Title: Discrete-Continuous Dual Families, Reciprocal Laws, Random Summation, and Mixtures of Gaussian Distributions

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Abstract: We provide a simple way of constructing dual continuous-discrete families of distributions, where the members of the discrete family are integer-valued random variables which arise by discretization of the members of the continuous family while the members of the continuous family are the weak limits of the (scaled) members of the discrete family. In addition, we propose a new notion of discrete reciprocal distribution and explain how these concepts connect with limiting distributions that arise in the random summation scheme and mixtures of Gaussian distributions. Multiple examples, involving classical continuous and discrete distributions as well as their new discrete and continuous analogs, illustrate the theoretical constructions discussed in the paper.

In particular, we consider a special case of location-scale mixtures of Gaussian distributions where the conditional mean and variance follow Pareto Type II (Lomax) distribution. We investigate fundamental properties of this model and its potential applications, particularly in handling heavy-tailed data. The proposed conditionally Gaussian hierarchical stochastic model provides a generalization of the Laplace probability distribution, which has already demonstrated its utility in various scientific disciplines. We present the model's basic properties and delve into related computational challenges, particularly those involving the inferential aspects of the model. This is a joint work with M. Ohemeng.