



NATIONAL OPEN UNIVERSITY OF NIGERIA
Plot 91, Cadastral Zone, Nnamdi Azikwe Express Way, Jabi-Abuja
FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS
July Examination 2017_1

Course Code: MTH304

Course Title: Complex Analysis I

Credit Unit: 3

Time Allowed: 3 Hours

Total: 70 Marks

INSTRUCTION: ANSWER QUESTION ONE AND ANY OTHER 4 QUESTIONS

1. (a) Given $z = 2 + i$, evaluate $3z^3 - 3z^2 + 4z - 8$ **(5 marks)**
(b) Find the real number x and y such that $3x + 2iy - ix + 5y = 7 + 5i$ **(6 marks)**
(c) Express the complex number $z = 2 + 2\sqrt{3}i$ in polar form **(5 marks)**
(d) Prove the identity $\sin \theta = \frac{3}{4} \sin \theta - \frac{1}{4} \sin 3\theta$ (Hint: $\sin \theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}$) **(6 marks)**

2. (a) Evaluate each of the following using theorems on limits

(i) $(z^2 - 5z + 10)$ **(6 marks)**
(ii) $\frac{(2z+3)(z-1)}{z^2-2z+4}$ **(6 marks)**

3. Using the definition of first principle, find the derivative of the followings at the point $z = z_0$

(i) $f(z) = z^3 - 2z$ **(6 marks)**
(ii) $f(z) = \frac{1+z}{1-z}$ **(6 marks)**

4. (a) Prove that $u = e^{-x}(x \sin y - y \cos y)$ is harmonic function **(8 marks)**

(b) Given $z = x + iy$, find $u(x, y)$ and $v(x, y)$ such that $f(z) = u + iv$ of the complex function of $f(z) = 2z^2 - 3iz$ **(4 marks)**

5. (a) Expand $f(z) = \ln(1 + z)$ in a Taylor series about $z = 0$ **(6 marks)**

(b) Prove that $\sin^2 z + \cos^2 z = 1$ **(6 marks)**

6. If $z_1 = r_1(\cos \theta_1 + i \sin \theta_1)$ and $z_2 = r_2(\cos \theta_2 + i \sin \theta_2)$, prove that

(i) $z_1 z_2 = r_1 r_2 (\cos (\theta_1 + \theta_2) + i \sin (\theta_1 + \theta_2))$ **(6 marks)**

(ii) $\frac{z_1}{z_2} = \frac{r_1}{r_2} (\cos (\theta_1 - \theta_2) + i \sin (\theta_1 - \theta_2))$ **(6 marks)**