

# భారతీయ సాంకేతిక విజ్ఞాన సంస్థ హైదరాబాద్ भारतीय प्रौद्योगिकी संस्थान हैदराबाद Indian Institute of Technology Hyderabad

# **Department of Mathematics - Courses**

# **Contents**

Credits: 1  Credits: 1  Credits: 1  Credits: 1  Credits: 1  Credits: 3  Credits: 3	5677
Credits: 1  Credits: 1  Credits: 1  Credits: 1  Credits: 3  Credits: 3	677
Credits: 1  Credits: 1  Credits: 1  Credits: 3  Credits: 3	6 7 7
Credits: 1  Credits: 3  Credits: 3	7 7
Credits: 3  Credits: 3	7 7
Credits: 3	7
Credits: 3	
	8
Credits: 1	8
Credits: 1	9
Credits: 3	9
Credits: 1	10
Credits: 1	10
Credits: 1	10
Credits: 3	11
Credits: 1	11
Credits: 1	12
Credits: 3	12
Credits: 3	13
Credits: 3	13
Credits: 3	13
Credits: 3	14
Credits: 1	14
	Credits: 1  Credits: 3  Credits: 1  Credits: 1  Credits: 3  Credits: 1  Credits: 3  Credits: 3

MA3163: Computational Algebra - I Mathematical Computing with Mathematica	a Credits: 2 15
MA3310: Basic Cryptography	Credits: 3 15
MA3320: Diophantine equations	Credits: 3 15
MA3610: Variational Calculus	Credits: 1 16
MA3620: Some special functions in Mathematical Analysis	Credits: 116
MA3615: Credited Research Project I	Credits: 316
MA4715: Credited Research Project II	Credits: 316
MA4815: Credited Research Project III	Credits: 316
MA3405: Internship	Credits: 616
MA3015: Honors Project I	Credits: 316
MA4015: Honors Project II	Credits: 3 16
MA3010/MA4010: Real Analysis	Credits:317
MA2550/MA4020: Linear Algebra	Credits: 317
MA2030/MA4030: Ordinary Differential Equations	Credits: 318
MA2540/MA4040: Probability Theory	Credits: 319
MA4051: Basics of Programming	Credits: 319
MA4053: Searching in Metric Spaces	Credits: 119
MA4060: Complex Analysis	Credits: 320
MA3070/MA4070: Algebra I - Groups and Rings	Credits: 3 21
MA4080: Measure and Integration	Credits: 3 21
MA4090: Multivariable Calculus	Credits: 3 22
MA4145: Design of Experiments	Credits: 222
MA4710: Topology and its applications	Credits: 123
MA4170: Linear algebra over commutative rings	Credits: 123
MA4113: Field and coding theory	Credits: 123
MA4110: Applied Galois theory	Credits: 1 24
MA4120: Advanced linear algebra	Credits: 124
MA4210: Algebraic curves and integer factorization	Credits: 2 24
MA4043: Algebro-Geometric Methods in Data Analysis: Theory, Applications and Algorit	thms Credits: 324
MA4033: Introduction to Algebro-Geometric Data Analysis	Credits: 2 25
MA4340: Probability Theory in Finance	Credits: 325
MA4220: Geometry of complex numbers	Credits: 2 26
MA4230: Advanced calculus	Credits: 226
MA4320: Representation of finite groups	Credits: 2 26
MA2570/MA4240: Applied Statistics	Credits: 3 26

MA4310: Topics in Number Theory	Credits: 327
MA4140: Homological Algebra I	Credits: 1 27
MA4150: Homological Algebra II	Credits: 3 27
MA4133: Computational Algebra on Polynomials and Ideals	Credits: 328
MA4610: Classical results in Analysis and applications	Credits: 1 28
MA4143: Introduction to Time Series Analysis	Credits: 3 29
MA4520: Positive definite matrices	Credits: 329
MA4540: Non-parametric Inference	Credits: 2 30
MA4550: Introduction to Modern Number theory	Credits: 3 30
MA4560: Introduction to Analytic number theory	Credits: 330
MA4570: Algebraic Coding Theory	Credits: 331
MA4740: Introduction to Bayesian Statistics	Credits: 331
MA4580: Coding theory on Algebraic curves	Credits: 3 31
MA4770: Commutative Ring Theory	Credits: 3 32
MA4050: Combinatorics and Graph Theory	Credits: 3 32
MA5020: Functional Analysis	Credits: 3 33
MA5030: Partial Differential Equations	Credits: 3 34
MA5040: Topology	Credits: 3 35
MA5050: Mathematical Methods	Credits: 335
MA3060/MA5060: Numerical Analysis	Credits: 336
MA5070: Modules and Fields	Credits: 337
MA5080: Advanced programming	Credits: 338
MA5090: Sets, Logics and Boolean Algebra	Credits: 3 38
MA5315: Project I	Credits: 3 39
MA5415: Project II	Credits: 339
MA5425: Project III	Credits: 339
MA5100: Introduction to Algebraic Topology	Credits: 339
MA5110: Fourier Analysis and Applications	Credits: 340
MA5120: Numerical Linear Algebra	Credits: 340
MA5130: Theory of Computation	Credits: 3 41
MA5150: Algebraic Number Theory	Credits: 341
MA5160: An introduction to Modular forms	Credits: 3 42
MA5170: Basic introduction to Algebraic Geometry	Credits: 3 42
MA5180: Advanced Measure Theory	Credits: 343
MA5190: Hyperbolic Conservation Laws	Credits: 3 43

MA5052: Advanced Mathematical Methods	Credits: 344
MA5142: Elliptic curve cryptography	Credits: 345
MA5220: Applied Functional Analysis	Credits: 3 45
MA5140: Introduction to stochastic Processes	Credits: 346
MA5363: Numerical methods for solving ODEs	Credits: 346
MA6040: Fuzzy Logic Connectives and their Applications	Credits: 3 47
MA6050: Wavelets and applications	Credits: 347
MA6060: Redundant and sparse representation theory	Credits: 348
MA6070: Approximation Theory	Credits: 3 48
MA6080: Measure Theoretic Probability	Credits: 3 48
MA6090: Operator Theory	Credits: 349
MA6100: Mathematics Behind Machine Learning	Credits: 3 50
MA6110: Convex Functions and their applications	Credits: 4 50
MA6120: An introduction to Operator Algebras	Credits: 351
MA6130: Banach Space Theory	Credits: 3 51
MA6140: Compressive Sensing	Credits: 152
MA6150: Discrete dynamical systems	Credits: 3 52
MA6160: Banach Algebras	Credits: 352
MA6190: Transcendental Number Theory	Credits: 353
MA6220: Distribution Theory and Sobolev Spaces	Credits: 3 53
MA6230: An introduction to variational methods	Credits: 354
MA6240: Differential Geometry	Credits: 3 54
MA6010: Topics in Analysis	Credits: 355
MA6020: Topics in Algebra	Credits: 3 55
MA6170: Topics in Differential Equations	Credits: 3 56
MA6180: Topics in Computational Mathematics	Credits: 3 56
MA6210: Curves and surfaces	Credits: 357
MA6260: Algebraic geometry I	Credits: 357
MA6270: Algebraic geometry II	Credits: 3 57
MA6116: Commutative Algebra	Credits: 358
MA6126: Combinatorial Commutative Algebra	Credits: 3 58
MA7020: Commutative Algebra II	Credits: 3 59
MA7040: Differential Topology	Credits: 3 59
MA7140: Statistical Reliability Theory	Credits: 3 59
MA5240: Mathematical Introduction to Elliptic Curves	Credits: 360

MA5510: Spectral Graph Theory	Credits: 360
MA50300: Algebraic Graph Theory	Credits: 361
MA5460: Applied and Computational Complex Analysis	Credits: 3 61
MA5230: Mathematical Fluid Dynamics	Credits: 3 62
MA6280: Vortex Dynamics	Credits: 3 62
MA5250: Drinfeld Modules	Credits: 3 64
MA4100: Elements of Matrix Theory	Credits: 3 64
MA4050: Finite Frame Theory	Credits: 3 65

\*

MA1000: Math Foundation Credits: 3

**Syllabus:** Statements, Logic, Proofs in Mathematics, Sets, Functions, Relations, Equivalence Relations, Partition of a Set, The Induction Principles, The Well ordering principles, Countability of Sets (finite and countable), Order Relations, Posets, Axioms of Choice.

## References:

- 1. **A. Kumar, S. Kumaresan and B. K. Sarma**: A foundation Course in Mathematics, Narosa Publishing House Pvt Ltd., 2018.
- 2. Paul R. Halmos: Naive Set Theory, Springer New York, NY, 1998.

MA1110: Calculus – I Credits: 1

**Syllabus:** Sequences and Series: Limit of a sequence, monotone and Cauchy sequences and properties of convergent sequences, examples. Infinite series, positive series, tests for convergence and divergence, integral test, alternating series, Leibnitz test. Differential Calculus: Continuity and differentiability of a function of a single variable, statement of Rolle's Theorem, Lagrange's mean value theorem and applications.

# **References:**

**1. George Thomas, Maurice Weir, and Joel Hass**: Thomas' Calculus: Early transcendentals, Pearson; 14th edition, 2018.

2. **James Stewart**: Calculus: Early Transcendentals, Cengage India Private Limited; 7th edition, 2017.

MA1130: Vector Calculus Credits: 1

**Syllabus:** Double and Triple Integrals: Calculations, Areas, Volumes, change of variables, Applications. Integrals of Vector Functions: Line integrals, Green's formula, path independence, Surface integral: definition, evaluation, Stoke's formula, Gauss-Ostrogradsky divergence theorem.

Prerequisite: MA1110

## References:

- **1. Jerrold Marsden, A.J. Tromba and Alan Weinstein:** Basic Multivariable Calculus, W.H.Freeman & Co Ltd; 3rd edition, 2001.
- 2. **Michael Corral**: Vector Calculus, Open Access Textbook, Available online at https://www.mecmath.net/VectorCalculus.pdf
- 3. **Murray R. Spiegel**: Vector Analysis and an Introduction to Tensor Analysis, McGraw-Hill, 1959.

Credits: 1

# MA1140: Elementary Linear Algebra

**Syllabus:** Vector spaces, Subspaces, basis and dimension, linear transformations, representation of transformations by Matrices, linear functionals, transpose of linear transformations, canonical forms. Linear functionals and adjoints, Bilinear forms, symmetric bilinear forms, skew symmetric bilinear forms.

## References:

- 1. Gilbert Strang: Linear Algebra and its Applications, 4th Edition. Cengage India, 2005.
- 2. **Sheldon Axler**, Linear algebra done right, Springer publications.
- 3. S. Kumaresan, Linear algebra A Geometric approach, Prentice Hall of India.
- 4. E. Kreyszig, Advanced engineering mathematics, John Wiley.

MA1150: Differential Equations Credits: 1

**Syllabus:** Ordinary Differential Equations: First order linear equations, Bernoulli's equations, Exact equations and integrating factor, Second order and Higher order linear differential equations with constant coefficients.

Prerequisite: MA1110, MA1140

### References:

- 1. **Erwin Kreyszig:** Advanced Engineering Mathematics, Wiley; 10th edition 2011.
- **2. Trench, William F.**, Elementary Differential Equations, Open Access Textbook, (2013), Earlier version by Brooks/Cole Thomson Learning, 2001.
- 3. **George F. Simmons:** Differential Equations with Applications and Historical Notes, Chapman and Hall/CRC; 3rd edition.

MA1220: Calculus – II Credits: 1

**Syllabus:** Integral Calculus: Definite Integrals as a limit of sums, Applications of integration to area, volume, surface area, Improper integrals. Functions of several variables: Continuity and differentiability, mixed partial derivatives, local maxima and minima for function of two variables, Lagrange multipliers.

Prerequisite: MA1110

# **References:**

- **1. George Thomas, Maurice Weir, and Joel Hass**: Thomas' Calculus: Early transcendentals, Pearson; 14th edition, 2018.
- 2. **James Stewart**: Calculus: Early Transcendentals, Cengage India Private Limited; 7th edition, 2017.
- **3.** Jerrold Marsden, A.J. Tromba and Alan Weinstein: Basic Multivariable Calculus, W.H.Freeman & Co Ltd; 3rd edition, 2001.

MA1230: Series of Functions Credits: 1

**Syllabus:** Functional Series: Pointwise and uniform convergence, basic aspects of Power series, Fourier series.

# **References:**

1. N. Piskunov: Differential And Integral Calculus Vol 1, 1996, Mir Publishers.

- 2. **N. Piskunov**: Differential And Integral Calculus Vol 2, 1996, Mir Publishers.
- 3. **Erwin Kreyszig**: Advanced Engineering Mathematics, 10th Edition, 2020, Wiley Publishers.
- 4. Thomas and Finney: Calculus, 2017, Addison-Wesley publishers.

# MA1240: Combinatorics

**Credits: 3** 

**Syllabus:** Mathematical induction, Pigeon-Hole principle, Elementary counting problems, Binomial and Multinomial theorem, Inclusion-exclusion principle, Recurrence relations and generating functions. Basics of Graphs, Eulerian trails, Hamiltonian cycles, Trees, Spanning trees, Bipartite graphs, Coloring of graphs, Matching and Philip Hall's theorem, Planar graphs, Euler's theorem for planar graphs. Partial orders and lattices.

# **References:**

- 1. **Miklós Bóna.** A walk through combinatorics. An introduction to enumeration and graph theory. Fourth edition. With a foreword by Richard P. Stanley. *World Scientific Publishing Co. Pte. Ltd., Hackensack, NJ,* 2017
- 2. **Richard P. Stanley.** Enumerative combinatorics. Vol. 1. With a foreword by Gian-Carlo Rota. Corrected reprint of the 1986 original. Cambridge Studies in Advanced Mathematics, 49. *Cambridge University Press, Cambridge*, 1997.
- 3. Kenneth H. Rosen. Discrete Mathematics & its Applications, 7th Edn., *Tata McGraw-Hill*, 2012.

# **MA1250: Introduction to Number Theory**

Credits: 3

**Syllabus:** Divisibility, Arithmetic functions and applications, Congruences (Chinese Remainder Theorem, Fermat/Euler theorems, Wilson's theorem, Lagrange's theorem), Primitive roots, Quadratic Reciprocity, Legendre/Jacobi symbols, Counting primes and their distribution, Zeta function and applications.

## **References:**

- 1. **Burton, David M.** Elementary number theory. Second edition. *W. C. Brown Publishers,* Dubuque, IA, 1989.
- 2. **Ireland, Kenneth; Rosen, Michael.** A classical introduction to modern number theory. Second edition. Graduate Texts in Mathematics, 84. Springer-Verlag, New York, 1990.
- 3. **Baker, Alan.** A concise introduction to the theory of numbers. *Cambridge University Press*, Cambridge, 1984.

MA1500: Math Foundation Credit: 1

**Syllabus:** Logic, Proofs in Mathematics, Sets, Functions, Relations, Equivalence Relations.

## **References:**

1. **A. Kumar, S. Kumaresan and B. K. Sarma**: A foundation Course in Mathematics, Narosa Publishing House Pvt Ltd., 2018.

Credits: 1

Credits: 1

Credits: 3

2. Paul R. Halmos: Naive Set Theory, Springer New York, NY, 1998.

# **MA1510: Introduction to Number System**

**Syllabus:** Countability of algebraic numbers, Transcendental numbers and construction of Liouville's number, Equivalence classes, construction of real numbers (using Cauchy sequences), Fermat's little theorem and using it for Miller-Rabin primality test, Wilson's theorem and Primitive root theorem.

## **References:**

- 1. **Burton, David M.** Elementary number theory. Second edition. *W. C. Brown Publishers,* Dubuque, IA, 1989.
- 2. **Ivan Niven M.** Numbers: rational and irrational. New Math. Library, 1 Random House, New York-Toronto, 1961.

# **MA2070: Introduction to Group Theory**

**Syllabus:** Symmetries of an equilateral triangle, Groups, Dihedral Groups, Symmetric Groups, Abelian Groups, Cyclic groups, Matrix Groups, Homomorphisms and Isomorphisms. Subgroups, Cosets, Equivalence relation, Lagrange's Theorem.

## **References:**

- 1. Michael Artin. Algebra. Prentice Hall, Inc., Englewood Cliffs, NJ. 1991 xviii+618.
- 2. Joseph A Gallian. Contemporary Abstract Algebra. 6th Edition. Houghton Mifflin. 2006.
- 3. John B. Fraleigh. A first course in abstract algebra. Addison-Wesley Publishing Co. 1967.

# **MA2101: Convex Optimization**

**Syllabus:** Basic of maxima and minima and convex optimization. Gradient methods, Classes of convex optimization problems. Least Squares, Convex sets and convex functions. Optimality conditions for constrained and unconstrained problems, Karush-Kuhn-Tucker Conditions, Duality

and examples. Linear programming, basics and examples. Simplex method, Interior point methods. Semidefinite programming.

## References:

- **1. Beck, Amir**. Introduction to nonlinear optimization. Theory, algorithms, and applications with MATLAB. MOS-SIAM Series on Optimization. 19. *Society for Industrial and Applied Mathematics (SIAM)* Philadelphia, PA. 2014 xii+282.
- **2. Tikhomirov, V. M.** Stories about maxima and minima. Mathematical World 1. *American Mathematical Society*, Providence, RI; 1990 xii+187
- 3. **Boyd, Stephen and Vandenberghe, Lieven**. Convex optimization. *Cambridge University Press*, Cambridge. 2004 xiv+716.
- 4. **Hiriart-Urruty, Jean-Baptiste and Lemarechal, Claude:** Convex analysis and minimization algorithms. Fundamental Principles of Mathematical Sciences. <u>305.</u> *Springer-Verlag, Berlin,* 1993. xviii+417 pp.
- 5. **Rockafellar, R. Tyrrell.** Convex Analysis. Princeton Mathematical Series, No. 28. *Princeton University Press*, Princeton, N.J. 1970 xviii+451.

Credits: 1

Credits: 1

# **MA2110: Introduction to Probability**

**Syllabus:** Sample space and events, definitions of probability, properties of probability, conditional probability. Random variables: distribution functions, discrete and continuous random variables, moments of random variables, conditional expectation, Chebyshev inequality, functions of random variables. Special Distributions: Bernoulli, Binomial, Geometric, Pascal, Poisson, Exponential, Uniform, Normal distributions, Limit Theorems: Law of large numbers.

# **References:**

- 1. Ross, Sheldon. A first course in Probability, Pearson Prentice Hall (2019).
- 2. **Chung and AitSahlia.** Elementary Probability Theory, Springer New York (NY) 2003 xiv+404pp.

# **MA2120: Transform Techniques**

**Syllabus:** Laplace and Inverse Laplace transform, linearity, Laplace transforms of Derivatives and Integrals, partial fractions, unit step function, shifting on the t-axis, periodic functions, applications of Laplace transform for solving differential equations. Fourier integral, Fourier Sine and Cosine transform, convolution, applications of Fourier transform for solving differential equations.

## References:

1. I.N. Sneddon, The Use of Integral Transforms, Tata Mc-Graw Hill (1974).

- **2. Schiff, Joel L.,** The Laplace transform: theory and applications. Springer Science & Business Media, 1999.
- 3. Dyke P. P. G., Introduction to Laplace Transform and Fourier Series, Springer.
- **4. Pinkus A. & Zafrany S.**, Fourier Series and Integral Transforms, Cambridge University Press.

Credits: 1

**Credits: 3** 

## **MA2140: Introduction to Statistics**

**Syllabus**: Fundamentals of Data: Collection, Summarization, and Visualization; Sampling and Sampling Distributions, Central Limit Theorem; Methods of Estimation, Unbiased estimators; Confidence Interval Estimation: Z-interval, t-interval; Hypothesis Testing, Types of Errors, Rejection Region Approach and p-value Approach.

Prerequisite: MA2110

## References:

- **1. Ross, S.M.** Introduction to Probability and Statistics for Engineers and Scientists *Academic Press.* 2014.
- **2. Hines, W.W., Montgomery, D.C. and Borror, D.M.G.C.M.E.** Probability and statistics in engineering. *John Wiley & Sons.* 2008.

# MA2142/MA4142: Introduction to Regression and Multivariate Analysis

**Syllabus:** Simple and multiple linear regression models. Estimation, tests and confidence regions of the

model parameters. Residual analysis, Regression diagnostics (assumptions check), Collinearity, outliers. Transformation of response variables, model selection, variable selection, stepwise regression, Sparse regression, L1 regularization (Lasso), Ridge regression.

MANOVA models, Factor Analysis, Cluster Analysis, k-means clustering, PCA.

Prerequisite: MA2140

- 1. N.R. Draper and H. Smith, Applied Regression Analysis, John Wiley and Sons (Asia) Pvt. Ltd., Series in Probability and Statistics, 2003.
- 2. D.C. Montgomery, E.A. Peck, and G.G. Vining, Introduction to Linear Regression Analysis, 5th Edition, John Wiley, NY.
- 3. Applied Multivariate Statistics R.A. Johnson and D.W. Wichern; 6th edition 2019.
- 4. Methods of Multivariate Analysis A.C. Rencher; 2nd edition 2002.

Software: R

**MA2130: Complex Variables** 

Credits: 1

**Syllabus:** Complex Functions limits, Continuity, Differentiability, analytic functions, Cauchy -Riemann equations, Laplace equations, Harmonic functions, conformal mapping, Cauchy integral theorem, Cauchy integral formula, derivations of an analytic function, Power series, Taylor series, Laurent series, zeros, singularities, residues, evaluation of real integrals.

Prerequisite: MA1110

# **References:**

- 1. **James Brown, and Ruel Churchill:** Complex Variables and Applications, McGraw Hill Higher Education; 9th edition, 2013.
- 2. Erwin Kreyszig: Advanced Engineering Mathematics, Wiley; 10th edition 2011.

**MA2150: Introduction to Metric Spaces** 

Credits: 1

**Syllabus:** Definition and examples of open balls and open sets, sequence in metric space; Cauchy sequence, convergence, bounded, dense sets, continuous functions and related properties, other topological properties.

## References:

- 1. **S. Kumaresan.** Topology of Metric Spaces. Second Edition, *Narosa Publishing house*, New Delhi, 2005. xii+152 pp. ISBN: 81-7319-656-7
- **2. Mícheál O'Searcoid.** Metric spaces. Springer Undergraduate Mathematics Series. *Springer-Verlag London,* Ltd., London, 2007. xx+304 pp

MA2233: Data Structures and Applications Lab

Credits: 3

**Syllabus:** This is a lab course for the Data Structures and Applications - ID2230.

Topics dealt with in ID2230 course, viz., Basic data types - Stacks, Queues, Trees, Dictionaries, Binary search trees, Balanced search trees, Hash tables, Heaps, Priority queues, Graphs.

Augments the above with many tailored data structures that have been necessitated in other areas of computing like query processing, nearest neighbour searches, graph kernel methods, stochastic hashing, dynamic programming, greedy algorithms, etc.

## References:

- 1. Cormen, Thomas H.; Leiserson, Charles E.; Rivest, Ronald L.; Stein, Clifford,. Introduction to Algorithms (4th ed.). MIT Press and McGraw-Hill, 2022.
- 2. **Jon M. Kleinberg, Éva Tardos,** Algorithm design. Addison-Wesley, 2006.
- Peter Yianilos: Data Structures and Algorithms for Nearest Neighbor Search in General Metric Spaces, Proceedings of the Fourth Annual ACM-SIAM Symposium on Discrete Algorithms, 1970.

## MA2580: Writing and Presenting Mathematics

**Syllabus:** The instructor may choose a book, like the ones listed below, and organize class discussions on mathematical topics and historical development. Students will then be assigned four-five formal writing assignments (of 8 to 10 pages each) related to these discussions. Students will also be made familiar with latex writing tools. Of the four, the last two assignments will be latex typed. Seminars by students will be an integral part of this course to improve the presentation skills for mathematics topics.

## **References:**

- 1. **J. Stillwell:** Mathematics and Its History, Springer New York UTM, (2013).
- 2. **W. Dunham**: Euler, The Master of Us All, Mathematical Association of America, (1999) xxviii + 185 pp.
- 3. W. Dunham: Journey Through Genius, Penguin Books; Reprint edition (1 January 1900)
- 4. **M. Aigner and M. Ziegler**: Proofs from THE BOOK, Springer Berlin, Heidelberg, (2018), viii + 326 pp.
- 5. **A. Weil**: Number Theory, An Approach Through History from Hammurapi to Legendre, Birkhäuser (2006).

# MA3050: Introduction to Lattice Theory

**Syllabus:** Posets, Diagrams, Lattices, Sublattice, Intervals, Distributive Lattices, Modular Lattices, Boolean Lattices, Boolean Algebras, Semilattice, Morphisms and Ideals, Congruence Relations, Lattice Polynomials, Universal Algebra, and Applications.

## References:

Credits: 3

Credits: 3

- **6. G. Birkhoff**: Lattice Theory; third edition, American Mathematical Society Colloquium Publications, vol. XXV, *Amer. Math. Soc.*, Providence, RI, 1967.
- 7. **George Grätzer**: Lattice Theory: Foundation; *Birkhäuser Basel*, 2011.
- 8. **Vijay K. Garg**: Introduction to Lattice Theory with Computer Science Applications; *John Wiley and Sons*, Ltd, 2015.

**Credits: 3** 

**Credits: 3** 

Credits: 1

# MA3120: Theory of Polynomials

**Syllabus:** Polynomials, factorization, Inequalities for roots, The resultant and the discriminant, Lagrange's series, Irreducibility criteria, Hilbert's irreducibility theorem, The cyclotomic polynomials, Chebyshev polynomials, Bernoulli polynomials, Hilbert's Seventeenth Problem

#### References:

- 1. Prasolov, Victor V. Polynomials. Algorithms and Computation in Mathematics, 11. Springer-Verlag, Berlin, (2010).
- 2. Barbeau, E. J. *Polynomials*. Springer-Verlag, New York, (1995).

# MA3140: Statistical Inference

**Syllabus:** Unbiasedness, Consistency, Efficiency, Fisher-Information, Cramer-Rao Lower Bound, UMVU estimators, Sufficiency, Minimal Sufficiency, Completeness; Hypothesis testing: Neyman-Pearson Lemma, UMP tests, UMP Unbiased Tests, UMP Unbiased Tests for Normal Populations, Invariant tests, Tests for Goodness of Fit, Sequential Probability Ratio test, Likelihood Ratio Tests, Confidence Interval Estimation.

## References:

- 1. **Rohatgi, V. K. and Saleh, A.M.E**. An introduction to probability and statistics. *John Wiley & Sons* 2015.
- 2. Casella, G. and Berger, R.L. Statistical inference (Vol. 2). *Pacific Grove, CA:* Duxbury. 2002.
- 3. **Lehmann, E.L. and Casella, G.** Theory of point estimation. *Springer Science & Business Media*. 2006.
- 4. **Lehmann, E.L. and Romano, J.P.** Testing statistical hypotheses. *Springer Science & Business Media*. 2006.
- 5. Kale, B.K.. A first course on parametric inference. Alpha Science Int'l Ltd. 2005.

## MA3143: Statistical Analysis using R

**Syllabus:** Data organization, Data import–export, Data production and manipulation, Graphical techniques, Conditional statements, Functions. Random variables, Distributions and simulation, Descriptive statistics, Confidence intervals and hypothesis testing, Basic regression analysis (linear and logistic), Analysis of variance.

Prerequisite: MA 2110, MA 2140

# **References:**

- 1. Pierre Lafaye de Micheaux, Rmy Drouilhet, and Benoit Liquet. The R Software: Fundamentals of Programming and Statistical Analysis. Springer Publishing Company, Incorporated, 1st edition, 2016.
- **2. D. M. Venables, W. N. & Smith**. *An Introduction to R. Development Core Team*, available at: http://cran.r-project.org/doc/manuals/R-intro.pdf, 2002.

# MA3163: Computational Algebra - I Mathematical Computing with Mathematica Credits: 2

**Syllabus:** Computations in Number Theory, Calculus, Linear Algebra; and modeling, Visualization and Geometry

## References:

1. Jonathan Borwein and Matthew P. Skerritt: An introduction to modern mathematical computing. Springer Undergraduate Texts in Mathematics and Technology. Springer, New York, (2011).

Credits: 3

# MA3310: Basic Cryptography

**Syllabus:** Elliptic Curves, Weierstrass and Edwards curves, Factoring using elliptic curves, Primality testing, discrete logarithm problem, Anamalous curves, A Cryptosystem based on the Weil and Tate-Lichtenbaum pairings, Miller's algorithm, Hyperelliptic curves, divisors, Cantor's algorithm.

- 1. Washington, Lawrence C. Elliptic curves. Number theory and cryptography. Discrete Mathematics and its Applications (Boca Raton). Chapman \& Hall/CRC, Boca Raton, FL, (2008).
- 2. Blake, I. F.; Seroussi, G.; Smart, N. P. Elliptic curves in cryptography. London Mathematical Society Lecture Note Series, 265. Cambridge University Press, Cambridge, (2000).

# MA3320: Diophantine equations

**Syllabus:** Lattices, quadratic forms, algebraic numbers, class group, class numbers, Diophantine aspects of elliptic curves, analytic tools.

Credits: 3

Credits: 1

Credits: 1

## References:

- 1. **Alaca, Şaban; Williams, Kenneth S.** *Introductory algebraic number theory. Cambridge University Press, Cambridge,* (2004).
- 2. Marcus, Daniel A. Number fields. Springer, Cham, (2018).

## MA3610: Variational Calculus

**Syllabus:** Extrema of functionals, Variation of a functional and its properties, Euler's equation, Field of extremals, Sufficient conditions for the Extremum of a Functional, Conditional extrema, Moving boundary problems, Ritz method

Prerequisite: MA1220

## References:

- 1. Lev D. Elsgolc, Calculus of Variation, Dover Publications, New York, (2007).
- 2. M. L. Krasnov, G. I. Makarenko, A. I. Kiselev, *Problems and exercises in the Calculus of Variation*, MIR Publications, Moscow, (975).
- 3. I.M Gelfand and S.V. Zomin, Calculus of Variation, Dover Publication, (2000).

# MA3620: Some special functions in Mathematical Analysis

**Syllabus:** Exponential and Logarithmic function. The Trigonometric functions. The Gamma function. The characterization of Gamma function. Introduction to Fourier Series and Fourier transform.

- 1. W. Rudin, Principle of Mathematical Analysis, McGraw-Hill, 1976
- 2. T. M. Apostol, Mathematical Analysis

MA3615: Credited Research Project I Credits: 3

MA4715: Credited Research Project II Credits: 3

MA4815: Credited Research Project III Credits: 3

MA3405: Internship Credits: 6

MA3015: Honors Project I Credits: 3

MA4015: Honors Project II Credits: 3

MA3010/MA4010: Real Analysis Credits:3

**Syllabus:** Real number system: Field properties, ordered properties, completeness axiom, Archimedean property, subsets of \$\mathbb{R}\$, infimum, supremum, extended real numbers. Finite, countable and uncountable sets, decimal expansion. Sequences of real numbers, Subsequences, Monotone sequences, Limit infimum, Limit Supremum, Convergence of Sequences.

Metric spaces, limits in metric spaces. Functions of single real variable, Limits of functions, Continuity of functions, Uniform continuity, Continuity and compactness, Continuity and connectedness, Monotonic functions, Limit at infinity. Differentiation, Properties of derivatives, Chain rule, Rolle's theorem, Mean-value theorems, L'Hospital's rule, Derivatives of higher order, Taylor's theorem. Definition and existence of Riemann integral, properties, Differentiation and integration.

Revision of Series, Sequences and Series of functions, Pointwise and uniform convergence, Uniform convergence of continuous functions, Uniform convergence and differentiability, Equicontinuity, Pointwise and uniform boundedness, Ascoli's theorem, Weierstrass approximation theorem, Fourier series

- 1. W. Rudin, Principles of Mathematical Analysis, McGraw-Hill international editions (Math Series), 3rd Edition, **1976.**
- 2. S.R. Ghorpade and B.V. Limaye. A course in calculus and real analysis. Undergraduate Texts in Mathematics. Springer, New York, Springer International Ed., New Delhi, **2006.**
- 3. R.R.Goldberg, Methods of Real Analysis, Oxford & IBH Publishing Co. Pvt Ltd, 1970.

4. Kenneth A. Ross, Elements of Analysis: The Theory of Calculus, Springer Verlag, UTM, 1980.

Credits: 3

Credits: 3

# MA2550/MA4020: Linear Algebra

**Syllabus:** System of Linear Equations, Elementary Operations, Row-Reduced Echelon Matrices, Gaussian Elimination. Vector Spaces, Subspaces, Direct Sums, Bases and Dimension, Linear Maps, Rank-Nullity Theorem, The Matrix of a Linear Map, Invertibility.

Eigenvalues and Eigenvectors, The Characteristic Polynomial, Cayley-Hamilton Theorem, The Minimal Polynomial, Invariant Subspaces, Upper-Triangular Matrices, Diagonal Matrices, Diagonalizability.

Inner Products, Norms, Orthonormal Bases, Gram-Schmidt process, Schur's theorem, Orthogonal Projections and Minimization Problems, Linear Functionals and Adjoints. Self adjoint and Normal operators, The Spectral Theorem for finite dimensional operators.

Generalized Eigenvectors, Canonical Forms, Rational and Jordan canonical forms.

## **References:**

- 1. K. Hoffman and R. Kunze, "Linear Algebra", PHI Learning, Second Edition, 2009.
- Sheldon Axler, "Linear Algebra Done Right", University Press, 2010.
- 3. Gilbert strang, Linear algebra and its applications, Thomson Brooks/Cole, Fourth Edition **2006**.
- 4. Friedberg H. Stephen, Insel J. Arnold, Spence E. Lawrence, "Linear Algebra" PHI Learning, Fourth Edition **2009.**

# MA2030/MA4030: Ordinary Differential Equations

**Syllabus:** Introduction: Mathematical modeling using ODE's, Definition of Linearity, Classification of ODE's, Notion of solutions, Methods of solution for first order linear differential equations: Separation of variables, integrating factor. Second order linear differential equations: Homogeneous and non homogeneous differential equations. Series solutions.

Initial Value Problem (IVP): Notion of solutions, wellposedness of IVP in the sense of Hadmard. Some examples on unique solution, infinitely many solutions and no solution of IVP – Lipschitz continuity, Gronwall's inequality and uniqueness of the solution of IVP. Picard's existence and uniqueness theorem for IVP. Peano existence theorem. Continuous dependence of solution on

initial data. Continuation of solution and maximal interval of existence.

Linear System Theory: Reduction of nth order scalar differential into a system of n first order ODE's. Fundamental matrix solution, space of all solutions as n-dimensional vector space Transition matrix and solution of IVP. Peano-Baker series for computation of transition matrix. Autonomous systems and matrix exponential. Computation of matrix exponential for diagonal matrices, Jordan blocks and other special matrices. Solution of nonhomogeneous IVP by Duhamel's principle.

Stability Theory: Stability theory for 2 \$\times\$ 2 systems, canonical form, equilibrium points, node, center and focus. Classification of equilibrium points of nonlinear systems. Lyapunov stability, asymptotic stability and exponential stability Poincar e-Bendixson theorem, Lienard's theorem.

Boundary value problems: Introduction to boundary value problems. Regular Sturm-Liouville problems. Green's function. Existence of eigen functions. Zeros of solutions. Oscillation results. Comparison theorems

## **References:**

1. G. F. Simmons, Ordinary Differential Equations with Applications and Historical Notes. Tata McGraw Hill Edition, **2003.** 

Credits: 3

2. G.F. Simmons and S.G. Krantz, Differential Equations Theory, Technique and Practice.

# MA2540/MA4040: Probability Theory

**Syllabus:** Probability Space, Independence and dependence, Random variables and distribution functions

Random variables and joint distributions, Functions of random variables

Expectation and moments, Conditional expectation, Characteristic functions, Sequences of random variables

Modes of Convergence, Weak and Strong laws of large numbers, Central Limit Theorems.

- 1. Sheldon M. Ross, "A first course in Probability," Prentice-Hall, 6ed, 2001.
- 2. P. Meyer, "Introductory probability and statistical applications," Oxford & IBH Publishing Co. PVT Ltd, **1970**.
- 3. P. Billingsley, "Probability & Measure," Wiley, 2012.

**MA4050: Combinatorics and Graph Theory** 

**Syllabus:** 

Basics of graphs, isomorphism, trees, Minimum spanning tree, Kruskal's algorithm.

Counting spanning trees, Prufer sequence, Cayley's formula, Matrix-tree theorem (without proof). Mantel's theorem. Bipartite graphs, Matching, Hall's theorem, System of distinct representatives. Eulerian graphs, Decompositions, Graham and Pollak theorem, Veblen's theorem. Hamiltonian graphs, Ore's theorem. Planar graphs, Euler's formula and its applications. Chromatic number, Five color theorem.

**Credits: 3** 

Credits: 3

Credits: 1

Basic Counting: sum rule, product rule and inclusion-exclusion principle. Pigeon hole principle, generalized pigeonhole principle and its applications. Permutation and Combinations. Permutation and combinations with repetitions, binomial and multinomial coefficients. Weak compositions, compositions, set partitions, Stirling numbers, Bell numbers. Integer partition, Ferrer's shape, conjugate partitions. Catalan numbers. Recurrence relations. Solving linear recurrence relations with constant coefficients. Ordinary generating functions, exponential generating functions. Solving recurrence relations using generating functions.

Polya's enumeration theory. Linear Algebraic methods in combinatorics.

**References:** 

- 1. Miklos Bona, A walk through combinatorics, Fourth edition, World Scientific.
- 2. Gary Chartrand, Introductory graph theory, Dover.
- 3. Richard A. Brualdi, Introductory combinatorics, Fourth edition, Pearson

**MA4051: Basics of Programming** 

**Syllabus:** Structure of a program, Input and Output Variables and Types, Arithmetic and Relational Operators, Control Structures, Functions, Arrays and Pointers, File Handling.

**References:** 

- 1. Bruce Eckel, "Thinking in C++", Pearson Education India, 2000.
- 2. B. Kernighan & D. Ritchie, "C Programming Language", Prentice Hall, 1988.
- 3. Stanley B Lippman, "C++ Programing Primer", Addison-Wesley, 2012.

MA4053: Searching in Metric Spaces

**Syllabus:** Basics: Introduction to metric spaces - Nearest Neighbour Searches - Some applications.

Efficacy of Searching in Metric Spaces: Interplay of different properties, like triangle inequality, order-invariance, etc. on the obtained search results. Role of separable metric spaces in 1-NN search

Efficiency of Searching in Metric Spaces: Computationally efficient search algorithms in metric spaces like Burkhard–Keller Tree, Bisector Trees, Geometric Near-Neighbour Access Trees, etc.

Prerequisite: MA4010, MA2150

#### References:

- 1. **Z. Wang and A. C. Bovik**, "Mean squared error: Love it or leave it? A new look at Signal Fidelity Measures," in IEEE Signal Processing Magazine, vol. 26, no. 1, pp. 98-117, 2009.
- 2. **E Chávez, G Navarro, R Baeza-Yates, JL Marroquín**, Searching in metric spaces, ACM computing surveys (CSUR) 33 (3), 273-321, 2001.
- 3. **T. Cover and P. Hart**, "Nearest neighbor pattern classification," in IEEE Transactions on Information Theory, vol. 13, no. 1, pp. 21-27, January 1967
- 4. **Piotr Indyk, Rajeev Motwani**, Approximate nearest neighbors: towards removing the curse of dimensionality, Proceedings of the thirtieth annual ACM symposium on Theory of computing, 1998.
- 5. **Sergey Brin**, Near Neighbor Search in Large Metric Spaces, Proceedings of the 21st International Conference on Very Large Data Bases, VLDB '95: 574–584, 1995.
- 6. **Charu C. Aggarwal, Alexander Hinneburg, Daniel A. Keim**, On the Surprising Behavior of Distance Metrics in High Dimensional Spaces. ICDT 2001: 420-434, 2001.

MA4060: Complex Analysis Credits: 3

**Syllabus:** Spherical representation of extended complex plane, Analytic Functions, Harmonic Conjugates, Elementary Functions, Cauchy Theorem and Integral Formula, Homotopic version

Linear fractional transformations, Power Series, Analytic Continuation and Taylor's theorem, Zeros of Analytic functions, Hurwitz Theorem, Maximum Modulus Theorem, Laurent's Theorem, Classification of singularities

Residue theorem and applications, Argument Principle, Theorem of Rouche, Schwarz-Christoffel Transformation.

- 1. J.W. Brown, R.V. Churchill, Complex Variables, McGraw Hill, 8th Edition, 2010.
- 2. J. B. Conway, Functions of one Complex Variables, Springer, 2nd Edition, 1978.

3. L. Ahlfors, Complex analysis, Mc Graw Hill, 1979.

**4.** A.R. Shastri, Basic Complex Analysis of one variable, Mc Millan, New Delhi, **2011.** 

MA3070/MA4070: Algebra I - Groups and Rings

**Credits: 3** 

**Syllabus:** Binary operation and its properties, Definition of Groups, Examples and basic properties. Subgroups, Coset of a subgroup, Lagrange's theorem. Cyclic groups. Normal subgroups, Quotient group. Homomorphisms, Isomorphism theorems. Permutation groups, Cayley's theorems. Direct and semidirect product of groups. Group actions and Sylow theorems.

Definition of Rings, Examples and basic properties, Zero divisors, Integral domains, Fields, Characteristic of a ring, Quotient field of an integral domain. Subrings, Ideals, Quotient rings, Isomorphism theorems. Ring of polynomials. Prime, Irreducible elements and their properties, Unique Factorization Domains, Principal Ideal Domains, and Euclidean domains. Prime ideal, Maximal ideal, Prime avoidance theorem, Chinese remainder theorem.

# **References:**

1. **D. S. Dummit and R. M. Foote**, Abstract Algebra, John Wiley & Sons Inc, 3rd Edition. 2004.

2. I. N. Herstein, Topics in Algebra, Wiley India Pvt Ltd, 2nd Edition, 2006.

3. Artin, M., Algebra, Prentice Hall of India, 1994.

**4.** Jacobson, N., Basic Algebra I, Hindustan Publishing Corporation, 2<sup>nd</sup> Edition, 1991.

MA4080: Measure and Integration

**Credits: 3** 

**Syllabus:** Sigma-algebra of measurable sets. Completion of a measure. Lebesgue Measure and its properties. Non-measurable sets.

Measurable functions and their properties. Integration and Convergence theorems. Lebesgue integral, Functions of bounded variation and absolutely continuous functions. Fundamental Theorem of Calculus for Lebesgue Integrals.

Product measure spaces, Fubini's theorem. L^p-spaces, duals of L^p spaces. Riesz Representation Theorem for C([a,b]).

Prerequisite: MA4010

References:

1. H. L. Royden, Real analysis. Third edition. Macmillan Publishing Company, New York,

1988.

2. W. Rudin, Real and complex analysis. Third edition. McGraw-Hill Book Co., New York,

1987.

3. G. B. Folland, Real Analysis; Modern Techniques and Their Applications (Second Edition)

4. K. R. Parthasarathy, Introduction to Probability and Measure, TRIM Series, Vol. 33,

Hindustan book agency, New Delhi, 2005.

**5.** Krishna B.Athreya and S. Lahiri, Measure theory and probability theory. Springer Texts in

Statistics, Springer Verlag, 2006.

MA4090: Multivariable Calculus

Credits: 3

Syllabus: Functions of several-variables, Directional derivative, Partial derivative, Total derivative, Jacobian, Chain rule and Mean-value theorems, Interchange of the order of differentiation, Higher derivatives, Taylor's theorem, Inverse mapping theorem, Implicit function theorem, Extremum problems, Extremum problems with constraints, Lagrange's multiplier

method.

Multiple integrals, Properties of integrals, Existence of integrals, iterated integrals, change of variables. Curl, Gradient, div, Laplacian cylindrical and spherical coordinate, line integrals,

surface integrals, Theorem of Green, Gauss and Stokes.

Prerequisite: MA4010

**References:** 

1. Apostol T.M., Mathematical Analysis; Narosa Book Distributors Pvt Ltd, 2000.

2. Jerrold E. Marsden, Anthony Tromba, Alan Weinstein, Basic multivariable analysis,

Springer Verlag 1993.

3. Ghorpade, Sudhir R.; Limaye, Balmohan V. A course in multivariable calculus and

analysis. Undergraduate Texts in Mathematics. Springer, New York, Springer International

Edition, New Delhi, 2010.

4. Fleming, Wendell Functions of several variables. Second edition. Undergraduate Texts in

Mathematics. Springer-Verlag, New York-Heidelberg, 1977.

5. J.E. Marsden, A. Tromba, and A.Weinstein, Basic Multivariable Calculus, Springer-Verlag,

1992.

# **MA4145: Design of Experiments**

**Syllabus:** Linear Models, One-way and two-way classification models. Standard designs such as CRD, RBD, LSD, BIBD. Confounding. Fractional factorial deigns. A brief introduction to mixed effects models. Cross-over and cluster designs. Response surface methodology.

Prerequisite: MA2110, MA2140

## **References:**

- 1. Kshirsagar, A. M., A First Course in Linear Models. Marcel Dekker, 1983
- 2. Montgomery, D. C., Design and Analysis of Experiments. 3rd Ed., John Wiley and Sons, 1991

# MA4710: Topology and its applications

**Syllabus:** Topological spaces, quotient topology, separation axioms, connectedness and compactness. (If time permits:) Brief introduction to topological data analysis.

#### References:

- 1. Munkres, *Topology*, Pearson, (2015).
- 2. M. A. Armstrong, Basic Topology, Springer, (1997).

# MA4170: Linear algebra over commutative rings

**Syllabus:** Commutative rings, modules and their homomorphisms, sub-modules and quotient modules, tensor product.

## **References:**

- 1. S. D. Dummit and M. R. Foote, *Abstract algebra*, Wiley, (2003).
- **2.** M.F. Atiyah and I.G. Macdonald, *Introduction to commutative algebra* Levant books, (2007).

# MA4113: Field and coding theory

**Syllabus:** Field extensions, degree of a field extension, algebraic closure of a field, introduction to finite fields and coding theory.

Credits: 2

Credits: 1

Credits: 1

Credits: 1

## References:

- 1. S. D. Dummit and M. R. Foote, Abstract algebra, Wiley, (2003).
- 2. I.N. Herstein, *Topics in algebra*, Wiley, (1975)

# MA4110: Applied Galois theory

**Syllabus:** Revision of concepts from field theory, normal extensions, separable extensions, fundamental theorem of Galois theory, cyclotomic extensions, impossibility of solving quintic equations.

Credits: 1

Credits: 1

Credits: 2

# References:

- 1. S. D. Dummit and M. R. Foote, Abstract algebra, Wiley, (2003).
- **2.** I.N. Herstein, *Topics in algebra*, Wiley, (1975)

# MA4120: Advanced linear algebra

**Syllabus:** Vector spaces, multilinear maps, tensor product of vectors, exterior product, tensor algebra and exterior algebra.

## **References:**

- 1. Werner Greub, Multilinear algebra, Springer, (1978).
- 2. D.G. Northcott, *Multilinear algebra*, Cambridge University Press, (1984).

# MA4210: Algebraic curves and integer factorization

**Syllabus:** Ideals in polynomial rings, Hilbert's nullstellensatz, projective varieties, algebraic curves. elliptic curve in projective plane, integer factorization using elliptic curve.

- 1. W. Fulton, *Algebraic curves*, Addison-Wesley (1989).
- 2. M. Ried, Undergraduate Algebraic Geometry, London Mathematical Society (1989).

# MA4043: Algebro-Geometric Methods in Data Analysis: Theory, Applications and Algorithms Credits: 3

**Syllabus:** Review of Rings and Field Extensions. **Algebraic Sets** - Affine Varieties - Irreducible Components - Hilbert's Nullstellensatz - Noether Normalisation. **Union of subspaces** - Representation as a set of polynomials - Generalised PCA. **Applications**: Algebraic methods in Motion estimation and other Image Processing / Computer vision. **Bases for Ideals** - Grobner Bases - Their computation and applications.

#### References:

- 1. Klaus Hulek, Elementary Algebraic Geometry, AMS Publication, 2003.
- 2. Rene Vidal, Generalised PCA A Tutorial.
- 3. Robert J. Holt, Thomas S. Huang, Arun N. Netravali: Algebraic methods for image processing and computer vision. IEEE Trans. Image Process. 5(6): 976-986 (1996).
- 4. David A. Cox, John Little, Donal O'Shea: Ideals, varieties, and algorithms an introduction to computational algebraic geometry and commutative algebra (2. ed.). Undergraduate texts in mathematics, Springer 1997.

Credits: 2

Credits: 3

# MA4033: Introduction to Algebro-Geometric Data Analysis

**Syllabus:** Review of Rings and Field Extensions. **Algebraic Sets** - Affine Varieties - Irreducible Components - Hilbert's Nullstellensatz - Noether Normalisation. **Union of subspaces** - Representation as a set of polynomials - Generalised PCA. **Applications**: Algebraic methods in Motion estimation and other Image Processing / Computer vision.

# **References:**

- 1. 1Klaus Hulek, Elementary Algebraic Geometry, AMS Publication, 2003.
- 2. Rene Vidal, Generalised PCA A Tutorial.
- 3. Robert J. Holt, Thomas S. Huang, Arun N. Netravali: Algebraic methods for image processing and computer vision. IEEE Trans. Image Process. 5(6): 976-986 (1996).
- 4. David A. Cox, John Little, Donal O'Shea: Ideals, varieties, and algorithms an introduction to computational algebraic geometry and commutative algebra (2. ed.). Undergraduate texts in mathematics, Springer 1997.

# **MA4340: Probability Theory in Finance**

**Syllabus:** Review of basic probability theory; Money, Interest rates, Market, stochastic processes, call options, hedging and arbitrages, martingales (discrete and continuous), martingale convergence, stochastic integration, the convergence of random variables, stochastic Riemann integral, Ito integral, Ito's lemma, Black-Scholes formula.

Prerequisite: MA4040

## References:

- 1. Probability theory in Finance, by Sean Dineen, American Mathematical Society, (2011).
- 2. Stochastic differential equations, by Bernt Oksendal, Springer (2004).

# MA4220: Geometry of complex numbers

Credits: 2

**Syllabus:** Holomorphic and meromorphic functions, compact Riemann surfaces, holomorphic maps, coverings, projective space and complex projective curves

# **References:**

- 1. Raghavan Narasimhan, Compact Riemann surfaces, Birkhauser, (1992).
- 2. Rick Miranda, Algebraic curves and Riemann surfaces, AMS (GSM 5), (1995).

## MA4230: Advanced calculus

Credits: 2

**Syllabus:** Differentiation, integration, inverse function theorem, implicit function theorem, manifolds, differential forms, Stokes' theorem.

## References:

- 1. Michael Spivak, Calculus on manifolds, Westview press, (1971).
- 2. James Munkres, Analysis on manifolds, Westview press, (1997).

## MA4320: Representation of finite groups

Credits: 2

**Syllabus:** Representation of finite groups, complete reducibility, Schurs lemma, characters, projection formulae, induced representation, Frobenius reciprocity.

## **References:**

- 1. S. D. Dummit and M. R. Foote, *Abstract algebra*, Wiley, (2003).
- **2.** J-P Serre, *Linear representations of finite groups*, Springer, (1996).

MA2570/MA4240: Applied Statistics Credits: 3

**Syllabus:** Collecting, Summarizing and Visualizing Data, Sampling Distributions, Point Estimation - Method of Moments and Maximum Likelihood Estimators, Confidence Interval Estimation for Population Mean, Variance and Proportion, Hypothesis Testing for One-Sample Mean, One-Sample Variance and One-Sample Proportion, Comparing two population parameters, Chi Square Test of Independence, Linear regression foundations.

Prerequisite: MA4040

## **References:**

- 1. **Rohatgi, V.K. and Saleh, A.M.E.**, An introduction to probability and statistics. *John Wiley & Sons*. 2015.
- 2. Milton, J.S. and Arnold, J.C. Introduction to Probability and Statistics, 3rd edn. 1995.
- 3. **Ross, S.M.** Introduction to probability and statistics for engineers and scientists. *Academic Press.* 2014.
- 4. **Hines, W.W., Montgomery, D.C. and Borror, D.M.G.C.M**. Probability and statistics in engineering. *John Wiley & Sons.* 2008.
- 5. **Walpole, R.E., Myers, R.H., Myers, S.L. and Ye, K.** Probability and statistics for engineers and scientists (Vol. 5). New York: Macmillan. 1993.

**Credits: 3** 

Credits: 1

# **MA4310: Topics in Number Theory**

**Syllabus:** Basic congruences, division algorithm, quadratic reciprocity, Chinese remainder theorem, primitive roots, Fermat's little theorem, Pythagorean triplets, primality testing, arithmetic functions, prime number theorem, Riemann-zeta function.

# **References:**

- 1. Burton, David M. *Elementary number theory*. Second edition. W. C. Brown Publishers, Dubuque, IA, (1989).
- **2.** Rosen, Kenneth H. *Elementary number theory and its applications*. Fourth edition. Addison-Wesley, Reading, MA, (2000).

# MA4140: Homological Algebra I

**Syllabus:** Categories, Functors, Chain complexes, Derived functors, Left/Right Exactness, Tor and Ext. Group homology and cohomology.

- 1. Weibel, Charles, *An introduction to homological algebra, Cambridge Studies in Advanced Mathematics*. 38, Cambridge University Press, Cambridge (1994)
- 2. S. D. Dummit and M. R. Foote, Abstract algebra, Wiley, (2003).
- **3.** D. Eisenbud, Commutative Algebra with a view toward algebraic geometry, Graduate Texts in Mathematics, 150, Springer-Verlag, New York, (1995).

Credits: 3

Credits: 3

Credits: 1

**Syllabus:** Category Theory: Categories, Functors, Natural Transformations, Abelian Categories, Limits and Colimits, Adjoint Functors. Chain complexes, Homological Dimension, Spectral Sequences.

## **References:**

- 1. **Weibel, Charles A.,** An introduction to homological algebra, Cambridge Stud. Adv. Math., Cambridge University Press, Cambridge, 1994. xiv+450 pp.
- 2. **Gelfand, Sergei I.; Manin, Yuri I.**, Methods of homological algebra. Second edition, Springer Monographs in Mathematics, Springer-Verlag, Berlin, 2003. xx+372pp.

# MA4133: Computational Algebra on Polynomials and Ideals

**Syllabus:** The Division Algorithm. Polynomial Rings. Basic Operations with Monomial Ideals and Modules. Term Orderings and Leading Terms. Gr\"obner Bases of Ideals and Modules, Buchberger's Algorithm. Computation of Colon Ideals and Annihilators, Computation of Intersections of ideals. Elimination. Diophantine Systems and Integer Programming. Systems of Polynomial Equations.

## **References:**

- 1. Martin Kreuzer and Lorenzo Robbiano: Computational Commutative Algebra. Vol 1 and 2, Springer, 2000 and 2005.
- **2.** D. A. Cox, J. Little, and D. O'Shea, *Ideals, varieties, and algorithms: An introduction to computational algebraic geometry and commutative algebra.* Undergraduate Texts in Mathematics. Springer-Verlag, New York. (1997).

# MA4610: Classical results in Analysis and applications

**Syllabus:** Implicit function theorem. Inverse function theorem. Stone-Weierstrauss theorem. Banach-Stone theorem. Arzela-Ascoli theorem. Mazur-Ulam theorem.

Prerequisite: MA1110, MA1220

- 1. W. Rudin, Principle of Mathematical Analysis, McGraw-Hill, 1976
- **2.** T. M. Apostol, *Mathematical Analysis*.

Credits: 3

Credits: 3

Prerequisite: MA2110, MA2140

## **References:**

- 1. The Analysis of Time Series: An Introduction- C. Chatfield; 6th edition 2003.
- 2. Analysis of Financial Time Series R.S. Tsay; 3rd edition 2010.
- 3. Introductory Time Series with R A.V. Metcalfe, P.S.P. Cowpertwait; 1st edition 2009.

**Note:** Relevant real data studies will be explored using statistical software such as R.

#### MA4520: Positive definite matrices

**Syllabus:** Positive matrices, characterizations, some basic results, block matrices, Schur product, Monotonicity, convexity. Positive linear maps: Representations, positive maps, properties, applications, tensor product of matrices, some applications, positive maps on operator systems. Completely positive maps: Basic examples, Choi-Krauss theorem, Stinespring's theorem, Arveson's extension theorem, Schwarz inequalities, positive completions and Schur product, the numerical radius, Applications.

Prerequisite: MA4020

- 1. **Zhang, Fuzhen;** Matrix theory. Basic results and techniques. Second edition. Universitext. *Springer, New York*, 2011. xviii+399 pp.
- 2. **Bhatia, Rajendra;** Positive definite matrices. Princeton Series in Applied Mathematics. *Princeton University Press,* Princeton, NJ, 2007. x+254 pp.

**3. Bhatia, Rajendra;** Matrix analysis. Graduate Texts in Mathematics, 169. *Springer-Verlag, New York*, 1997. xii+347 pp.

MA4540: Non-parametric Inference

Credits: 2

Credits: 3

**Syllabus:** Order statistics, Tests of randomness and Goodness of fit, one-sample and paired sample procedures, general two-sample problems, linear rank tests for location and scale problems, k-sample problems, measures of association for bivariate samples, asymptotic relative efficiency, concepts of Jackknife and Bootstrap methods.

Prerequisite: MA3140

#### References:

- 1. Gibbons, J.D. and Chakraborti, S., 2011. Non-parametric statistical inference. Springer Berlin Heidelberg.
- 2. Wasserman, L., 2006. All of nonparametric statistics. Springer Science & Business Media
- 3. Tsybakov, A.B., 2004. Introduction to nonparametric estimation. Springer.

MA4550: Introduction to Modern Number theory

**Syllabus:** Divisibility, Arithmetic functions and applications, Congruences, Counting primes and their distribution, Arithmetic of quadratic extensions (units and prime factorization), Structure of units, Dirichlet L-functions, Finite fields, Equations over finite fields, Quadratic Reciprocity law.

Prerequisite: MA1250, MA4020, MA4070

## References:

- 1. **Burton, David M.** Elementary number theory. Second edition. W. C. Brown Publishers, Dubuque, IA, 1989.
- **2. Ireland, Kenneth; Rosen, Michael.** A classical introduction to modern number theory. Second edition. Graduate Texts in Mathematics, 84. *Springer-Verlag*, New York, 1990.

MA4560: Introduction to Analytic number theory

Credits: 3

**Syllabus:** Averages of Arithmetical functions, Distribution of prime numbers, Congruences, Dirichlet's theorem on primes in Arithmetic progressions, Zeta function, Dirichlet Series, Partitions.

Prerequisite: MA1250

#### References:

- 1. **Apostol, Tom M.** Introduction to analytic number theory. Undergraduate Texts in Mathematics. *Springer-Verlag*, New York-Heidelberg, 1976.
- **2. Ireland, Kenneth; Rosen, Michael.** A classical introduction to modern number theory. Second edition. Graduate Texts in Mathematics, 84. *Springer-Verlag*, New York, 1990.

**Credits: 3** 

Credits: 3

# MA4570: Algebraic Coding Theory

**Syllabus:** Group codes, Polynomial codes, Hamming codes, Finite fields and BCH codes, Error-Correcting codes, Linear codes, Cyclic codes, Classical Goppa codes, Bounds on codes, Self-dual codes, Quadratic residue codes, Maximum distance separable codes, Hadamard matrices and Hadamard codes, Codes on curves.

# **References:**

- 1. **Vermani, L. R.** Elements of algebraic coding theory. Chapman and Hall Mathematics Series. *Chapman and Hall, Ltd., London,* 1996.
- 2. **van Lint, J. H.** Introduction to coding theory. Third edition. Graduate Texts in Mathematics, 86. *Springer-Verlag, Berlin,* 1999.
- **3. van Lint, Jacobus H.**; **van der Geer, Gerard**. Introduction to coding theory and algebraic geometry. DMV Seminar, 12. *Birkhäuser Verlag, Basel*, 1988.

# MA4740: Introduction to Bayesian Statistics

**Syllabus:** Basics of point estimation, maximum likelihood estimators, hypothesis testing, likelihood ratiotests, confidence intervals, Bayes rule, prior distributions, posterior distributions, conjugateprior, techniques of posterior inference, highest posterior density interval, predictive distributions, Bayesian inference for one-parameter models: Binomial, Poisson, Normal, Bayesian inference for the mean of the multivariate normal distribution, Hierarchical models, Model-checking, posterior predictive checking, Bayes factor, Bayesian hypothesis testing, Bayesian computation, Markov Chain Monte Carlo, Gibbs sampling.

- 1. Andrew Gelman et al. (2013): Bayesian data analysis, CRC press
- 2. Peter Hoff (2009): A first course in Bayesian statistical methods, Springer
- **3.** Ghosh, Delampady, Samanta (2007): *An introduction to Bayesian analysis*, Springer science and business media

**Syllabus:** Error-Correcting codes: Linear codes and their parameters, Bounds on codes, Examples and constructions, Finite fields, Counting points on curves over finite fields. Algebraic curves: Varieties, non-singular curves, divisors, Riemann-Roch theorem. Codes on Algebraic curves: Geometric Goppa codes, algebraic geometric codes: Construction and properties, codes on higher dimensional varieties.

Credits: 3

Credits: 3

#### References:

- 1. **Stepanov, Serguei A.** Codes on algebraic curves. *Kluwer Academic/Plenum Publishers, New York,* 1999.
- 2. **van Lint, Jacobus H.**; **van der Geer, Gerard**. Introduction to coding theory and algebraic geometry. DMV Seminar, 12. *Birkhäuser Verlag, Basel*, 1988.
- 3. **Niederreiter, Harald**; **Xing, Chaoping**. Algebraic geometry in coding theory and cryptography. *Princeton University Press, Princeton, NJ*, 2009.
- **4. Walker, Judy L.** Codes and curves. Student Mathematical Library, 7. IAS/Park City Mathematical Subseries. *American Mathematical Society, Providence, RI; Institute for Advanced Study (IAS), Princeton, NJ,* 2000.

# **MA4770: Commutative Ring Theory**

**Syllabus:** A quick revision of rings, ideals, and ring homomorphisms.

Polynomial rings. Prime and maximal spectrum of a ring, Affine algebraic varieties, and subvarieties. Correspondence between geometry and algebra. Geometric interpretation of contraction and extension.

Modules and module homomorphisms, Submodules, finitely generated modules, Nakayama's lemma, Exact sequences, Tensor product of modules. Localization. Chain Conditions (A.C.C and D.C.C), Noetherian and Artinian rings. Introduction to primary decompositions. The notion of minimal and embedded primes and their geometric analog.

# Prerequisite: MA4070

- 1. **H. Matsumura** (Translated by Miles Reid), *Commutative Ring Theory*, Cambridge University Press, (2012).
- 2. N S Gopala Krishnan, Commutative Algebra, 2nd edition, Universities Press, (2016).
- 3. **M.F. Atiyah, and I.G. MacDonald**, *Introduction to Commutative Algebra*, Sarat Book House (2007).

**MA5020: Functional Analysis** 

Syllabus: Normed linear spaces. Non-compactness of the unit ball in infinite dimensional

normed linear spaces. Product and quotient spaces. Banach spaces, Hilbert spaces.

Linear maps. Boundedness and continuity. Linear isometries, linear functionals. Examples.

Hahn-Banach extension theorem, applications. Banach-Steinhaus theorem, closed graph theorem, open mapping theorem and bounded inverse theorem, Spectrum of a bounded

operator.

Gram-Schmidt orthogonalization. Bessel's inequality, Riesz-Fisher theorem. Orthonormal basis, Parseval's identity, Projection, orthogonal decomposition. Bounded linear functionals on Hilbert

spaces.

Prerequisite: MA4010, MA4020

References:

1. B.V. Limaye, Functional Analysis, Second edition, New Age International, New Delhi,

1996.

2. J. B. Conway, A Course in Functional Analysis, Second edition, Graduate Texts in

Mathematics, Vol. 96, Springer, 1990.

3. E. Kreyszig, Introductory Functional Analysis with Applications. John Wiley & Sons, 1978.

4. P. D. Lax, Functional Analysis. Wiley-Interscience, 2002.

5. B. Bollabas, Linear Analysis, An Introductory Course. Cambridge Mathematical

Textbooks, 1990.

6. S. Kesavan, Functional Analysis.

**MA5030: Partial Differential Equations** 

Credits: 3

Credits: 3

Syllabus: Basic Concepts: Definition and order of a PDE. Classification of PDEs. Examples of some important equations and their significance. Classification into hyperbolic, parabolic, and

elliptic equations, Canonical forms.

First order PDE's: Method of characteristics (Charpit's method). Existence and uniqueness

results for the Cauchy problem for quasilinear and fully non-linear equations. Breakdown of classical solutions.

Wave Equations: d'Alembert's formula, uniqueness and stability of solutions to the initial value problem for one dimensional wave equation. Parallelogram identity, domain of dependence, range of influence, finite speed of propagation, conservation of energy. Inhomogeneous equation. Duhamel's formula. Characteristic triangle. Spherical means, Hadamard's method of descent. Huygens' principle. Duhamel's principle for solutions of non-homogeneous wave equation. Uniqueness using energy method.

Laplace Equations: Green's identities, Uniqueness of solutions, Green's function and Possion's formula, Harnack's inequality, Liouville's theorem, Weak maximum principle, Mean-value property, Strong maximum principle, Analyticity of harmonic functions. Dirichlet principle, Uniqueness using energy method for Dirichlet, Neumann, and Robin(mixed) boundary value problems, Hadamard's example illustrating non-uniqueness, instability of solutions to Cauchy problem for Laplace equation.

Heat Equations: Fundamental solution, Cauchy problem for homogeneous heat equation, infinite speed of propagation, Duhamel's principle for non-homogeneous heat equation, Uniqueness using energy method for initial boundary value problem, Maximum principle and uniqueness, Ill-posedness of backward heat equation.

Supplementary Topic: Fourier Series Method: Construction of Fourier series solutions to Laplace, Heat, and Wave equations using method of separation of variables and their convergence.

Prerequisite: MA4030

## References:

- 1. John F., Partial Differential Equations, 2nd Edition, Springer-Verlag. 1981.
- 2. Evans, Lawrence C. Partial differential equations. Graduate Studies in Mathematics, 19. *American Mathematical Society, Providence, RI,* 1998.
- 3. Ian Sneddon, Elements of Partial Differential Equations, Dover Publications, 2006.
- 4. Tyn MynT, U., and Loknath Debnath: Partial Differential Equations for Scientists and Engineers, North Holland Publisher, 3rd Edition, **1987.**
- **5.** Zachmanoglou, E.C. and Thoe, D.W., Introduction to Partial Differential Equations with Applications. Dover Publications, **1987**.

MA5040: Topology Credits: 3

**Syllabus:** Definition of Topologies in terms of open sets, neighborhood system, closed sets and closure operations and their equivalence, points of accumulation, interior, exterior and boundary points.

Base and subbase of a topology, subspace, product space, quotient space, continuous, open and closed maps, homeomorphism convergence of sequence and nets.

Separation axioms, Urysohn's Lemma, Tietze extension theorem, separability.

Compactness, local compactness, sequential and countable compactness, Tychonoff's theorem, Lindelof space. One point compactification

Connectedness and local connectedness.

Urysohn's metrization theorem.

# **References:**

- 1. J. R. Munkres. Topology: a first course. Prentice-Hall, Inc., Englewood Cliffs, N. J., 1975.
- 2. J. Dugundji, Topology. Reprinting of the 1966 original. Allyn and Bacon Series in Advanced Mathematics. Allyn and Bacon, Inc., Boston, Mass.-London-Sydney, **1978.**
- 3. Joshi, K. D. Introduction to general topology. John Wiley & Sons, Inc., New York, 1983.
- **4.** Kelly, J. L. General topology. Graduate Texts in Mathematics, No. 27. Springer-Verlag, New York-Berlin, **1975.**

# **MA5050: Mathematical Methods**

**Syllabus:** Integral Transforms: Laplace transforms: Definitions - properties - Laplace transforms of some elementary functions - Convolution Theorem - Inverse Laplace transformation - Applications.

**Credits: 3** 

Fourier transforms: Definitions - Properties - Fourier transforms of some elementary functions - Convolution theorems - Fourier transform as a limit of Fourier Series - Applications to PDE.

Integral Equations: Volterra Integral Equations: Basic concepts - Relationship between Linear differential equations and Volterra integral equations - Resolvent Kernel of Volterra Integral equation - Solution of Integral equations by Resolvent Kernel - The Method of successive approximations - Convolution type equations, solution of integral differential equations with the aid of Laplace transformation.

Fredholm Integral equations: Fredholm equations of the second kind, Fundamentals - Iterated Kernels, Constructing the resolvent Kernel with the aid of iterated Kernels - Integral equations with degenerate Kernels - Characteristic numbers and eigen functions, solution of homogeneous integral equations with degenerate Kernel - non homogeneous symmetric equations - Fredholm alternative.

Calculus of Variations: Extrema of Functionals: The variation of a functional and its properties - Euler's equation - Field of extremals - sufficient conditions for the Extremum of a Functional conditional Extremum Moving boundary problems - Discontinuous problems - one sided variations - Ritz method.

#### References:

- 1. J W Brown and R V Churchill: Fourier Series and Boundary Value Problems, McGraw Hill, 8th Edition, **2011**.
- 2. A Chakraborty: Applied Integral Equations, Tata McGraw Hill, 2008.
- 3. F G Tricomi: Integral Equations, Dover Publications, 1985.

# MA3060/MA5060: Numerical Analysis

**Syllabus:** • Floating point representation of numbers, floating point arithmetic, errors, propagation of error.

Solution of nonlinear equations: Iterative methods, Fixed point iteration method, convergence of fixed point iteration, Newton-Raphson method, complex roots and Muller's method.

Credits: 3

Interpolation: Existence and uniqueness of interpolating polynomial, error of interpolation - interpolation of equally and unequally spaced data — Inverse interpolation - Hermite interpolation.

Approximation: Uniform approximation by polynomials, data fitting, Least square, uniform and Chebyshev approximations.

Solution of linear systems: Direct and iterative methods, ill-conditioned systems, Eigen values and eigen vectors: Power and Jacobi methods.

Integration: Newton-cotes closed type methods; particular cases, error analysis - Romberg integration, Gaussian quadrature; Legendre, Chebyshev formulae.

Solution of Ordinary differential equations: Initial value problems: Single step methods; Taylor's, Euler method, modified Euler method, Runge-Kutta methods, error analysis.

References:

1. S D Conte and Carl de Boor: Elementary Numerical Analysis, An Algorithmic Approach.

McGraw Hill International Edition 3rd Ed. 1980.

2. F B Hildebrand: Introduction to Numerical Analysis, Dover Publications, 2<sup>nd</sup> Ed. **2008**.

3. K. D. Atkinson: Elementary Numerical Analysis, John Wiley & Sons, 3<sup>rd</sup> Edition, 2009.

4. Burden and Faires - numerical analysis 9th Edition

MA5070: Modules and Fields

Credits: 3

**Syllabus:** Review of Rings, Modules, Free modules, Cartesian products and direct sums of modules, quotient modules, Simple and semisimple modules, isomorphism theorems. Modules over principal ideal domains and applications. Noetherian and Artinian rings/Modules, Hilbert

basis theorem. Jordan-Holder theorem. Projective/Injective modules.

Field extensions. Algebraic/transcendental elements, Algebraic extensions. Finite fields, Cyclotomic fields. Splitting field of a polynomial. Algebraic closure of a field, Uniqueness. Normal, separable, purely inseparable extensions. Primitive elements, simple extensions. Fundamental theorem of Galois theory. Solvability by radicals - Solutions of cubic and quartic polynomials, Insolvability of quintic and higher degree polynomials. Geometric constructions

**References:** 

1. D. S. Dummit and R. M. Foote, Abstract Algebra, John Wiley & Sons Inc, 3rd Edition.

2004.

C. Musili, Introduction to Rings and Modules, Narosa Publishing House, New Delhi 1994.

3. S. Lang, Algebra, 3rd Edition, Springer-Verlag (India), 2004.

4. Jacobson, N., Basic Algebra II, Hindustan Publishing Corporation, 1991.

MA5080: Advanced programming

Credits: 3

**Syllabus:** Mathematical background, Model - What to Analyze.

Abstract Data Types (ADT's), The List ADT, The Queue ADT, The Stack ADT,

Preliminaries, Binary Trees, The Search Tree ADT, Binary Search Trees, AVL Tree,

Preliminaries, Insertion Sort, Shell Sort, Merge Sort, Quick Sort,

Definitions, Topological Sort and Minimal Spanning Tree.

#### References:

- 1. Mark Allen Weiss, "Data Structures and Algorithm Analysis in C++", Pearson Education (India), 3 rd Edition, **2007**.
- 2. Jean-Paul Tremblay and Richard B. Bunt, "Introduction to Computer Science, An Algorithmic Approach", Mcgraw Hill, Second edition, **1988**.
- 3. Thomas H. Cormen, Charles E. Leiserson, Ronald L. Rivest, and Clifford Stein, "Introduction to Algorithms", MIT Press, Third edition, **2009**.

Credits: 3

# MA5090: Sets, Logics and Boolean Algebra

**Syllabus:** Sets and Relations: Types of relations, Peano Axioms and Mathematical Induction, Cardinality, Recursion.

Boolean Algebra: Partially Ordered Sets, Lattices, Subalgebras, Direct Product, Homomorphisms, Boolean Functions, Representation and Minimization of Boolean functions.

Mathematical Logic: Connectives, Normal Forms, Theory of Inference for the Statement Calculus.

### **References:**

- 1. J. P. Trembley and R. Manohar, "Discrete Mathematical Structures with Applications", Tata McGraw-Hill, **2009**.
- 2. R. L. Causey, "Logic, Sets and Recursion", 2nd Ed. Jones and Bartlett, 2010.
- 3. Singh, "Logics for Computer Science", PHI, 2004.
- 4. G. Birkhoff and S. Maclane, "A Survey of Modern Algebra", Mc Millan Publishers, 4th Ed, 1977.
- 5. S. Givant and P. Halmos, "Introduction to Boolean Algebras", Springer, 2009.

MA5315: Project I Credits: 3

Mandatory MSc Project 3rd semester (2020 onwards)

MA5415: Project II Credits: 3

Mandatory MSc Project 4th semester (2020 onwards)

MA5425: Project III Credits: 3

Optional MSc Project 4th semester (2020 onwards)

MA5100: Introduction to Algebraic Topology

**Syllabus:** Homotopy, Fundamental group, The Fundamental group of the circle, Retractions and fixed points, Application to the Fundamental Theorem of Algebra, The Borsuk-Ulam theorem, Homotopy equivalence and Deformation retractions, Fundamental group of a product of spaces, and Fundamental group the torus, Sphere, and the real projective n-space.

**Credits: 3** 

Credits: 3

Free Products of Groups, The Van Kampen Theorem, Fundamental Group of a Wedge of Circles, Definition and construction of Cell Complexes, Application to Van Kampen Theorem to Cell Complexes, Statement of the Classification Theorem for Surfaces, Fundamental groups of the closed orientable surface of genus g.

Introduction to Covering spaces, Universal Cover and its existence, Unique Lifting Property, Galois Correspondence of covering spaces and their Fundamental Groups, Representing Covering Spaces by Permutations - Deck Transformations, Group Actions, Covering Space Actions, Normal or Regular Covering Spaces.

Prerequisite: MA5040

### References:

- 1. J. R. Munkres, Topology, 2nd Edition, Pearson Publishing Inc, **2000**.
- 2. J. R. Munkres, Elements of Algebraic Topology, Westview Press, 1 996.
- 3. Hatcher, Algebraic Topology, Cambridge University Press, 2002.
- 4. M. A. Armstrong, Basic Topology, Springer International Edition, **2004.**
- 5. W. S. Massey, Algebraic Topology: An Introduction, Springer, 1977.
- 6. J. J. Rotman, An Introduction to Algebraic Topology, Springer, 1988.

MA5110: Fourier Analysis and Applications

**Syllabus:** Definition, Examples, Uniqueness of Fourier series, Convolution, Cesaro summability and Abel summability of Fourier series, Mean square convergence of Fourier series, A continuous function with divergent Fourier series. Some applications of Fourier series, The isoperimetric inequality, Weyl's equidistribution theorem.

Fourier transform on the real line and basic properties, The Schwartz space, Approximate identity using Gaussian kernel, Solution of heat equation, Fourier inversion formula, \$L^2\$-theory.

Some basic theorems of Fourier Analysis, Poisson summation formula, Heisenberg uncertainty principle, Hardy's theorem, Paley-Wiener theorem, Wiener's theorem, Shannon sampling theorem.

The class of test functions, Distributions, Convergence, differentiation and convolution of distributions, Tempered distributions, Fourier transform of a tempered distribution.

Prerequisite: MA4010

#### **References:**

- 1. Bhatia, Rajendra. Fourier series. Texts and Readings in Mathematics. Hindustan Book Agency, New Delhi, *1993*.
- 2. Chandrasekharan, Komaravolu. Classical Fourier transforms. Universitext. Springer-Verlag, Berlin, **1989**.
- 3. Katznelson, Yitzhak. An introduction to harmonic analysis. Third edition. Cambridge Mathematical Library. Cambridge University Press, Cambridge, **2004**.
- 4. Stein, Elias M.; Shakarchi, Rami. Fourier analysis. An introduction. Princeton Lectures in Analysis, 1. Princeton University Press, Princeton, NJ, *2003*.

Credits: 3

# **MA5120: Numerical Linear Algebra**

**Syllabus:** Gaussian elimination and its variants. Sensitivity of system of linear systems. QR factorization and The least squares. The singular value decomposition. Computing Eigenvalues and Eigenvectors. Iterative methods for linear systems.

Prerequisite: MA4020

- 1. Iloyd N. Trefethen, David Bau III, "Numerical Linear algebra", SIAM, 1997.
- 2. Gene H. Golub, Charles F. Van Loan, "Matrix computations", Hindustan Book Agency, 3rd edition, *2007*.

3. Alston S. Householder, The theory of matrices in Numerical Analysis", Dover 1964.

4. J. W. Demmel, "Numerical Linear Algebra", SIAM, 1996.

### MA5130: Theory of Computation

Credits: 3

**Syllabus:** Regular Languages: Finite Automata, Non-determinism, Regular Expressions, Nonregular Languages.

Context-Free Languages: Context-free Grammars, Pushdown Automata, Non-context-free Languages

The Church-Turing Thesis: Turing Machines and Variants.

Decidability: Decidable Languages, The Halting Problem.

Reducibility: Undecidable Problems, Example, Mapping Reducibility

Time Complexity: Measuring Complexity, The classes of P and NP.

#### References:

1. Michael Sipser, Theory of Computation, Cengage Learning, 2007.

2. J. E. Hopcroft, R. Motwani, J. D. Ullman, Introduction to Automata Theory, Languages, and Computation, Addison Wesley, *2000*.

3. P.Linz, Introduction To Formal Languages And Automata, Narosa Pub. 1997.

4. Gyorgy E Revesz, Introduction to Formal Languages, Mcgraw-Hill Book Co., 1985.

# **MA5150: Algebraic Number Theory**

Credits: 3

**Syllabus:** Localisation, Integral ring extensions, Dedekind domains, discrete valuation rings, unique factorisation of ideals, ideal class groups, finiteness of class number, some class number computations, valuations and completions of number fields, Hensel's lemma, norm, trace, discriminant, different, Ramification theory of p-adic fields, Decomposition groups, Inertia groups, cyclotomic fields, Gauss sums, quadratic reciprocity, geometry of numbers, Ostrowski's theorem, Dirichlet's unit theorem.

Prerequisite: MA4070, MA5070

- 1) **Janusz, Gerald J.** Algebraic number fields. Second edition. Graduate Studies in Mathematics, 7. *American Mathematical Society, Providence, RI*, 1996.
- 2) Lang, Serge. Algebraic number theory. Second edition. Graduate Texts in Mathematics, 110. Springer-Verlag, New York, 1994.
- **3)** Marcus, Daniel A. Number fields. Universitext. *Springer-Verlag, New York-Heidelberg,* 1977.

Credits: 3

**Credits: 3** 

#### MA5160: An introduction to Modular forms

**Syllabus:** Modular group, congruence subgroups, modular forms, examples, Eisenstein series, lattice functions, Some number theoretic applications, space of modular functions, expansions at infinity, zeroes and poles using contour integrals, Hecke operators, Theta functions, Atkin-Lehner theory, Petersson inner product, Eigenforms, L-functions and some properties, relation between Modular forms and Elliptic curves.

Prerequisite: MA4060, MA4070

# **References:**

- 1) **Diamond, Fred; Shurman, Jerry**. A first course in modular forms. Graduate Texts in Mathematics, 228. *Springer-Verlag, New York*, 2005.
- 2) **Miyake, Toshitsune**. Modular forms. Translated from the Japanese by Yoshitaka Maeda. *Springer-Verlag, Berlin,* 1989.
- 3) **Serre, J.-P.** A course in arithmetic. Translated from the French. Graduate Texts in Mathematics, No. 7. *Springer-Verlag, New York-Heidelberg*, 1973.

# MA5170: Basic introduction to Algebraic Geometry

**Syllabus:** Algebraic curves in the plane, Singular points and tangent lines, local rings, intersection multiplicities, Bezout's theorem for plane curves, Max Noether's theorem and some of its applications. Affine spaces, Projective spaces, Affine and projective varieties, coordinate rings, morphisms and rational maps, local ring of a point, function fields, dimension of a variety, Zariski's main theorem.

Prerequisite: MA4070, MA5070

- 1) **Shafarevich, Igor R.** Basic algebraic geometry. 1. Varieties in projective space. Third edition. Translated from the 2007 third Russian edition. *Springer, Heidelberg*, 2013.
- 2) **Musili, C.** Algebraic geometry for beginners. Texts and Readings in Mathematics, 20. *Hindustan Book Agency, New Delhi,* 2001.
- 3) **Smith, Karen E.**; **Kahanpää, Lauri**; **Kekäläinen, Pekka**; **Traves, William**. An invitation to algebraic geometry. Universitext. *Springer-Verlag, New York,* 2000.

4) **Abhyankar, Shreeram S.** Algebraic geometry for scientists and engineers. Mathematical Surveys and Monographs, 35. *American Mathematical Society, Providence, RI,* 1990.

5) **Daniel Bump**, Algebraic Geometry, World Scientific Publishing Co., Inc., River Edge, NJ, 1998. x+218 pp.

# **MA5180: Advanced Measure Theory**

Credits: 3

**Syllabus:** Revision on Radon-Nikodym Theorem, Radon-Nikodym derivative and their applications.

Complex measure and its various properties, Complex analogue of Radon-Nikodym

Theorem. Dual of  $C^0(X)$ , the space of all complex valued continuous functions vanishing at infinity on a locally compact Hausdorff X.

A revision on the spaces  $L^p(\mu)$  for a  $\sinh \pi$  finite measure  $\mu$ . Dual of  $\mu$ . Dense subclasses of  $\mu$ .

Modes of convergence: pointwise convergence, convergence in measure, convergence almost uniformly. Egoroff's Theorem.

Fundamental Theorem of Calculus for Lebesgue Integrals. Derivative of an integral.

Derivative of a measure: The Lebesgue Differentiation Theorem. Functions of Bounded Variation and Rectifiable curves in the plane. Absolutely continuous functions.

Prerequisite: MA5030

### References:

- 1. Sheldon Axler: Measure Integration and Real Analysis, GTM 282, Springer, Cham, 2020, xviii+411 pp.
- 2. J. Diestel and J. Uhl: Vector Measures, Math. Surveys No 15, American Mathematical Society, Providence, RI, 1977, xiii+322 pp
- 3. Walter Rudin: Real and Complex Analysis, McGraw-Hill Book Co., New York, 1987, xiv+416 pp.

# MA5190: Hyperbolic Conservation Laws

Credits: 3

**Syllabus:** Review: Quasi-linear PDE, Cauchy problem, higher order PDE, classification, wave equation, heat equation, Laplace equation. Introduction to non-linear waves: 1-D linear equation, basic non-linear equations, expansion wave, centered expansion wave, breaking and

examples. Shock waves, discontinuous shocks, equal area rule, asymptotic behavior, shock structure, Burgers equation, Thomas equation. Second order systems: the equations of shallow water theory, method of characteristics, waves on a sloping beach, linear and nonlinear theory, conservation equations and boundary value problems, exact solutions for certain nonlinear equations.

Prerequisite: MA4080, MA5030

#### References:

1. Whitham, G. B. (1974): Linear and Nonlinear Waves. Wiley-Interscience, New York.

### MA5052: Advanced Mathematical Methods

**Syllabus:** Introduction: Ordering symbols, 'O and o' notation, Asymptotic Sequence, Asymptotic Analysis, Applications.

Credits: 3

Basic Complex Analysis: Singularities of complex functions, Cauchy's residue and other important theorems, Jordan's lemma, Plemlj formulae.

Series Solution: Singular points – classification, Properties near ordinary and regular singular points, Frobenius solution for ordinary differential equations, Behaviour near irregular singular points, Method of dominant balance and some special functions: Airy functions, Gamma Function.

Matched Expansions, Boundary Layer Theory: Regular and singular perturbation theory, uniform approximations, Interior boundary layer analysis with examples.

Generalised Functions: Introduction, derivatives of generalised functions, applications to singular integrals.

Integral Transforms: Fourier Transform, Laplace Transform, Mellin Transform, Riemann-Lebesgue Lemma and analytic continuation of Mellin Transforms.

Asymptotic Expansion of Integrals: Use of Mellin transform for asymptotic expansion of integrals, Laplace method, stationary phase, method of steepest and decent.

Weiner-Hopf Method: Conformal mapping, critical points, Schwartz-Christoffel formula, Bilinear maps-Mobius transformation, Riemann-Hilbert problems and the Wiener-Hopf method.

Prerequisite: MA4030, MA4060, MA5050

- **1. C. M. Bender & S.A. Orszag,** Advanced Mathematical Methods for Scientists and Engineers, 1991 Springer
- 2. N. Bleistein & R.A. Handelsman, Asymptotic Expansions of Integrals. 1986, Dover

M. J. Ablowitz & A. S. Fokas Complex variables, introduction and applications, 2003, Cambridge

Credits: 3

**Credits: 3** 

# MA5142: Elliptic curve cryptography

**Syllabus:** Elliptic curves, the group law, Weierstrass and Edwards curves, Efficient computation. Integer arithmetic, Finite field arithmetic.

The Discrete logarithm problem, the Index calculus, General attacks on discrete logs, Attacks with pairings, Anamalous curves, Diffie-Hellman key exchange, Massey-Omura encryption, ElGamal public key encryption, ElGamal digital signatures.

The Digital signature algorithm, Public key scheme based on factoring, A Cryptosystem based on the Weil pairing, Factoring using elliptic curves, Primality testing. The Weil and Tate-Lichtenbaum pairings, Miller's algorithm, Hyperelliptic curves, divisors, Cantor's algorithm

Prerequisite: MA4060, MA4070

#### References:

- 1. Washington, Lawrence C. Elliptic curves. Number theory and cryptography. Discrete Mathematics and its Applications (Boca Raton). Chapman \& Hall/CRC, Boca Raton, FL, 2008
- 2. Blake, I. F.; Seroussi, G.; Smart, N. P. Elliptic curves in cryptography. London Mathematical Society Lecture Note Series, 265. *Cambridge University Press, Cambridge*, 2000.
- **3.** Hankerson, Darrel; Menezes, Alfred; Vanstone, Scott. Guide to elliptic curve cryptography. Springer Professional Computing. *Springer-Verlag, New York,* 2004.

# **MA5220: Applied Functional Analysis**

**Syllabus:** Review of normed linear spaces, Banach and Hilbert spaces. Orthogonal systems in Hilbert spaces, Representation through harmonic and nonharmonic bases, Redundant representations, Sampling theorems, Issues with under-sampling and over-sampling, Applications in signal analysis.

- 1. L. Debnath and P. Mikusinski, "Hilbert spaces with applications," 3rd ed, Elsevier Press, 2005
- 2. G. F. Simons, "Introduction to topology and modern analysis," Krieger Publications, 2003.
- 3. Ole Christensen, "An introduction to Frames and Riesz bases," Springer, 2002.
- 4. I. Daubechies, "Ten lectures on wavelets, "SIAM, 1992.

Syllabus: Basics of probability, Definition, Classification and properties, Markov processes, Gaussian process, Stationary processes, Discrete and continuous-time Markov chains, Classification of states, Limiting distribution, Poisson process, Steady-state and transient distributions, Queuing models.

Prerequisite: MA4040, MA4080

#### References:

1) Stochastic Processes- S. M. Ross, 2nd Edition 1996

2) Stochastic Processes- J. Medhi; 4th edition 2019.

MA5363: Numerical methods for solving ODEs

Credits: 3

**Syllabus:** Numerical Solution of Ordinary differential equations:

First order initial value problems: Single step methods; Explicit and implicit methods; Taylor series method, Picard's method of successive approximation, Euler method, Backward Euler methods, Modified Euler method, Euler-Cauchy (Heun) method, Runge-Kutta methods, Error, convergence and Stability analysis

Multistep methods; Explicit multistep methods; Adams-Bashforth method, Nystrm method, Implicit multistep methods; Adams-Moulton method, Milne-Simpson method, General multistep methods, Predictor-Corrector methods, Error, convergence and Stability analysis, Stiff system

Second and higher order initial value problems: Reduction to the system of first order initial value problems, Taylor series method, Runge-Kutta method of second and fourth order, Stormer's methods, Cowell's methods.

Boundary value problems: Two points boundary value problems: Shooting methods; Linear BVP, Nonlinear BVP; Secant and Newton-Raphson method, Finite difference methods; Linear BVP, Solution of Tridiagonal system, Thomas Algorithm, Nonlinear BVP; Iteration and Newton-Raphson method, Error, convergence and stability analysis, Higher order BVP.

- 1. **D. Kincaid and W. Cheney**, Numerical Analysis: Mathematics of Scientific Computing, 3rd Ed., AMS, 2002.
- 2. **K. E. Atkinson**, An Introduction to Numerical Analysis, Wiley, 1989.
- 3. **S. D. Conte and C. de Boor,** Elementary Numerical Analysis An Algorithmic Approach, McGraw-Hill, 1981.

### MA6040: Fuzzy Logic Connectives and their Applications

**Credits: 3** 

**Syllabus:** Fuzzy Logic Connectives: T-norms: Classes and their generation process, Algebraic and analytical properties, related conjunctions.

Fuzzy implications: Classes and their generation process, Algebraic and analytical properties.

Fuzzy Measures and Integrals: An Introduction.

Applications: Including but not limited to: Approximate Reasoning, Clustering and Data Analysis, Image Processing

#### References:

- 1. "Triangular Norms", by E.P. Klement, R. Mesiar and E. Pap, Trends in Logic series, Vol. 8, Kluwer Academic Publishing, 2000.
- 2. "Fuzzy Cluster Analysis", **F. Höppner, F. Klawonn, R. Kruse, T. Runkler,** Wiley, Chichester 1999.
- 3. "Handbook of Measure Theory", E. Pap, North Holland Publishers, 2002.
- 4. "Fuzzy sets and Fuzzy Logic: Theory and Applications", George J Klir and Bo Yuan, Prentice -Hall of India, 1997.
- 5. "Fuzzy Implications", M. Baczynski and B. Jayaram, Studies in Fuzziness and Soft Computing Series, Vol 231, Springer Verlag, 2008.

# MA6050: Wavelets and applications

Credits: 3

**Syllabus:** Fourier transform - Continuous wavelet transform, frames - Multiresolution analysis, discrete wavelets, - Spline, orthogonal and biorthogonal wavelets - Applications in Image processing, Numerical analysis

- 1. Daubechies, "Ten Lectures on Wavelets," CBMS-NSF Regional Conference Series in Applied Mathematics 61, SIAM Philadelphia, 1992.
- 2. S. Mallat, "A wavelet tour of signal processing," Academic Press, 1999.
- 3. C. K. chui, "An introduction to wavelets," Vol.1, Academic press, 1992.

**Syllabus:** Redundant representations, Orthogonal, nonorthogonal and frame type bases, Sparsity, Coherence, Uncertainty Principle, L1 minimization, Probabilistic and deterministic approaches, Convex and iterative methods, Applications in analog-to-digital conversion, Nyquist sampling theory, Low-rank matrix recovery, Dictionary design, Recent develop.

**Credits: 3** 

Credits: 3

Credits: 3

#### **References:**

- 1. Michael Elad, "Sparse and Redundant Representations" First edition, Springer, 2010.
- 2. Recent research papers

# **MA6070: Approximation Theory**

**Syllabus:** The Theorems of Weierstrass, Bernstein, Fejer, and Korovkin, Stone's Approximation Theorem and the Stone-Weierstrass Theorem, Some applications, Best approximation in normed spaces: some basic notions and results, Degree of uniform approximation by algebraic and trigonometric polynomials - Modulus of continuity and modulii of smoothness - Jackson's theorems - Bernstein's inequality for trigonometric polynomials - Inverse theorems for uniform trigonometric approximation, Bernstein and Markov inequalities for algebraic polynomials, Characterizations of best uniform approximants - Theorems of Collatz and Schewdt, Collatz and Kolmogorov - Haar systems and the Haar-Kolmogorov Theorems - Chebyshev's Alternation Theorem and some applications.

#### References:

- 1. R. G. Bartle, The Elements of Real Analysis
- 2. E. W. Cheney, Introduction to Approximation Theory
- 3. G. G. Lorentz, Approximation of Functions
- 4. G. Meinardus, Approximation of Functions: Theory and Numerical Methods
- 5. I. P. Natanson, Constructive Function Theory, Volume I
- 6. T. J. Rivlin, An introduction to the approximation of functions

#### MA6080: Measure Theoretic Probability

**Syllabus:** Classical Probability and Preliminaries: Discrete Probability, Conditional Probability, Expectation, Theorems on Bernoulli Trials. Basic definitions of algebraic structures, few facts about Banach Spaces; Measure Theory: Sigma Algebra, Measurable functions, Positive and Vector valued measures, Total Variation of a measure, Spaces of measures, Lebesgue Measure on R, Completion, Caratheodory's theorem,

Lebesgue Integration: Abstract Integral, Convergence theorems of Lebesgue and Levi, Fatou's Lemma, Radon-Nikodym Theorem, Modes of convergence of measurable functions; Product Spaces: Finite Products, Fubini's Theorem, Infinite Products, Kolmogorov's Extension Theorem; Independence: Random Variables, Distributions, Independent Random Variables, Weak and Strong Law of Large Numbers, Applications.

#### References:

- 1. L. Ambrosio, N. Fusco, and D. Pallara.
- 2. Functions of bounded variation and free discontinuity problems. Oxford mathematical monographs. Clarendon Press, 2000. ISBN 9780198502456.
  - URL http://books.google.com/books?id=7GUMIh6-5TYC.
- 3. R.B. Ash and C. DolÂ'ans-Dade. Probability and measure theory. Harcourt/Academic Press, 2000. ISBN 9780120652020.
  - URL http://books.google.com/books?id=GkqQoRpCO2QC.
- 4. D. Khoshnevisan. Probability. Graduate studies in mathematics. American Mathematical Society, 2007. ISBN 9780821842157.
  - URL http://books.google.co.in/books?id=KDj49iyN6GcC.
- 5. A.N. Kolmogorov. Foundations of the theory of probability. Chelsea Pub. Co., 1950. URL http://books.google.com/books?id=puRLAAAAMAAJ
- 6. G.G. Roussas. An introduction to measure-theoretic probability. Elsevier Academic Press, 2005. ISBN 9780125990226.
  - URL <a href="http://books.google.co.in/books?id=J8ZRgCNS-wcC">http://books.google.co.in/books?id=J8ZRgCNS-wcC</a>

MA6090: Operator Theory Credits: 3

**Syllabus:** Operators on Hilbert spaces: Basics of Hilbert spaces; Bounded linear operators, Adjoint of operators between Hilbert spaces; Self-adjoint, normal and unitary operators; Numerical range and numerical radius; compact operators, Hilbert-Schmidt operators. Spectral results for Hilbert space operators: Eigen spectrum, approximate eigen spectrum; Spectrum and resolvent; Spectral radius formula; Spectral mapping theorem; Riesz-Schauder theory; Spectral results for normal, self-adjoint and unitary operators; Functions of self-adjoint operators. Spectral representation of operators: Spectral theorem and singular value representation for compact self-adjoint operators; Spectral theorem for self-adjoint operators. Unbounded Operators: Basics of unbounded closed Operators in Hilbert spaces, Cayley transform, Spectral theorem for unbounded self-adjoint operators.

- 1. B. V. Limaye, Functional analysis, second edition, New Age, New Delhi, 1996. MR1427262(97k:46001
- 2. G. Bachman and L. Narici, Functional analysis, reprint of the 1966 original, Dover, Mineola, NY, 2000. MR1819613 (2001k:46001)
- 3. R. Bhatia, Notes on functional analysis, Texts and Readings in Mathematics, 50, Hindustan Book Agency, New Delhi, 2009. MR2477477 (2010a:46001)

- 4. A. W. Naylor and G. R. Sell, Linear operator theory in engineering and science, second edi-tion, Applied Mathematical Sciences, 40, Springer, New York, 1982. MR0672108 (83j:46001)
- 5. M. Reed and B. Simon, Methods of modern mathematical physics. I. Functional analysis, Academic Press, New York, 1972. MR0493419 (58 #12429a)
- 6. Reed, Michael; Simon, Barry Methods of modern mathematical physics. II. Fourier analysis, self-adjointness. Academic Press [Harcourt Brace Jovanovich, Publishers], New York-London, 1975. xv+361 pp.

Credits: 3

Credits: 4

# MA6100: Mathematics Behind Machine Learning

**Syllabus:** Data Representation: Eigenvalues - Eigenvectors - PCA - SVD - Fischer Discriminant; Functionals - Hilbert Spaces - Riesz Representation Theorem - Kernel Trick - Kernel PCA - Kernel SVM; Norm Minimization - LLE - Sparse Representation Theory - Dimensionality Reduction.

Supervised Learning: Convex Optimisation - Primal-Dual Transformations - Karush-Kuhn-Tucker Conditions - SVM; Probability and Measures - Types of Convergences - Statistical Learning Theory - VC dimension and Capacity - Some bounds.

Unsupervised Learning: Expectation Maximization - EM-based Clustering - C-means clustering - Fuzzy CM clustering; Operator Theory - Decomposition of Operators and Subspaces - Subspace Clustering.

#### References:

- 1. David C. Lay, Linear Algebra and Its Applications, Addison Wesley, 2003.
- 2. Christopher Bishop, Pattern Recognition and Machine Learning, Springer, 2007.
- 3. Schölkopf B Person and Smola AJ, Learning with Kernels: Support Vector Machines, Regularization, Optimization, and Beyond, MIT Press, Cambridge, **2002**.
- 4. G. Bachman and L. Narici, Functional Analysis, Dover Publications Inc, 2nd ed., 2003.
- 5. Stephen Boyd and Lieven Vandenberghe, Convex Optimization, Cambridge University Press, London, **2004**.

# **MA6110: Convex Functions and their applications**

**Syllabus:** Basic properties of convex functions; Convex functions on a normed linear spaces; Various notions of differentiability of a convex function on a normed linear space; Monotone operators, Asplund spaces and Radon Nikodym property; A smooth variational principle and more on Asplund spaces.

- 1. Convex functions, monotone operators and differentiability, Lecture notes in Mathematics 1364. Springer verlag R. R. Phelps.
- 2. Convex functions and their applications. A contemporary approach, CMS Books in Mathematics. Constantin Niculescu and Lars Erik Persson.
- 3. A course in optimization and best approximation, Lecture notes in Mathematics 257. Springer verlag Richard B. Holmes.
- 4. Banach space theory, Basis for linear and non-linear analysis, CMS Books in Mathematics/Ouvrages de Mathematiques de la SMC, Springer Verlag, New York, 2011. Marian Fabian, Petr Habala, Petr Hajek, Vicente Montesinos, Vaclav Zizler.

Credits: 3

Credits: 3

# MA6120: An introduction to Operator Algebras

**Syllabus:** Banach Algebras: Banach Algebras & invertible group; spectrum; multiplicative linear functionals; Gelfand transform & applications; maximal ideal spaces; Non-unital Banach Algebras.

C\*-algebras: C\*-algebras; commutative C\*-algebras; the spectral theorem and applications; polar decomposition; positive linear functional and states; The GNS Construction; non unital C\*-algebras

von Neumann Algebras: Topologies on B(H); Existence of projections; the Double Commutant Theorem; the Kaplansky density theorem; the Borel functional calculus; Abelian von Neumann algebras; the La functional Calculus; equivalence projections; Type decompositions.

#### References:

- 1. R. V. Kadison and J. R. Ringrose, Fundamentals of the theory of operator algebras. Vol. I, reprint of the 1983 original, Graduate Studies in Mathematics, 15, Amer. Math. Soc. Providence, RI, 1997.
- 2. G. J. Murphy, C\*-algebras and operator theory, Academic Press, Boston, MA, 1990.
- 3. K. H. Zhu, An introduction to operator algebras, Studies in Advanced Mathematics, CRC, Boca Raton, FL, 1993.
- 4. R. G. Douglas, Banach algebra techniques in operator theory, second edition, Graduate Texts in Mathematics, 179, Springer, New York, 1998.
- 5. W. Z\_ elazko, Banach algebras, translated from the Polish by Marcin E. Kuczma, Elsevier Publishing Co., Amsterdam, 1973.
- 6. W. Arveson, An invitation to C\*-algebras, Springer, New York, 1976.

# MA6130: Banach Space Theory

**Syllabus:** Basic properties of Banach spaces; Classical Banach spaces and their various properties; Linear operators in Banach spaces; Schauder bases; Convexity and smoothness.

#### References:

- 1. A course in Functional analysis--J. B. Conway.
- 2. An introduction to Banach space theory--Robert E. Megginson.
- 3. A short course in Banach space theory--N.L. Carothers.
- 4. Banach space theory, Basis for Linear and Nonlinear Analysis--M. Fabian, P. Habala, P. Hajek, V. Montesinos, V. Zizler.

# **MA6140: Compressive Sensing**

**Syllabus:** Nyquist Sampling Theorem, Under-determined linear systems, Classical solution techniques, IO, I1 and I2 norm minimization problems, Theoretical guarantees for sparse recovery, Greedy and Convex optimization techniques, Dictionary Learning, Applications in Signal Processing.

#### **References:**

- 1. Michael Elad, "Sparse and Redundant Representations" First edition, Springer, 2010.
- 2. Recent research papers

# MA6150: Discrete dynamical systems

**Syllabus:** Phase portraits, Topology of the Real numbers, periodic points and stable sets, Sarkovskii's theorem, Families of dynamical systems, bifurcation, The logistic function, Cantor sets and chaos, topological conjugacy. period-doubling cascade. Symbolic dynamics. Newton's method. Complex dynamics, quadratic family, Julia sets, Mandelbrot set.

Prerequisite: MA4010, MA4060, MA5040

#### References:

- 1. Holmgren, R. M., A First Course in Discrete Dynamical Systems, Springer-Verlag, 1996.
- 2. Devaney, Robert L., Introduction to Chaotic Dynamical Systems, Addison-Wesley, 1989.
- 3. Brin, M and Stuck G., Introduction to Dynamical Systems, Cambridge University Press, 2002.

# MA6160: Banach Algebras

**Syllabus:** Preliminaries on functional analysis, Banach spaces and Hilbert spaces.

Banach algebras: Definition, homomorphism, spectrum, basic properties of spectra, Gelfand-Mazur

theorem, spectral mapping theorem, group of invertible elements.

Credits: 1

Credits: 3

Credits: 3

Commutative Banach algebras and Gelfand theory: Ideals, maximal ideals and homomorphism, semi-simple Banach algebra, Gelfand topology, Gelfand transform, involutions.

Banach\$^\*\$-algebras, Gelfand-Naimark theorem, applications to non-commutative Banach algebras. A characterization of Banach\$^\*\$-algebras.

Prerequisite: MA4010, MA4020, MA4060, MA5020, MA5040

#### References:

- 1. Allan, G. R., Introduction to Banach Spaces and Algebras, Oxford Graduate Texts in Mathematics 20, Oxford University Press, 2011.
- 2. Sunder, V. S., Functional Analysis: Spectral Theory, TRIM Series, No. 13, Hindustan Book Agency, Delhi, 1997,

Credits: 3

Credits: 3

3. Rudin, W., Functional Analysis, second edition, McGraw-Hill, 1991.

# **MA6190: Transcendental Number Theory**

**Syllabus:** Irrational Numbers: Decimal representation of real numbers, repeating decimals and rational numbers, irrationality of \$k\$-th root of an integer, irrationality of \$e\$, \$\pi\$, irrationality of various trigonometric functions at rational arguments, irrationality of \$\zeta(3)\$.

Transcendental Numbers: Liouville's construction of transcendental numbers, transcendence of \$e\$ and \$\pi\$, Lindemann's theorem on algebraic independence of exponentials of algebraic numbers and its corollaries, Gelfond - Schneider theorem on transcendence of algebraic exponents of algebraic numbers and its corollaries, linear forms in logarithms - Baker's theorem with application to the Catalan's conjecture, Mahler's construction of transcendental numbers.

Prerequisite: MA4010, MA4060, MA4070, MA5070.

#### References:

- 1. Alan Baker, Transcendental Number Theory, Cambridge University Press.
- 2. Serge Lang, Introduction to Transcendental Numbers, Addison Wesley, Reading, Mass.
- 3. Michel Waldschmidt, Transcendence Methods, Queen's University Press.

# MA6220: Distribution Theory and Sobolev Spaces

**Syllabus:** Distributions: Test functions and Distributions, Convolution of Distributions, Fundamental solutions, The Fourier transforms, The Schwartz space \$\mathcal{S}\$, Tempered Distributions.

Sobolev spaces: Definition and basic properties, Approximations by smooth functions, Traces,

Sobolev inequalities, Compactness, Other spaces of functions, Dual spaces, Fractional order spaces and trace spaces.

Weak solutions of elliptic boundary value problems: Definitions of weak solutions, Existence, The Lax-Milgram theorem, Regularity, Galerkin method, Maximum principle, eigenvalue problems, Introduction to finite element methods.

#### References:

- 1. S. Kesavan, Topics in Functional Analysis and Applications
- 2. Robert A. Adams and J. F. Fournier, Sobolev Spaces
- 3. Haim Brezis, Functional Analysis, Sobolev Space and Partial Differential Equations.
- 4. W Rudin, Functional Analysis

### MA6230: An introduction to variational methods

**Syllabus**: Preliminaries: Differential calculus, The local and the global inversion theorems, Function spaces, Nemitski operators, Elliptic equations.

Credits: 3

Credits: 3

Topological methods: Bifurcation results, The Lyapnov-Schmidt reduction, Bifurcation from the simple eigenvalue. Brouwer degree and its properties, The LeraySchauder degree, Some applications to elliptic equations, The Krasnoselski bifurcation theorem, The Rabinowitz global bifurcation theorem.

Critical points and extrema: Functionals and critical points, Gradients, Existence of extrema, Differentiable manifolds, an outline, Constrained critical points, Manifolds of codimension one, Natural constraints.

Deformations and the PalaisSmale condition: Deformations of sublevels, The steepest descent flow, The PalaisSmale condition, Existence of constrained minima, The mountain pass theorem, Applications.

# **References:**

- 1. Ambrosetti and Malchiodi, Nonlinear Analysis and Semilinear Elliptic Problems
- 2. Michael Struwe, Variational Methods: Applications to nonlinear PDEs and Hamiltonian Systems.

# MA6240: Differential Geometry

**Syllabus:** Differentiable manifolds, Tensor fields, Differential forms, Riemannian metrics and induced metrics on tensor bundles, Connections and covariant differentiation, Geodesics and Curvature.

References:

1. John M. Lee, Riemannian Manifolds: An Introduction to Curvature, Springer (1999).

2. Peter Petersen, Riemannian Geometry, GTM, Springer (2016).

3. M. P. do Carmo, Riemannian Geometry.

MA6010: Topics in Analysis

**Credits: 3** 

**Syllabus:** Real Analysis: Review of real numbers, sequences and series.

Basic topology, continuity, differentiation, Riemann-Stieltjes integral, Sequence and series of functions.

Complex Analysis: Analytic functions, Harmonic conjugates, Cauchy theorems and consequences, Power series, Maximum modulus theorem, Phragmen Lindelof theorem, Singularities, Laurent series, Residues. Mobius transformations.

References:

1. W. Rudin, Principles of Mathematical Analysis, Mc-Graw Hill, 1976.

2. L. Ahlfors: Complex Analysis, 2nd ed., McGraw-Hill, New York, 1966.

3. W. Rudin, Real and Complex analysis, Tata McGraw Hill, 2009.

**4.** J.B. Conway: Functions of one Complex Variables, 2nd edition, Springer-Verlag, **1978.** 

MA6020: Topics in Algebra

Credits: 3

**Syllabus:** Review of vector spaces, bases, dimension, Linear transformations, The rational and Jordan forms, Inner product spaces, Bilinear forms. \\ Review of Group Theory, Jordan Holder theorem, Rings, Modules and Fields.

References:

1. Sheldon Axler, Linear Algebra Done Right, University Press, 2010.

2. K. Hoffman and R. Kunze, Linear Algebra, PHI Learning, Second Edition, 2009.

3. Nathan Jacobson, Basic Algebra I, Hindustan Publishing Corporation, 2<sup>nd</sup> Edition, 1991.

4. D. S. Dummit and R. M. Foote, Abstract Algebra, John Wiley & Sons Inc, 3rd Edition. **2004**.

**MA6170: Topics in Differential Equations** 

**Credits: 3** 

**Syllabus:** 

**Ordinary Differential Equations:** Existence and uniqueness of solutions of first order ODE, system of first order equations and the nth order ODE. The method of successive approximations.

Variations of solutions with respect to initial conditions and parameters. Linear Differential equations and asymptotic behaviour of the solutions of certain linear systems problem.

Linear systems with isolated singularities: Singularities of the first kind and singularities of the second kind.

**Partial Differential Equations:** First order PDE: Pfaffian differential equation, Quasi-linear PDE's, Cauchy Problem, Compatible systems, non-linear PDE's, Monge Cone Method.

Higher order PDE: Classification, canonical form, Heat equation, Wave equation, Laplace equation, Uniqueness theorems.

# **References:**

- 1. E. A. Coddington and N. Levinson: Theory of Ordinary Differential equations. Tata McGraw- Hill, 2006
- 2. I.N. Sneddon: Elements of Partial Differential Equations. Dover 2006
- 3. Fritz John: Partial Differential Equations, Springer 2011.

**MA6180: Topics in Computational Mathematics** 

Credits: 3

# **Syllabus:**

**Basics of Programming:** Structure of a Program - Variables and Data Types - Conditional Statements - Loops - Functions - Arrays.

# **Boolean Logic:**

**Propositional Logic:** Syntax of PL - Semantics of PL - Normal Forms - Some Applications - Resolution Proof Procedure - Proofs in PL - Axiomatic System of Predicate Calculus - Soundness and Completeness of PL

**First Order Logic:** Syntax and Semantics - Proofs in FL - Axiomatic System of First Order Calculus - Soundness and Completeness of FL

**Recurrence Relations:** Growth of Functions - Asymptotic Notations - The Substitution Method - The Recursion-Tree Method - The Master Method.

#### References:

- 1. The C Programming Language, Kernighan and Ritchie, PHI.
- 2. The C++ Programming Language, Bjarne Stroustrup, 4th Edition, Addison Wesley, 2013.
- 3. Logics for Computer Science, Arindama Singh, PHI, 2003.
- 4. Introduction to Algorithms, Cormen et al, PHI, 2001.

#### MA6210: Curves and surfaces

**Syllabus:** Curves in two and three dimensions, curvature and torsion for space curves, Serret-Frenet formula for space curves, Surfaces in R3, Tangent spaces and derivatives of maps between manifolds, geodesics, first fundamental form, orientation of a surface, Second fundamental form and the Gauss map.

**Credits: 3** 

**Credits: 3** 

#### **References:**

- 1. Andrew Pressley, Elementary Differential Geometry, Springer, (2010).
- 2. M. P. Do Carmo, Differential Geometry of Curves and Surfaces, Prentice Hall, (1976)

# MA6260: Algebraic geometry I

**Syllabus:** Sheaves, Schemes and morphisms, First properties of schemes, Finiteness conditions on morphisms, Separated and proper morphisms, Sheaves of modules, Coherent sheaves, Divisors, Projective morphisms, Blowing up, Differentials, Tangent and normal bundles.

- 1. Robin Hartshorne, *Algebraic Geometry*, Springer (1976).
- 2. David Mumford, Algebraic Geometry II, Hindustan Book Agency (2015).
- 3. Igor R. Shafarevich, Basic Algebraic Geometry 1 and 2, Springer (2013).
- **4.** Rick Miranda, *Algebraic Curves and Riemann Surfaces*, American Mathematical Society (1995).

MA6270: Algebraic geometry II

Syllabus: Derived Functors, Cohomology of sheaves, Cohomology of noetherian affine schemes, Cech cohomology, Cohomology of projective space, Ext groups and sheaves, Serre duality

theorem, Higher direct images of sheaves, Flat morphisms, Smooth morphisms.

References:

1. Robin Hartshorne, Algebraic Geometry, Springer (1976).

2. David Mumford, Algebraic Geometry - II, Hindustan Book Agency (2015).

**MA6116: Commutative Algebra** 

Credits: 3

**Credits: 3** 

Syllabus: Modules, ideals, prime ideals, maximal ideals. Noetherian rings; Hilbert basis theorem. Minimal primes. Localization. Polynomial rings and algebraic sets. Weak Nullstellensatz. Nilradical and Jacobson radical; strong Nullstellensatz. Integral extensions. Prime ideals in integral extensions. Noether Normalization Lemma. Krull dimension; dimension of an

affine algebra.

Prerequisite: MA4070

References:

1. M.F. Atiyah and I.G. Macdonald, Introduction to commutative algebra Levant books, (2007).

2. D. Eisenbud, The geometry of syzygies, A second course in commutative algebra and algebraic geometry, Graduate Texts in Mathematics 229 (2005).

3. S. D. Dummit and M. R. Foote, Abstract algebra, Wiley, (2003). Balwant Singh, Basic Commutative Algebra, World Scientific Publishing Co. Ltd (2011)

4. N.S Gopalakrishnan, Commutative Algebra, Orient Blackswan Private Limited - New Delhi; Second edition (2015)

MA6126: Combinatorial Commutative Algebra

**Credits: 3** 

Syllabus: Monomial ideals and simplicial complexes. The theory of Gr\"obner bases. Hilbert functions. Resolutions of monomial ideals. Multigraded Betti numbers. Cellular resolutions. Alexander duality. Toric varieties and lattice ideals.

Prerequisite: MA4070

**References:** 

1. J. Herzog and T. Hibi: Monomial ideals, Graduate Texts in Mathematics. Springer-Verlag

London, Ltd., London, (2011).

- 2. E. Miller and B. Sturmfels: *Combinatorial commutative algebra,* Graduate Texts in Mathematics. 227, Springer-Verlag, New York, (2005)
- 3. Igor R. Shafarevich: Basic Algebraic Geometry 1 and 2, Springer (2013).
- 4. M.F. Atiyah and I.G. Macdonald, *Introduction to commutative algebra* Levant books, (2007).
- 5. D. Eisenbud, The geometry of syzygies, A second course in commutative algebra and algebraic geometry, Graduate Texts in Mathematics 229 (2005).
- **6.** D. A. Cox, J. Little, and D. O'Shea, *Ideals, varieties, and algorithms: An introduction to computational algebraic geometry and commutative algebra.* Undergraduate Texts in Mathematics. Springer-Verlag, New York. (1997).

Credits: 3

Credits: 3

# MA7020: Commutative Algebra II

**Syllabus:** Regular sequences and depth: Regular sequences, Grade and depth, Depth and projective dimension, Free resolution. Torsion-free and reflexive modules, Ideal of minors, Acyclicity criterion, Graded rings and modules, The Koszul complex. Cohen-Macaulay rings: Cohen-Macaulay rings and modules, Regular rings and normal rings, complete intersections, Gorenstein rings, Injective resolution. Hilbert functions: Hilbert functions over homogeneous rings, Macaulay's theorem on Hilbert functions, Regularity, Hilbert functions over graded rings.

#### **References:**

- W. Bruns and J. Herzog, Cohen-Macaulay Rings (Cambridge Studies in Advanced Mathematics). Cambridge: Cambridge University Press. (1998). doi:10.1017/CBO9780511608681.
- **2.** D. Eisenbud, Commutative Algebra with a view toward algebraic geometry, Graduate Texts in Mathematics, 150, Springer-Verlag, New York, (1995).

# **MA7040: Differential Topology**

**Syllabus:** Differentiable manifolds and maps, Inverse and implicit function theorem, Sub-manifolds, immersions and submersions. Differential forms, Exterior differential, closed and exact forms, Poincare's lemma, Integration on manifolds, Stokes theorem. Tangent space, Vector bundles, Tangent and Cotangent bundle as a vector bundle, Vector fields, flows, Lie derivative, De-Rham cohomology.

# Prerequisite: MA4090

- **1. Brickell, F. and Clark, R. S.,** *Differentiable Manifolds,* Van Nostrand Reinhold Co., London, 1970.
- **2. Guillemin, V. and Pollack, A.,** *Differential Topology,* Prentice Hall, 1974.
- 3. Kosniowski, C., A, First Course in Algebraic Topology, Cambridge Univ. Press, 1980.

- **4. Milnor, John W.,** *Topology from the Differentiable Viewpoint,* Princeton Landmarks in Mathematics, Princeton Un
- **5.** iv. Press, 1997.
- **6.** Munkres, J. R., Elements of Algebraic Topology, Addison-Wesley, 1984.

# MA7140: Statistical Reliability Theory

**Syllabus:** Coherent Systems - structural properties and reliability, bounds on system reliability, notion of ageing and stochastic orders, parametric families of life distributions, life distributions of coherent systems, poisson process and shock models, mixture of distributions, partial orderings of life distributions, reliability bounds.

Credits: 3

Credits: 3

#### **References:**

- **1. Shaked, M. and Shanthikumar, J.G., 2007.** Stochastic Orders, Springer Science & Business Media.
- **2. Barlow, R.E. and Proschan, F., 1975.** *Statistical theory of reliability and life testing: probability models* (Vol. 1). New York: Holt, Rinehart and Winston.
- **3. Belzunce, F., Riquelme, C.M. and Mulero, J., 2015.** *An introduction to stochastic orders.* Academic Press.

# **MA5240: Mathematical Introduction to Elliptic Curves**

**Syllabus:** Plane curves, Bezout's theorem, Basic Theory of Elliptic Curves. Reduction modulo p, Torsion points. Elliptic curves over the complex numbers, Lattices and bases, Doubly periodic functions. Heights, Mordell-Weil theorem, rank of E(Q), Neron–Tate pairing, Nagell-Lutz Theorem, Elliptic curves over finite fields and local fields, Elliptic Curves and its relation with modular forms.

#### Prerequisite: MA4070

- 1. **Silverman, Joseph H.** The arithmetic of elliptic curves. Second edition. Graduate Texts in Mathematics, 106. Springer, Dordrecht, 2009.
- 2. **Husemöller, Dale**. Elliptic curves. Second edition. Graduate Texts in Mathematics, 111. Springer-Verlag, New York, 2004.
- 3. **Knapp, Anthony W. Elliptic curves**. Mathematical Notes, 40. Princeton University Press, Princeton, NJ, 1992.

**Syllabus:** Review of matrix theory: Spectral theorem for symmetric matrices, positive definite matrices, Interlacing theorem, Perron-Frobenius theory (All these results will be reviewed without proof). Adjacency matrices of graphs: Eigenvalues of some graphs, Harray's determinant formula, Sachs coefficient theorem, Inverses of trees. Bounds for the eigenvalues, Wilf's theorem, Hoffman's ration bound, Cvetkovic's inertia bounds. Regular graphs, adjacency algebra of a regular graph, Strongly regular graphs and Friendship theorem. Laplacian matrices of graphs: Eigenvalues of some graphs, Matrix-Tree theorem. Algebraic connectivity: classification of trees, Monotonicity properties of Fielder vector. Normalized Laplacian: Basic properties, Cheegar's inequality, Expander mixing lemma. Random walks on graphs: stationary distribution, mixing time. Expander graphs: Basic properties, Ramanujan graphs.

#### References:

- 1. Graphs and Matrices, Ravindra B. Bapat, TRIM series, 2014, Second edition.
- 2. Spectra of Graphs Andries E.Brouwer and Willem H. Haemers, Springer, 2012, First edition.
- 3. Algebraic Graph Theory Chris Godsil and Gordon F. Royle, Springer, 2001, First edition.
- 4. Algebraic and Spectral Graph Theory Lecture notes by Dan Speilman, Yale University. (Available in his webpage)

MA50300: Algebraic Graph Theory Credits: 3

**Syllabus:** Automorphism of graphs, vertex-transitive and edge-transitive graphs, symmetric and vertex transitive graphs. Distance transitive graphs. Distance regular graphs. Intersection arrays. Krein parameters. Bose-Mesner algebra, Association Scheme. Equitable partition and applications. Line graphs and eigenvalues: Generalized quadrangles, root systems, graphs with minimum eigenvalue at least -2. Cayley graphs: Examples and properties. Eigenvalues of Cayley graphs. Design graphs. Incidence graph. Paley graphs.

- 1. Algebraic Graph Theory –Norman Bigs, Cambridge University Press, 1993, Second edition.
- 2. Algebraic Graph Theory Chris Godsil and Gordon F. Royle, Springer, 2001, First edition.
- 3. Algebraic Combinatorics Chris Godsil, Chapman & Hall, 1993, First edition.
- 4. A Brief introduction to spectral graph theory Bogdan Nica, European Mathematical Society, 2018, First edition.

**Syllabus:** Two dimensional fluid flows, Schwarz functions, applications of conformal mapping, function theory in multiply connected domains, asymptotic evaluation of integrals, introduction to Riemann-Hilbert problems, Painleve equations.

Credits: 3

Credits: 3

#### **References:**

- 1. Ablowitz, M., & Fokas, A. (2003). Complex Variables: Introduction and Applications (2nd ed., Cambridge Texts in Applied Mathematics). Cambridge: Cambridge University Press. doi:10.1017/CBO9780511791246
- 2. Henrici, Peter, and W. R. Kenan. 1993. Applied and Computational Complex Analysis, 3 Volume Set. Nashville, TN: John Wiley & Sons.
- 3. Needham, Tristan. 1998. Visual Complex Analysis. London, England: Oxford University Press
- 4. Nehari, Zeev. 2012. Conformal Mapping. Dover Publications.
- 5. Davis, Philip J. 1978. Schwarz Function and Its Applications. Washington, D.C., DC: Mathematical Association of America.

# **MA5230: Mathematical Fluid Dynamics**

**Syllabus:** Derivation of the compressible Navier—Stokes and compressible Euler equations. Buckingham pi theorem, dimensionless quantities and their importance, particular types of flow including two dimensional flow, irrotational flow, incompressible flow, steady flow, Stokes flow. Selected exact solutions. Vorticity and Helmholtz theorems. Flow past obstacles in two and three dimensions, boundary layers. Waves in fluid dynamics, free boundary problems.

Further selection of topics among: existence and uniqueness of solutions, non-Newtonian fluid flow, numerical methods for fluid dynamics, turbulence.

- 1. **Batchelor, G.**: An Introduction to Fluid Dynamics (Cambridge Mathematical Library). Cambridge: Cambridge University Press. doi:10.1017/CBO9780511800955, (2000).
- 2. Landau, L. D., and E. M. Lifshitz: Fluid Mechanics: Volume 6. Translated by J. B. Sykes and W. H. Reid. 2nd ed. Oxford, England: Butterworth-Heinemann, 1987.
- 3. **Aris, Rutherford**: Vectors, Tensors and the Basic Equations of Fluid Mechanics. Dover Publications, 2012.
- 4. **Chorin, Alexandre J., and Jerrold E. Marsden**: A Mathematical Introduction to Fluid Mechanics. 3rd ed. New York, NY: Springer, 2013.
- 5. **Giga, Yoshikazu, and Antonin Novotny, eds.**: Handbook of Mathematical Analysis in Mechanics of Viscous Fluids 1st ed. Basel, Switzerland: Springer International Publishing, 2018.

MA6280: Vortex Dynamics Credits: 3

**Syllabus:** Navier—Stokes and Euler equations in vorticity form, notion and importance of a vortex in fluid mechanics, vortices in the classical theory of fluid dynamics, two-dimensional vortices, inviscid and viscous vortices, point vortices, compressible vortices, vortex equilibria and relations to mathematical physics, vortices on a sphere and other surfaces, stability of vortex solutions, vortices in other areas of science.

#### **References:**

- 1. **Saffman, P.**: Vortex Dynamics (Cambridge Monographs on Mechanics). Cambridge: Cambridge University Press. doi:10.1017/CBO9780511624063 (1993).
- Majda, A., & Bertozzi, A: Vorticity and Incompressible Flow (Cambridge Texts in Applied Mathematics). Cambridge: Cambridge University Press. doi:10.1017/CBO9780511613203 (2001).
- 3. Truesdell, Clifford: The Kinematics of Vorticity. Mineola, NY: Dover Publications, 2018.
- 4. **Newton, Paul K:** The N-Vortex Problem: Analytical Techniques. 1st ed. New York, NY: Springer, 2001.

MA5250: Drinfeld Modules Credits: 3

**Syllabus:** Algebraic Preliminaries, Non-Archimedean Fields, Basic Properties of Drinfeld Modules, Algebraic and Analytic Theory of Drinfeld Modules, Drinfeld Modules over Finite, Local and Global Fields.

#### References:

- **1. Papikian, Mihran.** Drinfeld modules. Graduate Texts in Mathematics, 296. Springer, Cham, 2023 (1st edition).
- **2. Dinesh S. Thakur.** Function field arithmetic, World Scientific Publishing Co., Inc., River Edge, NJ, 2004 (2nd edition).
- 3. E.-U. Gekeler, M. van der Put, M. Reversat, and J. Van Geel (eds.). Drinfeld modules, modular schemes and applications, World Scientific Publishing Co., River Edge, NJ, 1997.
- **4. Ernst-Ulrich Gekeler.** Drinfeld modular curves, Lecture Notes in Mathematics, vol. 1231, Springer-Verlag, Berlin, 1986.

# MA4100: Elements of Matrix Theory Credits: 3

**Syllabus:** Review of Linear Algebra, Block Matrices, Orthonormal basis and Discrete Fourier Transform, Unitary Matrices, Orthogonal Complements and Orthogonal Projections, Eigenvalues and similarity of matrices, Jordan Canonical Forms, Singular value decomposition.

#### References:

- 1. Matrix Mathematics, A Second Course in Linear Algebra, By Stephan Ramon Garcia, Roger A. Horn, Cambridge 2023.
- 2. Matrix Analysis and Applied Linear Algebra, Second Edition, Carl D. Meyer, SIAM.
- 3. Matrix Theory, Basic Results and Techniques, Fuzhen Zhang, Second edition, Springer 2011.

MA4050: Finite Frame Theory

Credits: 3

**Syllabus:** Review of Linear algebra, Finite dimensional spectral theory, Introduction to frames, Frames in R^2, Properties of frames, Dual and orthogonal frames, Frame operator and its decompositions.

- 1. Han, D., Kornelson, K., Larson, D., & Weber, E. Frames for undergraduates. American Mathematical Society, 2007
- 2. Christensen, Ole. Frames and Bases: An Introductory Course. Birkhäuser, 2008.
- 3. Casazza, Peter G., and Gitta Kutyniok, eds. Finite Frames: Theory and Applications. New York: Springer, 2013.
- 4. Gröchenig, Karlheinz. Foundations of Time-Frequency Analysis. Boston: Birkhäuser, 2001.
- 5. Zhang Z., and Palle Jorgensen. Frame theory in Data science. Springer, 2023.