: $a^2 + b^2 = c^2$



This theorem can be seen on the right wherein the squares of the Two legs (3 and 4) are equal to the Square of the hypotenuse (5)

$3^2 + 4^2 = 5^2$

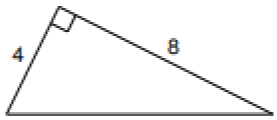
$9 + 16 = 25$

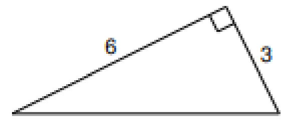
$25 = 25$

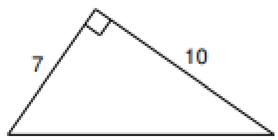
This theorem can also be used to backtrack from a hypotenuse and a single leg to find the other leg. This can be found through the algebraic rearrangement: $c^2 - b^2 = a^2$

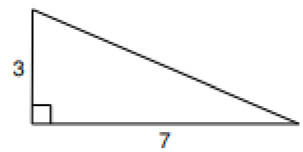
The application of this theorem shows up in any situation where you have 2 known distances and an unknown. Lets look at a number of practice problems and then expand into word problems before refreshing 10^{th} grade trig!

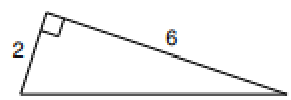
Practice Problems Type 1: Find the length of the missing side in the following shapes.

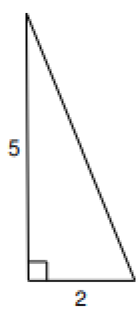


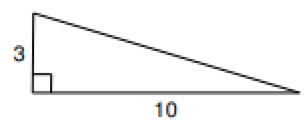


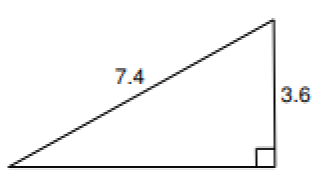


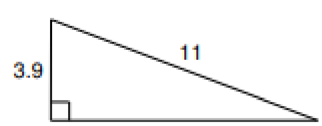


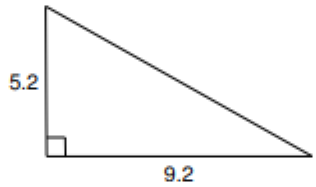


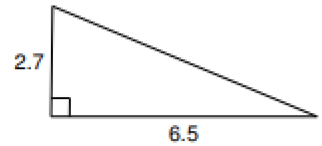


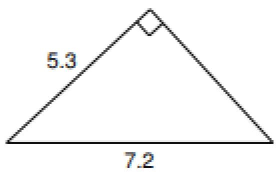


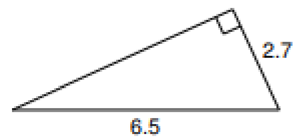






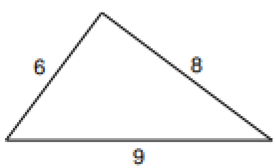






Practice Problems Type 2: Verify if the following figures are right angle triangles using the pythagorean theorem.

1)



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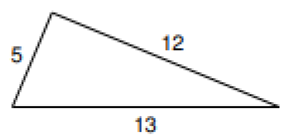

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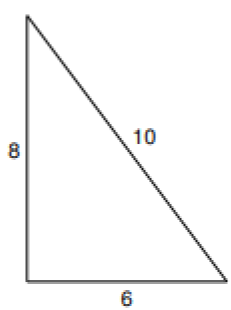

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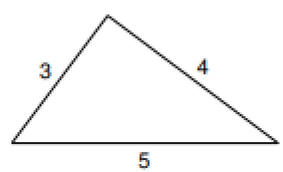


Practice Problems Type 3: Solve the following word problems. Ensure you create a diagram of the situation before calling for help.

1. The bottom of a ladder must be placed 3 feet from a wall. The ladder is 10 feet long. How far above the ground does the ladder touch the wall?

2. A soccer field is a rectangle 100 meters wide and 130 meters long. The coach asks players to run from one corner to the other corner diagonally across. What is that distance?

3. How far from the base of the house do you need to place a 15-foot ladder so that it exactly reaches the top of a 12- foot tall wall?

4. What is the length of the diagonal of a 10 cm by 15 cm rectangle?

5. The diagonal of a rectangle is 25 in. The width is 15 inches. What is the length?

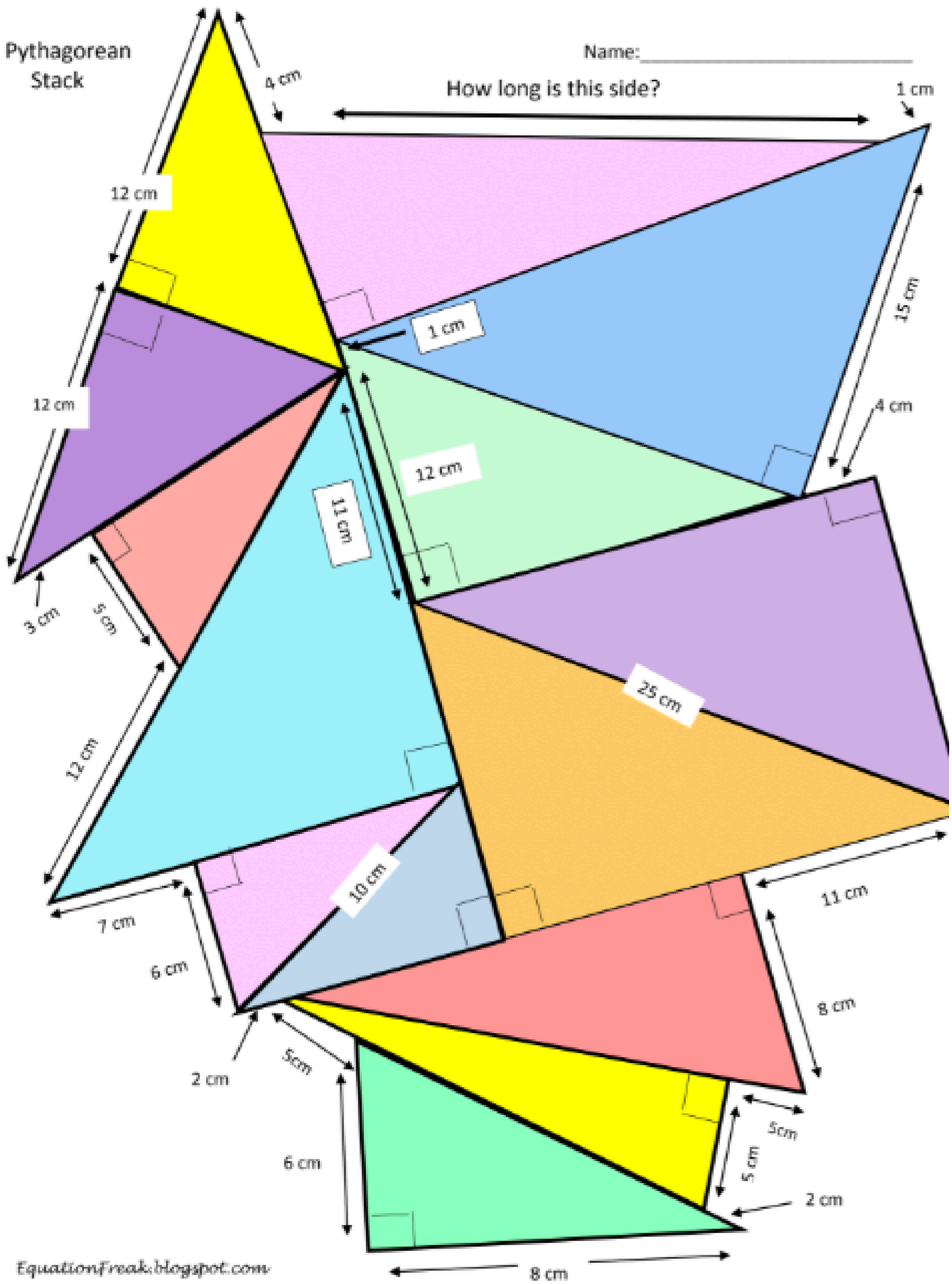
6. Two sides of a right triangle are 8 and 12. Find the hypotenuse.

7. A baseball diamond is a square that is 90 feet on each side. What is the distance a catcher has to throw the ball from home to second base?

8. David leaves the house to go to school. He walks 200m west and 125m north. How far away is he from his starting point? (the diagonal)

9. A park is in the shape of a rectangle 8 miles long and 6 miles wide. How much shorter is your walk if you walk diagonally across the park than along the two sides of it?

Practice Problems Type 4: Complex Stacks.

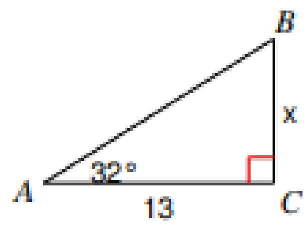


Topic 1.2: Naming a Triangle

The Pythagorean Theorem has a major drawback.

It is unable to deal with Angles.

Consider the diagram below. . .



In order to find the missing side using the pythagorean theorem we need at least 2 sides. We do not have two sides, but we do have a side and an angle.

This is where we reach into the depths of primary trig. The first step in using the trigonometric functions is to be able to name a triangle.

Naming a triangle

Opposite: ____

____

Adjacent: ____

____

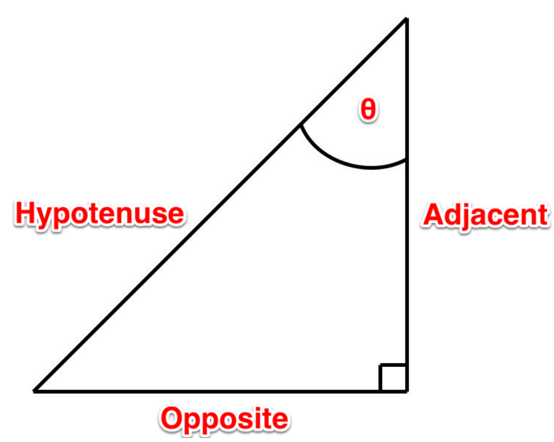
Hypotenuse: ____

____

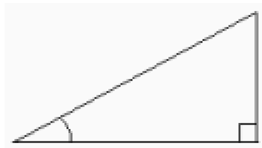
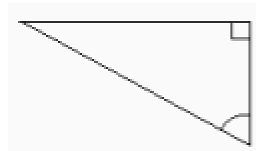
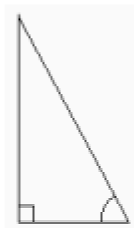
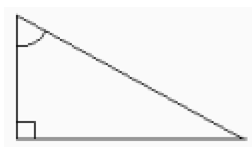
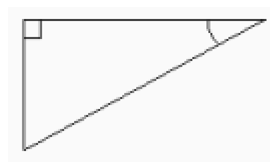
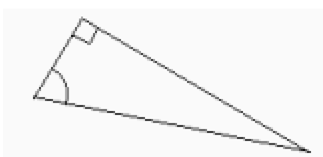
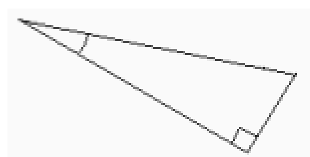
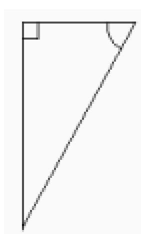
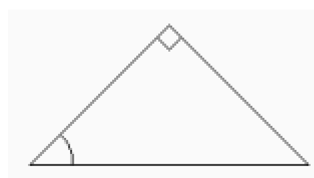
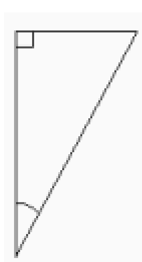
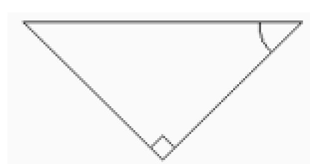
The IMPORTANT reminder is that these names of Opposite or Adjacent, are dependent on the angles location.

They can change based on your perspective of the actual figure.

We can see in the figure to the right that the angle with the partial circle drawn in and the "θ" is the angle we are measuring from.



Practicing your skill: Name the sides of the figures shown below.

1. 	7. 
2. 	8. 
3. 	9. 
4. 	10. 
5. 	11. 
6. 	12. 

Topic 1.3a: Primary Ratios

Once a triangle is appropriately named, we need to *do* something with it.

The *doing* starts with utilizing something known as the primary trigonometric ratios known as:

Sine, Cosine and Tangent.
Or
Sin, Cos, and Tan

These are all buttons on your calculator, but it is important to know that these are *functions* not values. Using the “sin” button is the same as using a “+” or a “-” it does something to a number.

In this lesson we will figure out what when we are supposed to use a given function and describe what the function actually *is*.

What is “sin, cos and tan”

These functions describe the ratio of two sides of an already named right triangle. We have a handy phrase to remember them that goes

SOH - CAH - TOA

Where each word describes the ratio

SOH means → Sinθ = $\frac{Opposite}{Hypotenuse}$ So you use the Sin function when you are working with opposites AND hypotenuse

CAH means → Cosθ = $\frac{Adjacent}{Hypotenuse}$ So you use the Cos function when you are working with adjacents AND hypotenuse

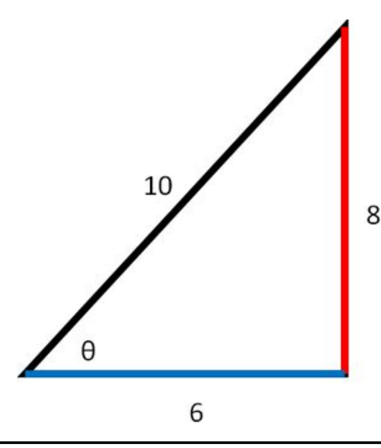
TOA means → Tanθ = $\frac{Opposite}{Adjacent}$ So you use theTan function when you are working with opposites AND adjacents

Lets try it out! In the triangle provided. . . .

Find the SIN:

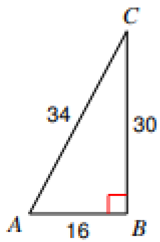
Find the COS:

Find the TAN:

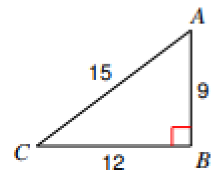


Building the Skill - complete the practice below.

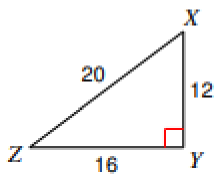
1) $\tan A$



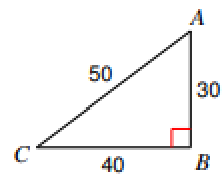
2) $\cos C$



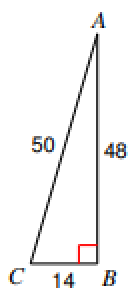
3) $\sin Z$



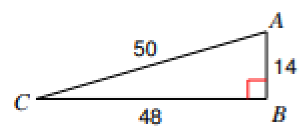
4) $\sin C$



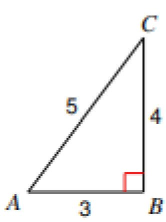
5) $\sin C$



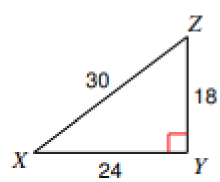
6) $\sin C$



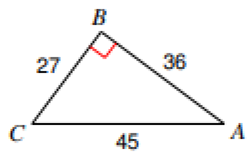
7) $\cos A$



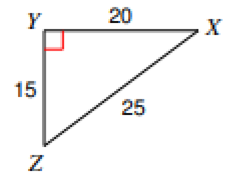
8) $\cos X$



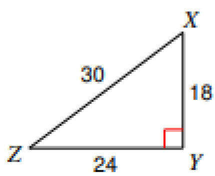
9) cos A



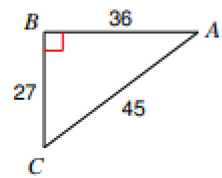
10) cos Z



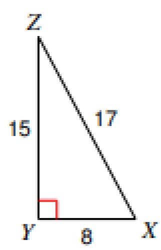
11) sin Z



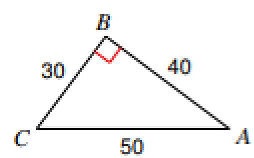
12) sin C



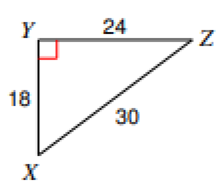
13) cos X



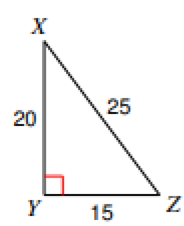
14) tan A



15) tan X



16) tan X



Topic 1.3b: Primary Ratios

The SIN, COS, and TAN buttons have one additional secret. If you have a triangle that features angles but not sides you can still find that same information.

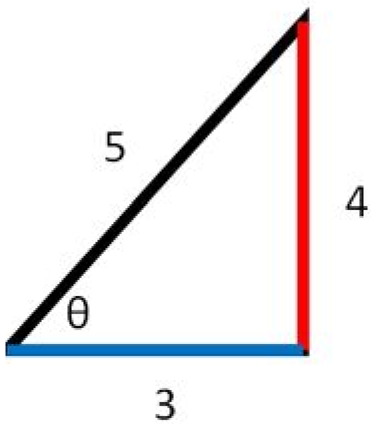
Let me show you.

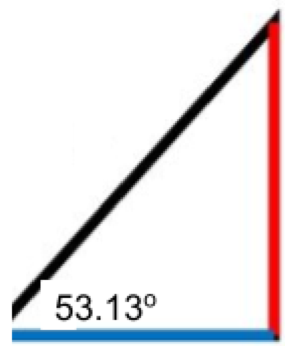
Previously we could find the cos of this triangle but now we can utilize a triangle with an angle but missing sides.

$$cos\theta = \frac{Adj}{Hyp}$$

$$cos\theta = \frac{3}{5}$$

$$cos\theta = 0.6$$





$$cos\theta = \frac{Adj}{Hyp}$$

$$cos53.13 = \frac{Adj}{Hyp}$$

$$cos53.13 = 0.6$$

Important

Your calculator can only help you if it is in degree mode. Make sure to check it over, if you do not you may end up with wrong answers everywhere.

Use a calculator to find the value of each to the nearest ten-thousandth.

17) sin 21°

18) tan 22°

19) cos 20°

20) sin 77°

21) tan 17°

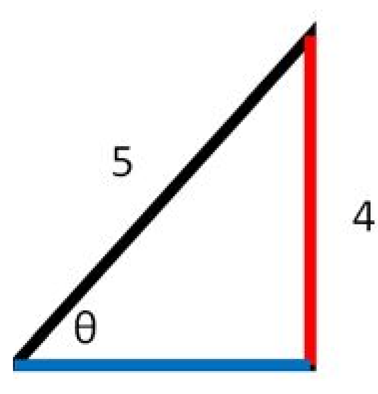
22) cos 87°

Topic 1.4: finding Angles - inc

Now that we have our primary ratios we can use them to solve for a missing angle.

To do this you will first select the second function of your needed ratio. If you have opposite and adjacent, you use the tangent ratio, if you have adjacent and hypotenuse you would use cosine

You then use the 2^{nd} function of that function, and insert your ratio into the bracket of your calculator. See below!



With the triangle given you have an opposite and a hypotenuse. This means that you can use the sine function.

Start with 2^{nd} function sin

sin^{-1}(

Then insert your ration of $\frac{opp}{hyp}$

sin^{-1}($\frac{4}{5}$)

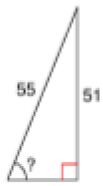
Your calculator will then be ready to hit “enter”. At that time the result will be the angle that you based your naming around in degrees.

Important

Your calculator can only help you if it is in degree mode. Make sure to check it over, if you do not you may end up with wrong answers everywhere.

Practice the skill!

7)



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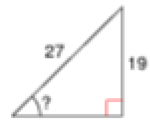

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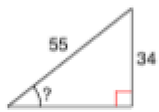

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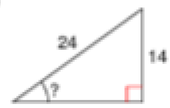

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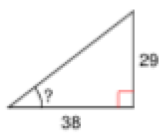

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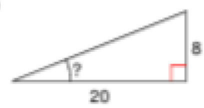

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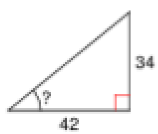
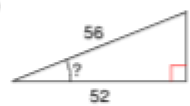


14)



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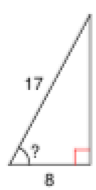

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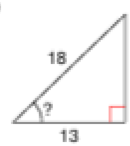

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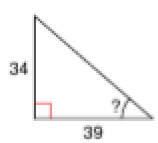

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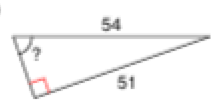

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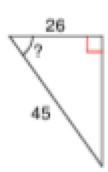

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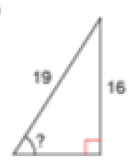

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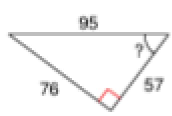

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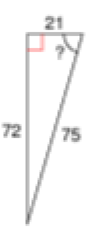
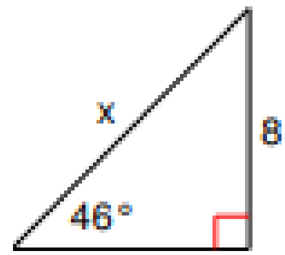


<u>Topic 1.5: Finding Sides</u>

At this point we feel more than comfortable finding the ratios, and angles. The last skill to build is the process of finding the missing side when given an angle and a side in a right triangle.

Exploring this skill, we will still need the same first steps.

1. Label the triangle
2. Select the appropriate function
3. Set up the equation



The big difference when solving for a missing side is to appropriately set up the equation as we now need to navigate an equals sign.

Lets use the example triangle on the right.

When labelled we see that we have an opposite, and an indicated hypotenuse.

Using SOH CAH TOA, we see that we will need to use a sine function. The equation needs to be set up, to show that the missing side is equal to the trig ratio. Look below.

$sin46 = \frac{8}{x}$ ← Here we see that we have the sin 46 as an equivalence to the ratio.

We now can solve for x in one of two ways.

Using Algebra

Utilizing algebra, we need to ensure that we isolate the x above the fraction bar. We do this first by multiplying both sides by x

$$x \cdot sin(46) = \frac{8}{x} \cdot x$$

With the x over the fraction bar, we can now isolate it, by dividing both sides by sin46

$$\frac{x \cdot sin(46)}{sin(46)} = \frac{8}{sin46}$$

This isolates x and can now be calculated by a calendar to solve for x, which in this case . x= 11.12

Using Cross multiples

When we look at our equation we can utilize our understanding of cross multiples to solve the problem.

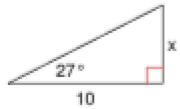
The sin46, although not written, is always over a 1, as anything divided by 1 leads to the same number. Knowing this we can use our cross multiply and divide methodology to solve for the missing value

$$\frac{sin46}{1} = \frac{8}{x}$$

X = 11.12

Practice the skill!

1)



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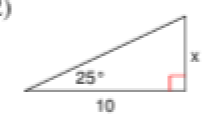

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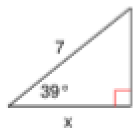

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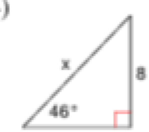

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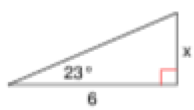

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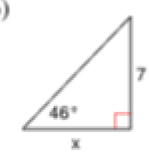

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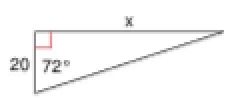

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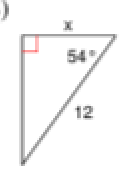

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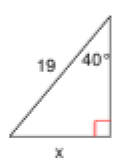

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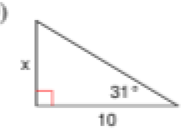

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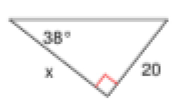

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