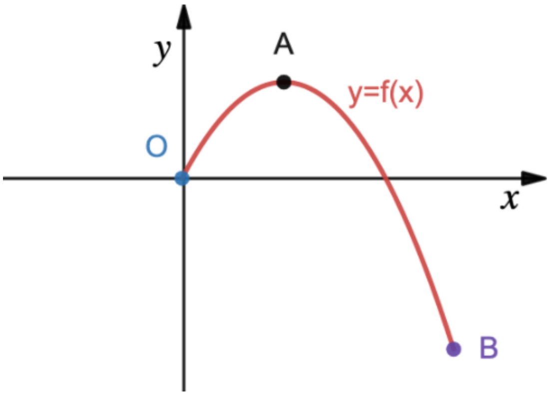
1.



Function $\mathrm{f}(x)$ has domain $0 \leqslant x \leqslant 6$. The graph $y = \mathrm{f}(x)$ passes through the origin, has a maximum point at $A(3, 8)$ and ends at $B(6, -4)$

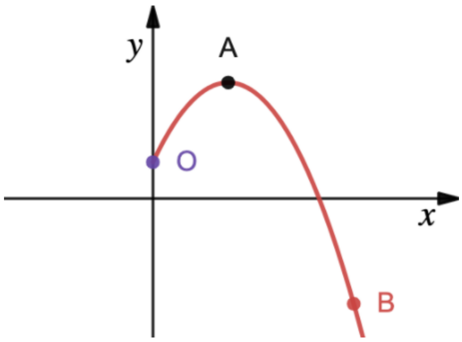
a. State the range of $\mathrm{f}(x)$

$-4 \leqslant y \leqslant 8$

(1)

b. Sketch on separate aces

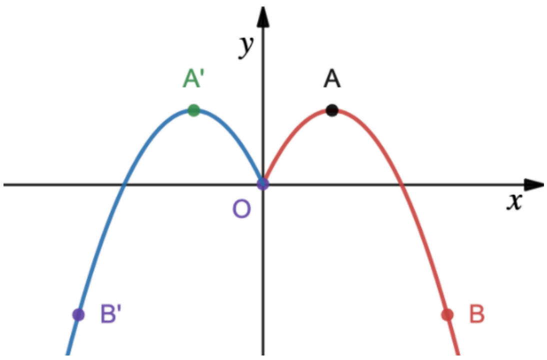
i. $y = 2\mathrm{f}(x) + 1$



$O = (0, 1),\ A = (3, 17),\ B = (6, -7)$

(4)

ii. $y = \mathrm{f}(|x|)$



$O = (0, 0),\ A = (\pm 3, 8),\ B = (\pm 6, -4)$

(4)

in each case indicating the coordinates of the points to which O, A and B have been transformed.

c. State will a reason whether $\mathrm{f}^{-1}(x)$ exists

Not exists, no at 1 to 1 function.

(2)

2.

A sequence is given by $x_{n+1} = 3 - \dfrac{9}{x_n}$ with $x_1 = 4$

a. Determine the period of the sequence

$$x_2 = 3 - \frac{9}{4} = \frac{3}{4}$$
$$x_3 = 3 - \frac{9}{\frac{3}{4}} = -9$$
$$x_4 = 3 - \frac{9}{-9} = 4$$
$$\text{sequence} = 4, \frac{3}{4}, -9......$$
$$\text{period} = 3$$

(4)

b. Find x_{32}

$$x_{30} = -9,\ x_{31} = 4,\ x_{32} = \frac{3}{4}$$

(2)

c. Evaluate $\sum_{i=1}^{32} x_i$

$$\sum_{i=1}^{32} x_i = 10\left(4 + \frac{3}{4} - 9\right) + 4 + \frac{3}{4}$$
$$= -\frac{151}{4}$$

(1)

3.

Find the coordinates of the point of intersection of the curve $y = 2^{1-x}$ and $y = 5^x$ giving your answers to 2 significant figures.

$$2^{1-x} = 5^x$$
$$1 - x = x\log_2 5$$
$$x = \frac{1}{1 + \log_2 5}$$
$$= 0.30\ (2\text{sf})$$
$$y = 1.6$$
$$= (0.3, 1.6)$$

(3)

4.

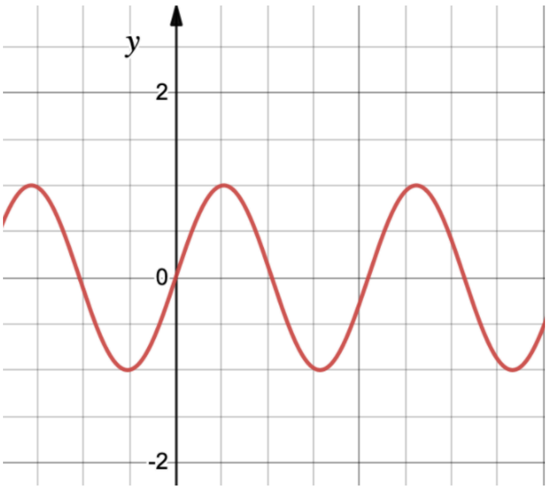
A curve is given parametrically by the equations

$$x = 6\cos t + 2 \qquad y = 3\cos 2t, \qquad 0 \leqslant t \leqslant 2\pi$$

Find the cartesian equation of the curve in the form $y = \mathrm{f}(x)$ and state the range and domain of function $\mathrm{f}(x)$

(4)

5.

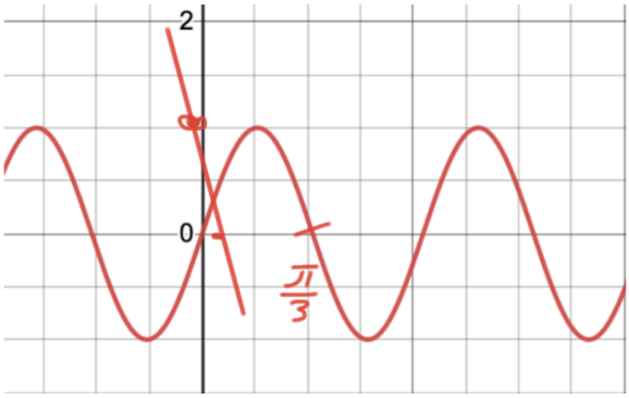


The graph shows part of the curve $y = \sin 3x$ where x is in radians

a. By drawing a suitable line on the graph on the insert sheet, explain why the equation

$$\sin 3x + 8x - 1 = 0$$

has only one real root.



the straight line $y = 1 - 8x$ cuts the curve $y = \sin 3x$ once only.

(2)

The root has a small numerical value.

b. Use a suitable trigonometrical approximation to estimate the value of this root.

$$\begin{aligned} \sin 3x + 8x - 1 &= 0 \\ \sin 3x &= 1 - 8x \\ \lim_{x \to 0} \sin 3x &= 1 - 8x \\ 3x &= 1 - 8x \\ x &= \frac{1}{11} \end{aligned}$$

(3)

6.

$\mathrm{f}(x) = x + \frac{1}{x}, \quad x \neq 0$

a. Find, in terms of *k*, the roots of the equation $\mathrm{f}(x) = 2k$

$$\begin{aligned} x + \frac{1}{x} &= 2k \\ x^2 - 2kx + 1 &= 0 \\ (x-k)^2 - k^2 + 1 &= 0 \\ x &= k \pm \sqrt{k^2 - 1} \end{aligned}$$

(3)

b. For what range of *k* are these roots real?

$$\begin{aligned} k^2 - 1 &\geq 0 \\ (k+1)(k-1) &\geq 0 \\ k &\geq 1 \\ k &\leq -1 \end{aligned}$$

(2)

7.

In the expansion of $(1+ax)^n$ the coefficients of x and x^2 are 3 and 4 respectively. Find the values of *a* and *n*.

$$\begin{aligned} (1+ax)^n &= 1 + nax + \frac{n(n-1)}{2!}(ax)^2 \\ na &= 3 \\ \frac{n(n-1)}{2!}a^2 &= 4 \\ n^2a^2 - a^2n &= 8 \\ 9 - 3a &= 8 \\ a &= \frac{1}{3} \\ n &= 9 \end{aligned}$$

(5)

8.

$f(x) \equiv 0.2x^2 - 0.2x + 1.05$

a. Write $f(x)$ in the form $a(x+b)^2 + c$

$$f(x) = 0.2(x^2 - x) + 1.05$$
$$= 0.2\left[(x-0.5)^2 - 0.25\right] + 1.05$$
$$= 0.2(x-0.5)^2 + 1$$

(3)

b. Find the coordinates of the maximum point on the curve

$$y = \frac{1}{0.2x^2 - 0.2x + 1.05}$$

y is maximum when the denominator is minimum , the minimum value of $f(x) = 1$ when $x = 0.5$

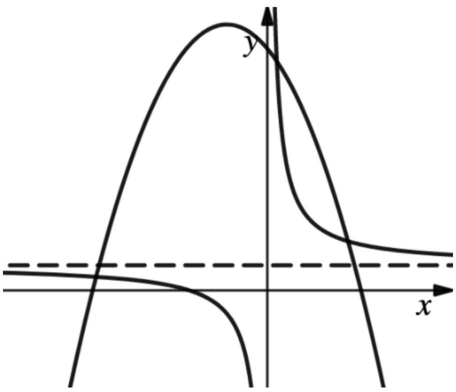
$\therefore$ maximum point =(0.5,1)

(2)

9.

a. Sketch, on the same axes, the curves

$y = \frac{2}{x} + 1$ and $y = 10 - 2x - x^2$



*coordinates of intersection of axes and asymptotes is not required in this question

(3)

b. with reference to your sketch, state the number of roots of the equation

$$9 - 2x - x^2 = \frac{2}{x}$$
$$10 - 2x - x^2 = \frac{2}{x} + 1$$

3 intersections, 3 solutions.

(2)

Given that one of the roots is $x = 2$

c. Find the other two roots.

$$9x - 2x^2 - x^3 = 2$$
$$x^3 + 2x^2 - 9x + 2 = 0$$
$$(x - 2)(x^2 + 4x - 1) = 0$$
$$(x + 2)^2 - 5 = 0$$
$$x = -2 \pm \sqrt{5}$$

(4)

10.

$\mathrm{f}(x) = x\mathrm{e}^{\cos 2x} - 4$

a. Show that if $x = a$ is a root of $\mathrm{f}'(x) = 0$

then $a = \dfrac{1}{2\sin 2a}$

$$\mathrm{f}'(x) = (1)\mathrm{e}^{\cos 2x} + x\mathrm{e}^{\cos 2x}(-2\sin 2x)$$
$$0 = \mathrm{e}^{\cos 2x}(1 - 2x\sin 2x)$$
$$\mathrm{e}^{\cos 2x} \neq 0$$
$$1 = 2x\sin 2x$$
$$x = \frac{1}{2\sin 2x}$$

(3)

b. Use the iterative formula $x_{n+1} = \dfrac{1}{2\sin 2x_n}$ with $x_0 = 0.6$ to find the values of $x_1,\ x_2$ and x_3 correct to 4 decimal places.

$$x_1 = 0.5365$$
$$x_2 = 0.5691$$
$$x_3 = 0.5507$$

(2)

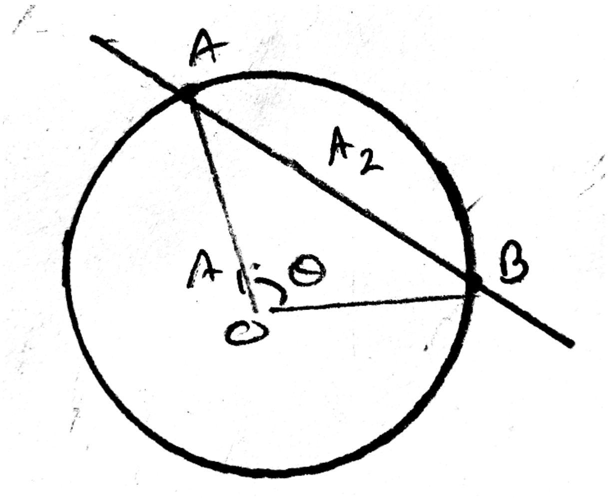
c. Prove that $a = 0.557$ is correct to 3 decimal places.

$$\mathrm{f}'(0.5565) = 0.0039$$
$$\mathrm{f}'(0.5575) = -0.0028$$

change signs and $\mathrm{f}(x)$ is continuous in the interval, the $x = 0.557$ is the root of $\mathrm{f}'(x) = 0$ correct to 3 decimal places.

(3)

11.



A chord AB divides a circle into two arcas A_1 and A_2

Given $\angle AOB = \theta$ and $A_2 = \frac{1}{3}A_1$, show that

$$2\theta = \pi + 2\sin\theta$$

$$\begin{aligned}
A_2 &= \frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta \\
A_1 &= \frac{1}{2}r^2(2\pi - \theta) + \frac{1}{2}r^2\sin\theta \\
\frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta &= \frac{1}{6}\left[r^2(2\pi - \theta) + r^2\sin\theta\right] \\
6\theta - 6\sin\theta &= 4\pi - 2\theta + 2\sin\theta \\
8\theta &= 4\pi + 8\sin\theta \\
2\theta &= \pi + 2\sin\theta
\end{aligned}$$

(5)

12.

a. Express $5\sin x - 2\cos x$ in the form $R\sin(x-\alpha)$, where $R > 0,\ 0 < \alpha < 90^\circ$

$$\begin{aligned} 5\sin x - 2\cos x &= R\sin x\cos\alpha - R\cos x\sin\alpha \\ 5 &= R\cos\alpha \\ 2 &= R\sin\alpha \\ 5^2 + 2^2 &= R^2\left(\cos^2\alpha + \sin^2\alpha\right) \\ R &= \sqrt{29} \\ \tan\alpha &= \frac{2}{5} \\ \alpha &= 21.8^\circ\ (1\text{dp}) \end{aligned}$$

(3)

b. Solve in the range $0 \leqslant \theta \leqslant 360^\circ$

i. $5\sin\theta - 2\cos\theta - 3 = 0$

$$\begin{aligned} \sqrt{29}\sin(\theta - 21.8) &= 3 \\ \theta - 21.8 &= \sin^{-1}\frac{3}{\sqrt{29}} \\ \theta - 21.8 &= 33.85, 146.15 \\ \theta &= 55.7,\ 167.9^\circ\ (1\text{dp}) \end{aligned}$$

(3)

ii. $5\tan\theta - 3\sec\theta = 2$

$$\begin{aligned} \frac{5\sin\theta}{\cos\theta} - \frac{3}{\cos\theta} &= 2 \\ 5\sin\theta - 3 &= 2\cos\theta \\ 5\sin\theta - 2\cos\theta - 3 &= 0 \\ \theta &= 55.7,\ 167.9^\circ\ (1\text{dp}) \end{aligned}$$

(2)

Insert