ZHONGHUA SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2020
SECONDARY 4E/4N/5N

Candidate's Name	Class	Register Number
Marking Scheme		

MATHEMATICS
PAPER 2

4048/02
02 September 2020
2 hours and 30 minutes

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use paper clips, glue or correction fluid.

Answer **all** questions.

If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is **100**.

For Examiner's Use		
Marks Obtained		**100**
Marks Deducted		
Final Total	**/100**	

This question paper consists of **23** printed pages (including this cover page)

[Turn Over

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone = πrl

Surface area of a sphere = $4\pi r^2$

Volume of a cone = $\frac{1}{3}\pi r^2 h$

Volume of a sphere = $\frac{4}{3}\pi r^3$

Area of triangle ABC = $\frac{1}{2}ab\sin C$

Arc length = $r\theta$, where θ is in radians

Sector area = $\frac{1}{2}r^2\theta$, where θ is in radians

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

Standard deviation = $\sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$

1 **(a)** $p = 4q(r-2)$

(i) Evaluate p when $q = 2$ and $r = -3$.

$p = 4(2)(-3-2)$

$= -40$

Answer [1]

(ii) Express r in terms of p and q.

$p = 4qr - 8q$ M1 – expansion

$p + 8q = 4qr$

$r = \dfrac{p+8q}{4q}$ A1

Answer [2]

(b) Simplify $\dfrac{9-6x}{3-2x+3y-2xy}$.

$\dfrac{9-6x}{3-2x+3y-2xy} = \dfrac{3(3-2x)}{3-2x+y(3-2x)}$

$= \dfrac{3(3-2x)}{(1+y)(3-2x)}$ B1 – factorisation by grouping

$= \dfrac{3}{1+y}$ A1

Answer [2]

1 (c) (i) Express $7-4x+x^2$ in the form $(x-p)^2+q$.

$7-4x+x^2=(x-2)^2-(2)^2+7$

$=(x-2)^2+3$

B2

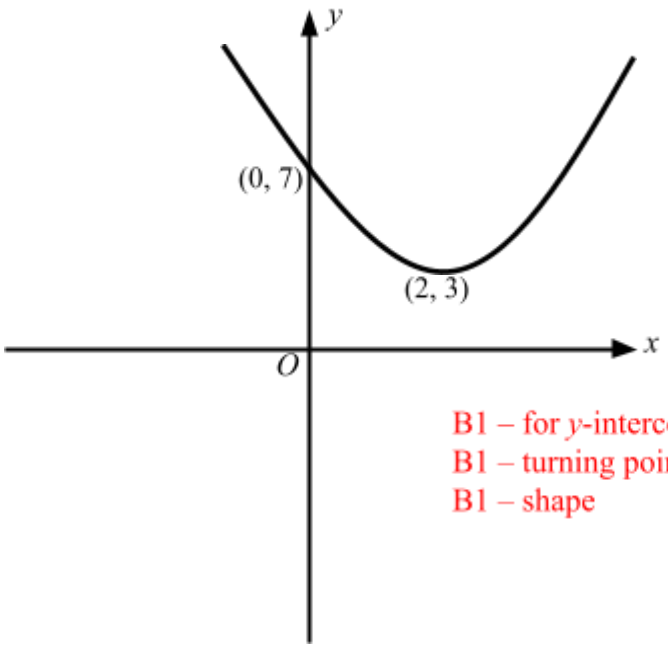
B1 – for either p or q

Answer [2]

(ii) Sketch the graph of $y=7-4x+x^2$.

Indicate clearly the coordinates of the points where the graph crosses the axes and the turning point on the graph.

Answer



B1 – for y-intercept
B1 – turning point
B1 – shape

[3]

(iii) Write the equation of line of symmetry of the graph $y=7-4x+x^2$.

$x=2$ B1

Answer [1]

1 (d) Solve $\dfrac{x}{2-x}+\dfrac{5}{3x-1}=\dfrac{5+10x}{(3x-1)(2-x)}$.

$x(3x-1)+5(2-x)=5+10x$ M1 – removing denominators

$3x^2-x+10-5x=5+10x$

$3x^2-16x+5=0$ M1 – quadratic eqn "= 0"

$(3x-1)(x-5)=0$ M1

$x=\dfrac{1}{3}$ (rej.) or $x=5$ A1

Answer $x=$ [4]

2 The table shows electricity consumption, in watt hours, in Singapore in 2018.

Households	Commerce	Industrial	Others	Total
72×10^{11}	186×10^{11}	215×10^{11}	32×10^{11}	505×10^{11}

(a) 1000 watt = 1 kilowatt

Convert the total amount of electricity consumed into kilowatt hours (kWh).
Give your answer in standard form.

B2
B1 – either 5.05 or 10^6

Answer 5.05×10^{10} kWh [2]

(b) Calculate the percentage of the total electricity that was consumed by commerce in 2018.

$$\% \text{ required} = \frac{186\times10^{11}}{505\times10^{11}}\times100\% \qquad \text{M1}$$
$$= 36.8\% \qquad \text{A1}$$

Answer % [2]

(c) The electricity consumed by households in 2008 was $p \times 10^{12}$ watt hours.

Find the ratio of electricity consumed by the household in 2008 and 2018 in terms of p.

$$\text{ratio required} = p\times10^{12} : 72\times10^{11}$$
$$= 10p\times10^{11} : 72\times10^{11} \qquad \text{M1 – both in multiple of } 10^{11} \text{ or } 10^{12}$$
$$= 5p : 36 \qquad \text{A1 – accept answers in fractions or decimals}$$

Answer : [2]

2 **(d)** The table shows the total residents in households of Singapore in 2008 and 2018.

Year	2008	2018
Total residents in households	4.84×10^6	5.64×10^6

(i) Calculate the mean amount of electricity consumed per resident per day in 2018. Give your answer correct to the nearest hundred.

$$\text{mean required} = \frac{72 \times 10^{11}}{5.64 \times 10^6 \times 365}$$
$$= 3497.5225$$
$$= 3500$$

M1 – divide by 5.64×10^6
M1 – divide by 365

A1

Answer [3]

(ii) A student, Albert, made the following claim:

> *The total residents in households of Singapore increased between 2008 and 2018.*
>
> *Thus, on average, the electricity consumed per resident per day in 2008 would be larger than in 2018.*

What is wrong with his claim?

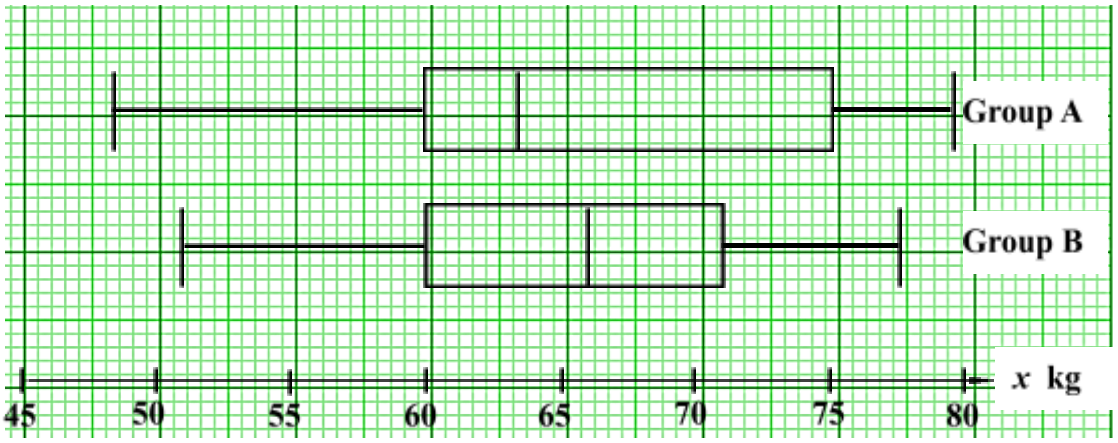
Answer

the claim assumes that the total electricity consumed in 2008 is the same as in 2018.

A1

[1]

3 Two groups of staff were surveyed.
Their weights are shown in the box-and-whisker diagrams.



(a) Find the range of masses for Group A.

Answer 31 kg B1 [1]

(b) Find the interquartile range for Group B.

interquartile range = 71 − 60

= 11 kg B1

Answer kg [1]

(c) Make two comparisons between the weights of the two groups of staff.

Answer

(1) On average, the staff in Group B are heavier because their median weight (66 kg) is larger than the staff in Group A (63.5 kg)

(2) The weights in Group B is more consistent because their interquartile range (11 kg) is smaller than the weights in Group A (15 kg) [2]

B1 − using medians to compare and interpret with values stated

B1 − using interquartile range (or range) to compare and interpret with values stated

4 **(a)** Eng bought a mobile phone at \$2020.
A week later, he decided to sell it in an online platform where every customer is entitled for 5% discount.
Eng would like to earn a profit of 10% of the cost.

Calculate the selling price of the mobile phone in the online platform.
Give you answer correct to the nearest dollar.

Case 1: The online platform excluded 5% discount from the selling price.

$\text{selling price} = 110\% \times 95\% \times 2020$ M1 – multiply by 110%

$= \$2111$ (nearest dollar) A1

Case 2: The online platform factored in 5% discount in the selling price.

$\text{selling price} = \dfrac{110\% \times 2020}{95\%}$ M1 – multiply by 110%

$= \$2339$ (nearest dollar) A1

Answer \$ [2]

(b) Forty-five staff needs to work for 8 hours a day to complete a project in a week.
Five workers were transferred to another department before the project begins.

Find the number of extra hours the remaining staff need to work in a day to complete the project in a week.

Let S be the number of staff and H be the number of hours.

$$S = \frac{k}{H}$$

$$45 = \frac{k}{8}$$

$k = 360$ B1

When $S = 40$,

$$40 = \frac{360}{H}$$

$H = 9$

$\therefore$ extra hour $= 1$ A1

Answer [2]

4 **(c)** Gina wishes to exchange her Singapore dollars (SGD) into Australia dollars (AUD).
At a money changer, the exchange rate SGD 1 = AUD 0.987.
However, the exchange rate in a bank is AUD 1 = SGD 1.12.

Where should Gina exchange her SGD 1200 into AUD?
Support your answer with suitable justification.

In the bank, the exchange rate is equivalent to

$$\text{SGD } 1 = \text{AUD } \frac{1}{1.12} \qquad \text{M1 – division shown}$$
$$= \text{AUD } 0.89285$$

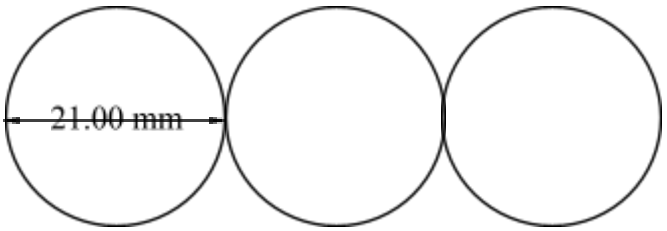
In the bank, the exchange rate is equivalent to

$$\text{SGD } 1 = \text{AUD } \frac{1}{1.12} \qquad \text{M1 – division shown}$$
$$= \text{AUD } 0.89285$$

Answer money changer because 1 Singapore dollar worth more than in the bank. B1

[2]

(d) A hospital is raising money by collecting 20-cent coins.
The target is to collect sufficient coins so that they would be one kilometre long when they are placed edge-to-edge in a straight line.



NOT TO SCALE

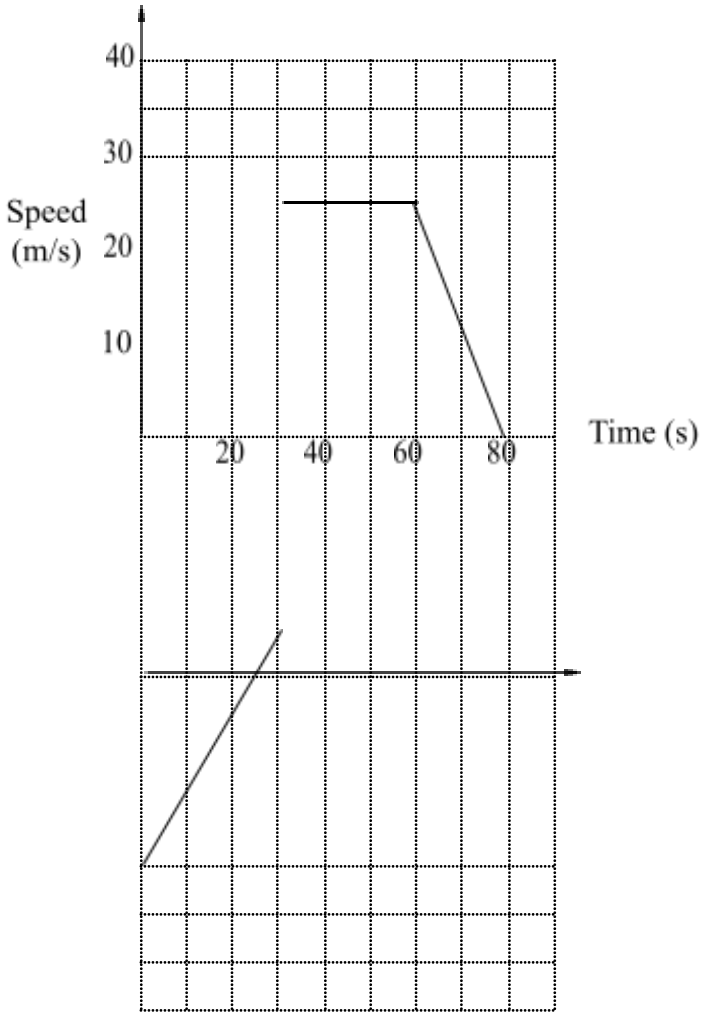
A 20-cent coin is 21.00 mm in diameter.
The hospital meets their target of one kilometre.
The local bank charges 12 cents per coin deposited into the saving accounts.

Calculate the amount of money raised after the deposit to the bank.
Give you answer in dollars.

$$\text{number of coins required} = \frac{1000000}{21} \qquad \text{B1 – conversion to mm}$$
$$= 47619.04762$$
$$= 47620 \qquad \text{A1}$$
$$\text{amount of money raised} = (0.2 - 0.12) \times 47620$$
$$= \$3809.60 \qquad \text{A1}$$

Answer $ [3]

4 **(e)** The diagram shows the speed-time graph for a bus's journey between two bus-stops.



Find

(i) the acceleration of the bus 10 seconds before reaching the next stop,

acceleration = gradient of line segment

$$= \frac{0-25}{80-60}$$

$$= -1.25 \text{ m/s}^2$$ B1

Answer m/s^2 [1]

(ii) average speed of the bus for the journey between the two bus-stops.

$$\text{average speed} = \frac{0.5(80+30)(25)}{80}$$
$$= 17.1875 \text{ m/s}$$

M1 – using area under graph to find distance travelled

A1

Answer m/s [2]

5 The variables x and y are connected by the equation $y = \frac{x}{5}\left(4 + 4x - x^2\right)$.

Some corresponding values of x and y correct to 1 decimal place, are given below.

x	−2	−1	−0.5	0	1	2	3	4	5
y	3.2	0.2	p	0	1.4	3.2	4.2	3.2	−1

(a) Find the value of p.

Answer −0.175 B1 – accept −0.2 [1]

(b) On the grid given, draw the graph $y = \frac{x}{5}\left(4 + 4x - x^2\right)$ for $-2 \le x \le 5$. [3]

P2 – all points plotted correctly P1 – at least 7 points plotted S1 – all 9 points are connected

(c) Use your graph to write an inequality in x to describe the range of values where $y > 3$.

$1.9 < x < 4.1$ or $x < -1.95$ B1

Answer [1]

(d) By drawing a tangent, estimate the gradient of the curve when $x = -1$.

Points are (−2, 1.6) and (0, −1.2) B1 – suitable tangent seen

$$\text{gradient} = \frac{1.6 - (-1.2)}{-2 - 0}$$
$$= -1.4$$

A1 – with working seen

Answer [2]

(e) The equation $4x + 4x^2 - x^3 = 20$ only has only 2 solutions for $-2 \le x \le 5$.

Explain how this can be seen from your graph.

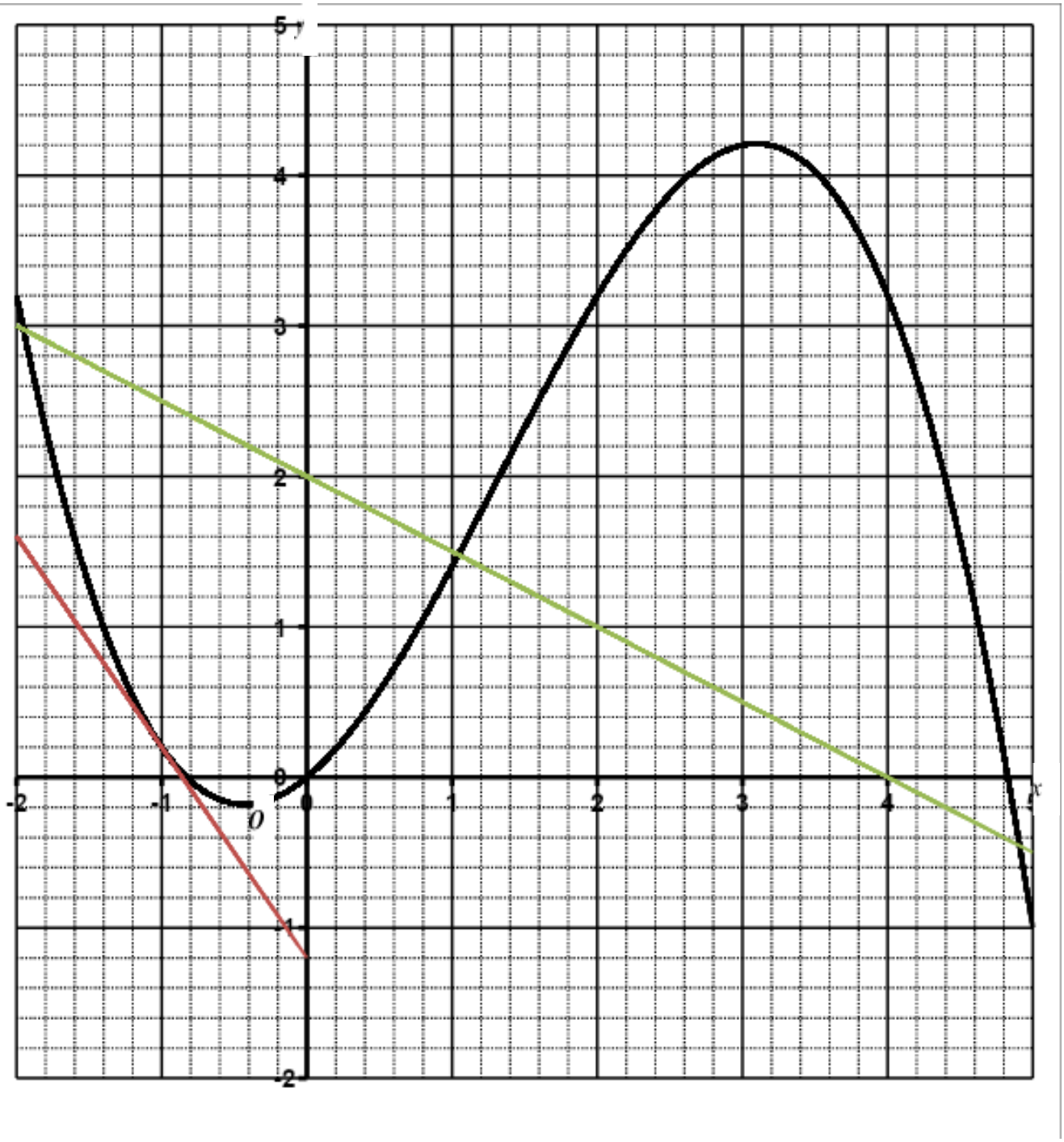
Answer

$\frac{x}{5}\left(4+4x-x^2\right)=4$

The line $y = 4$ intersects the curve at only 2 points for $-2 \le x \le 5$ therefore there are only 2 solutions in the equation $4x+4x^2-x^3=20$.

[2]

B1 – "$y = 4$" seen B1 – intersect at 2 points with concluding remark


y
x
O
5
4
3
2
1
0
-1
-2
-2
-1
0
1
2
3
4

5 **(f)** **(i)** On the same grid, draw the graph $2y + x = 4$ for $-2 \leq x \leq 5$.

B1 – straight line with gradient −0.5
B1 – straight line with y-intercept 2

[2]

(ii) Write down the x-coordinates of the points when the line and the curve intersect.

Answer $x = -1.95,\ 1.05,\ 4.9$ A1 [1]

(iii) The value of x obtained in **(e)(ii)** are solutions of the equation

$$2x^3 + Ax^2 + Bx + 20 = 0.$$

Determine the values of A and B.

$$\text{Sub } y = -\frac{1}{2}x + 2 \text{ into } y = \frac{x}{5}(4 + 4x - x^2),$$

$$-\frac{1}{2}x + 2 = \frac{x}{5}(4 + 4x - x^2)$$

M1 – solving simultaneous eqns

$$-5x + 20 = 8x + 8x^2 - 2x^3$$

$$2x^3 - 8x^2 - 13x + 20 = 0$$

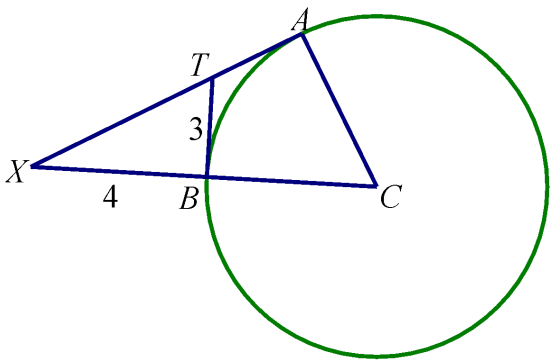
$$A = -8 \quad B = -13$$

A1

Answer $A =$

$B =$ [2]

6 **(a)**



TA and *TB* are tangents to a circle with centre *C*.
AT produced meets *CB* produced at *X*.
XB = 4 cm and *TB* = 3 cm.

Prove that triangle *BTX* is similar to triangle *ACX*.

Answer
In triangle *BTX* and triangle *ACX*,

angle *BXT* = angle *AXC* (common angle)
angle *TBX* = angle *CAX* (tangent perpendicular to radius)
angle *BTX* = angle *ACX* (angles sum of triangle)

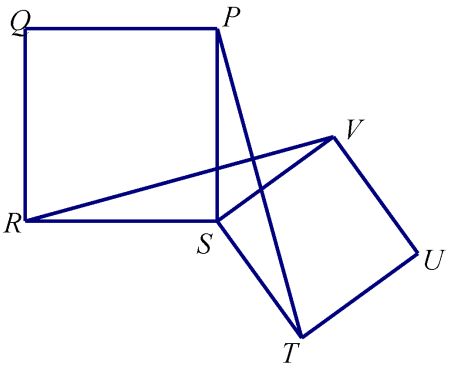
B2 – all 3 pairs of angles with reasons
B1 – 2 pairs of angles with reasons

Therefore, both triangles are similar because all three corresponding angles are equal.

[3]

A1 – with all reasons stated

(b) In the diagram, *PQRS* and *STUV* are both squares.



Prove that triangle *PST* is congruent to triangle *RSV*.

Answer
In triangle *PST* and triangle *RSV*,

PS = *RS* (sides of square *PQRS*)
ST = *SV* (sides of square *STUV*)
angle *BTX* = angle *ACX* (90° + angle *RSV*)

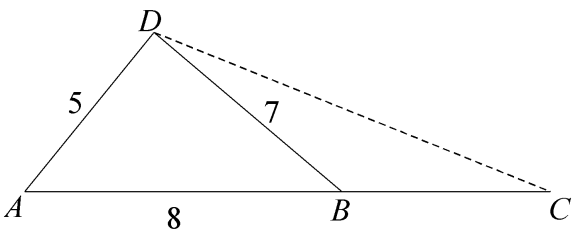
B1 – sides of square used
B1 – 90° + angle *RSV*

Therefore, both triangles are congruent because two pairs of corresponding sides and their included angles are equal.

A1 – with all reasons stated

[3]

7



North

A, B, C and D are points on horizontal ground.
$AB = 8$ m, $BD = 7$ m, $AD = 5$ m.

(a) Show that angle $ABD = 38.2°$, correct to 1 decimal place.

Answer

$$\cos \angle ABD = \frac{7^2 + 8^2 - 5^2}{2(7)(8)} \quad \text{M1}$$

$$\angle ABD = 38.213° \quad \text{A1}$$

$$= 38.2° \quad \text{AG}$$

[2]

(b) Calculate the area of triangle ABD.

$$\text{area of triangle } ABD = \frac{1}{2}(8)(7)\sin 38.213° \quad \text{M1}$$

$$= 17.320$$

$$= 17.3 \text{ m}^2 \quad \text{A1}$$

Answer m² [2]

(c) Point C is due east of point B and C is 12 m away from D.

Find the bearing of D from C.

$$\frac{\sin(180° - 38.213°)}{12} = \frac{\sin \angle ABD}{7} \quad \text{M1}$$

$$\angle ABD = 21.151 \quad \text{A1}$$

$$\text{Bearing required} = 270° + 21.151$$

$$= 291.151°$$

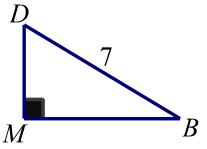
$$= 291.2° \quad (1 \text{ dp}) \quad \text{A1}$$

Answer [3]

7 A 4.5 m tall vertical pole is erected at D.
A camera is placed at the top of the pole.
Fanny walks along the path AC.

(d) **(i)** Calculate the shortest distance Fanny is from D as she walks along AC.

Let M the point on AC such that MD is perpendicular to AC.
In triangle BMD,



$$\sin 38.213° = \frac{MD}{7}$$ M1

$$MD = 7 \sin 38.213°$$
$$= 4.33 \quad \text{m (3sf)}$$ A1

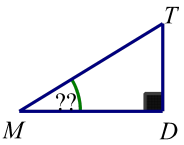
Answer m [2]

(ii) Calculate the greatest angle of depression of Fanny from the camera.

In triangle MDT,

$$\tan \angle TMD = \frac{4.5}{7 \sin 38.213°}$$
$$\angle TMD = 46.102°$$

M1 – tangent ratio with shortest distance used



The greatest angle of depression is 46.1°. A1

Answer [2]

8 An open container is made of a hollow cylinder and a cone, shown in Diagram I.

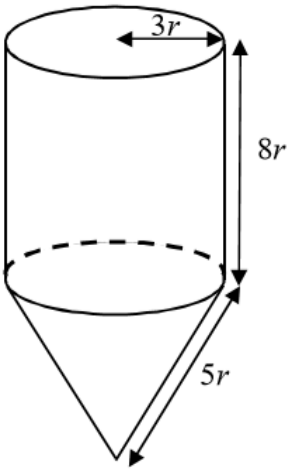


Diagram I

Both cylinder and cone have radius $3r$ cm.
The height of the cylinder is $8r$ cm.
The slant height of the cone is $5r$ cm.
The conical part of the container can hold 96π cm^3 of water.

(a) Show that $r = 2$.

Answer

$$\text{height of cone} = \sqrt{(5r)-(3r)^2} \qquad \text{M1}$$
$$= 4r$$
$$\text{Volume of cone} = \frac{1}{3}\pi(3r)^2(4r) \qquad \text{M1 – apply the volume of cone}$$
$$96\pi = \frac{1}{3}\pi(3r)^2(4r)$$
$$96 = 12r^3$$
$$r^3 = 8 \qquad \text{A1}$$
$$r = 2 \qquad \text{AG}$$

[3]

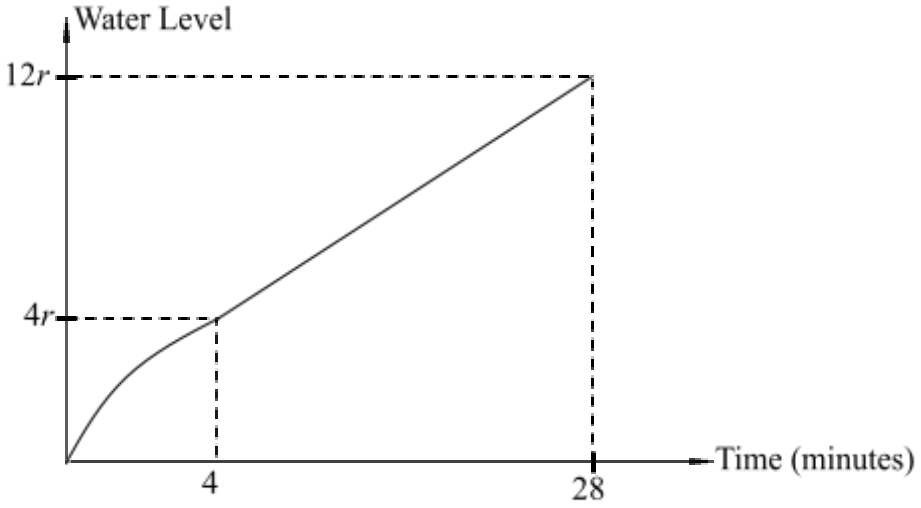
(b) Find the height of the water when 600 cm^3 of water is poured into the container.

Volume of water in cylinder $= 600 - 96\pi$

$$\pi(6)^2 h = 600 - 96\pi \qquad \text{M1}$$
$$h = \frac{600-96\pi}{36\pi}$$
$$\text{height of water} = 4(2) + \frac{600-96\pi}{36\pi}$$
$$= 10.638$$
$$= 10.6 \text{ cm} \qquad \text{A1}$$

Answer cm [2]

8 c) On the grid below, sketch the graph that depicts how the water level increases over time when the container in Diagram I is completely filled with water in 28 minutes. Indicate clearly the time when the water level reaches 4*r*.



B1 – concave down to height 4*r*

B1 – straight line from height 4*r* to 12*r* B1 – 4 minutes to reach 4*r*

Answer On the diagram [3]

(d) In Diagram II, a solid sphere of radius 3 cm is placed into the container such that it touches the curved surface area of the cone.

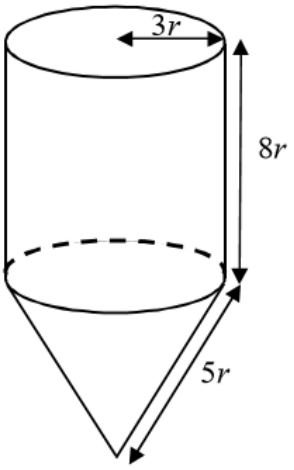


Diagram I

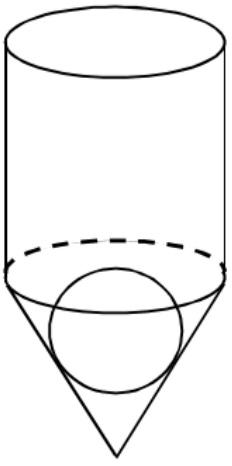


Diagram II

(i) Find, in terms of π, the volume of the sphere.

$$\text{Volume of sphere} = \frac{4}{3}\pi(3)^3$$
$$= 36\pi \quad \text{cm}^3$$

Answer cm^3 [1]

(ii) The container contains 600 cm^3 of water before the placement of the sphere.

Calculate the rise of the height of the water when the sphere is placed in the container.

Volume of sphere = Volume of water with increased height

$$\frac{4}{3}\pi(3)^3 = \pi(6)^2 H \qquad \text{M1}$$

$$H = 1$$

The rise is 1 cm. A1

Answer cm [2]

9 Three integers, a, b and c, are such that $a < b < c$.

The three integers are said to form Pythagorean triple (PT) if $c^2 = a^2 + b^2$ or $c^2 - b^2 = a^2$.

For example,

3, 4, 5 form a PT because $5^2 - 4^2 = (5-4)(5+4) = (1)(9) = 9 = 3^2$

and 5, 12, 13 form a PT because $13^2 - 12^2 = (13-12)(13+12) = (1)(25) = 25 = 5^2$

(a) Form a Pythagorean triple

(i) in which the last two integers are 40 and 41,

9, 40, 41 B1

Answer [1]

(ii) in which the first integer is 11.

11, 60, 61 B2

B1 – for answer given as $61^2 = 11^2 + 60^2$

Answer [2]

It is also possible to form a Pythagorean triple in which the last two integers differ by 2.

(b) **(i)** Simplify $(4n^2+1)^2 - (4n^2-1)^2$ and hence express it as a perfect square.

$(4n^2+1)^2 - (4n^2-1)^2 = \left(4n^2+1-4n^2+1\right)\left(4n^2+1+4n^2-1\right)$ M1

$= 16n^2$

$= (4n)^2$ A1 – perfect square form

Answer [2]

(ii) Form a Pythagorean Triple in which the first integer is 400 and the other two integers differ by 2.

$4n = 400$ M1 – determining the value of n

$n = 100$

400, 39999, 40001 A1

Answer [2]

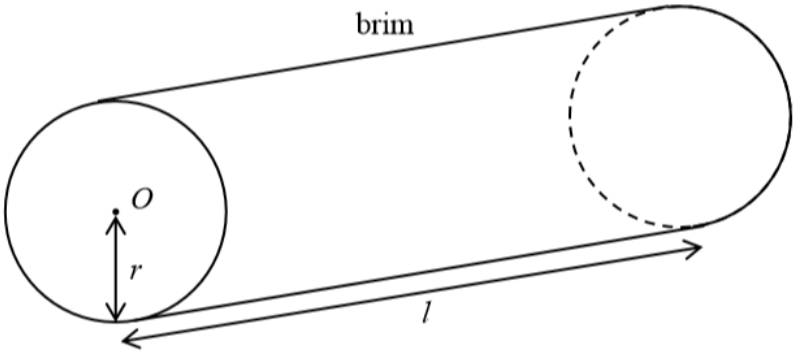
10 The tables below show the specifications of an empty oil tanker and a pumping pipe.

Specifications of Oil Tanker



(Picture taken from: http://www.cnasia.com/products/liquid-gas-rail-road-tankers/refined-sugar-tankers/)

- Radius (r) : 0.9 m
- Length (l) : 10.4 m
- Model : cylindrical as shown below. Point O is the centre of circular cross section



- Safety information : the oil tanker can only be filled up to 15 cm from its brim

Specifications of Pumping Pipe

- Radius (r) : 4.8 cm
- Maximum flow of oil : 350 cm/s
- Model : cylindrical

(a) Calculate the cross sectional area, in square metres, of the tank.

cross sectional area $= \pi(0.9)^2$

$= 0.81\pi$

$= 2.54 \quad m^2$ B1

Answer m^2 [1]

10 **(b)** To provide an extra protection to the container, the external area has to be painted by a special paint.
A tin of paint can be used to polish an area of 8.5 m^2.

Find the number of tins required to paint the container.

total surface area $= 2(0.81)\pi + 2\pi(0.9)(10.4)$ M1 – total surface area

$= 20.34\pi$

number of tins required $= \dfrac{20.34\pi}{8.5}$ M1 – their TSA

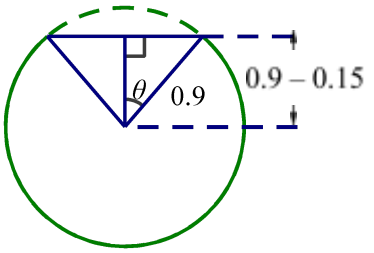
$= 7.5176$

$= 8$ (rounded up to nearest integer) A1

Answer [3]

(c) A pumping pipe is used to fill the container with oil.

Calculate the minimum time, in minutes, needed to fill the container to its safe volume.



$$\cos\theta = \frac{0.9-0.15}{0.9} \qquad \text{M1}$$
$$\theta = 33.557°$$

$$\text{area of major segment} = \frac{1}{2}(0.9)^2 \sin(2\times 33.557°) + \frac{360° - 2\times 33.557°}{360°} \times \pi(0.9)^2 \quad \text{M1}$$
$$= 2.44340$$

$$\text{Volume of fuel required} = 2.44340 \times 10.4 \qquad \text{M1 – area of segment × 10.4}$$
$$= 25.41136$$

$$\text{Maximum rate flow} = \pi(0.048)^2(3.5) \qquad \text{B1}$$

$$\text{Minimum time taken} = \frac{25.41136}{\pi(0.048)^2(3.5)} \qquad \text{M1 – even the denominator is in cm}$$
$$= 1003.0613 \text{ seconds}$$
$$= 16.7 \text{ minutes (3sf)} \qquad \text{A1}$$

Answer minutes [6]