

Setting up an Interactive Notebook:

	Lesson # Date
	<i>Interactive Math Notebook</i> <i>Goal: To improve our flexibility with number, and improve our ability to show what we understand by focusing on our math communication</i>

A NOTE ABOUT INTERACTIVE NOTEBOOKS...

Students write notes, or do the 'learning' of new content on the Right-hand page. Then, to work with that new content, students reflect or complete practice tasks on the Left-hand page.

This allows students to see the 'how-to' at the same time as working on the task.

Left	Right
Practice, reflection questions, and critical thinking	Class notes & foldables, model questions, & vocabulary

These Right-hand notes should be:

1. Neatly labeled at the top with the lesson number and date
2. Carefully written to ensure accuracy and neatness
3. Complete!

The in-class notes are not optional. If you find it hard to get them down accurately within the 100-minute math-block, you can:

1. Start in class, and finish copying the notes from the web site later in the day,
2. Start in class, and finish copying the notes from the web site later at home as homework
3. Print the notes-page and paste it into your book

Your interactive notebook is your take-home textbook. You will be allowed to use it as a tool during tests. It is in your best interest to have it as complete as possible.

1.0 INTERACTIVE NOTEBOOK TYPES OF NUMBERS: (LEFT SIDE)

Where do you see [Vertical Number](#) Lines all the time?

Paste
Tab of
Vertical
Number
Line
Here

Where have you seen real-world
Horizontal Number Lines?

Insert [Horizontal Number Line](#) Here:

Task: Create a Way to help you remember the TYPES of numbers. You could try an Acrostic, a different drawing, or anything to help you remember.

Ps. Don't forget to add the new math definitions to your BOK!

1.0 TYPES OF NUMBERS (RIGHT SIDE)

Learning goal: Be able to define 6 types of numbers

Natural Numbers: Whole numbers from '1' and up Eg. 4, 7, 11

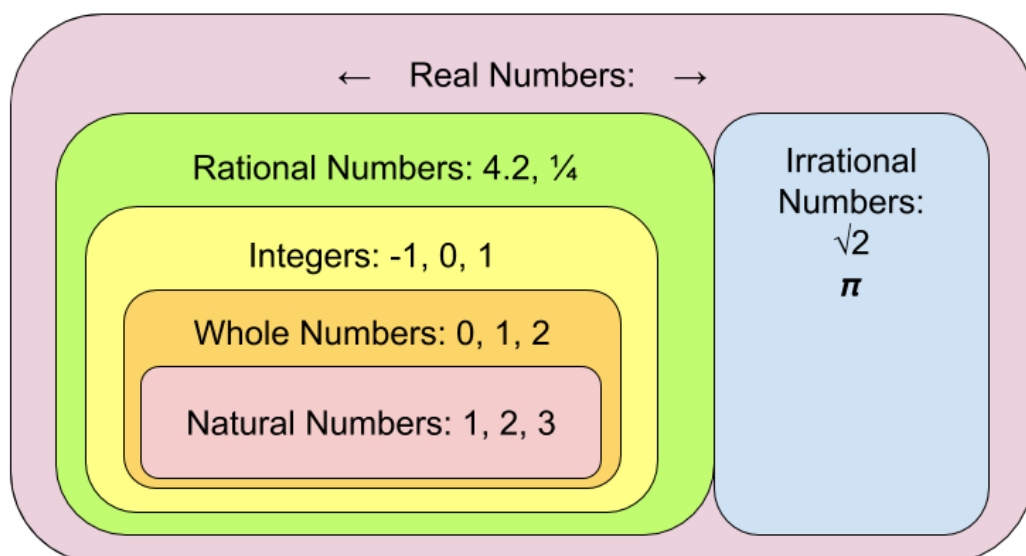
Whole Numbers: a number with no fractional or decimal part; can NOT be negative Eg. 0, 1, 2, 3, 4

Integers: All Positive and Negative numbers Eg. -4, -1, 9, 14

Rational Numbers: Any number that can be written as a fraction or a ratio Eg. $\frac{3}{4}$, 4.68 (ie. $\frac{468}{1000}$), $\frac{1}{3}$ (0.3333333 repeating)

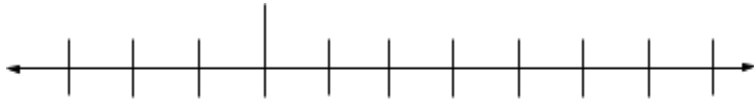
Irrational Numbers: Numbers that can't be written as a simple fraction because their decimal goes on forever WITHOUT repeating Eg. 3.141592... or $\sqrt{2}$

Real Numbers: All the numbers that can be placed on a number line: -4, 32, $\sqrt{8}$

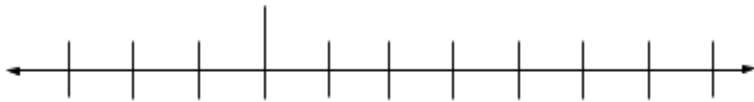


1.1 COMPARING AND ORDERING: (LEFT SIDE)

Task 1: Place the following numbers on an open number line: 2, -3, 0, 3, 6



Task 2: Place the following on an open number line: -1, $3\frac{1}{2}$, $-2\frac{1}{2}$, $5\frac{3}{4}$, 0
-2 $\frac{1}{2}$ -1 0 $3\frac{1}{2}$ $5\frac{3}{4}$



Task 3: Compare the following using equal (=) greater than (>) less than (<)

4 6 42 24 $5\frac{1}{2}$ 5.5 7.25 7.174

17.50 $17\frac{1}{2}$ $3\frac{1}{2}$ $7/2$ $5\frac{1}{2}$ $23/4$ 3^3 5^2

Task 4: Express the following in words:

$$6 > 4$$

$$9 = 27/3$$

$$42 < 45$$

$$4.50 = 4.5$$

$$3.9 < 3.99$$

1.1 COMPARING AND ORDERING: (RIGHT SIDE)

Greater Than $>$ Ex: $6 > 2$ means "6 is greater than 2"

Less Than $<$ Ex: $4 < 7$ means "4 is less than 7"

Equal to $=$ Ex: $3 = \frac{3}{1}$ means "3 is equal to 3 wholes"

On a Number Line:

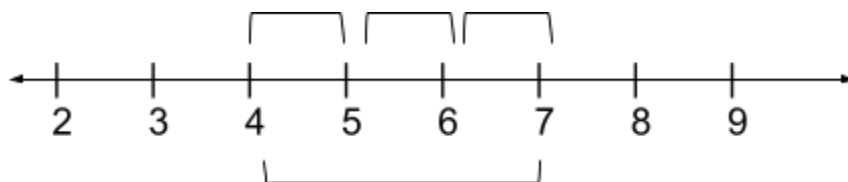
Numbers that represent a greater quantity go to the Right of the number line ("robust to the Right")

Numbers that represent a smaller quantity go to the Left of the number line ("little to the left")

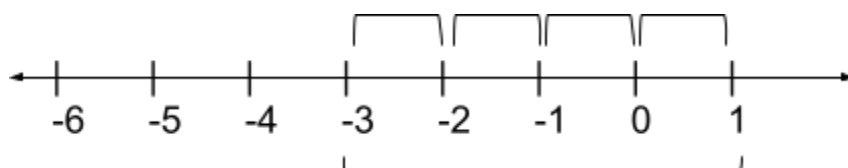
Smaller Valued numbers $\longleftrightarrow$ Larger Valued Numbers

Comparing numbers on a Number Line:

1. The spaces between two points on a number line is the difference between them:



"The difference between 7 and 4 is 3"



"The difference between 1 and -3 is 4"

1.2 INTERACTIVE NOTEBOOKS (LEFT SIDE)

What I know: (What do you know at the start of the question)
- invent one Q of each type with parts labelled

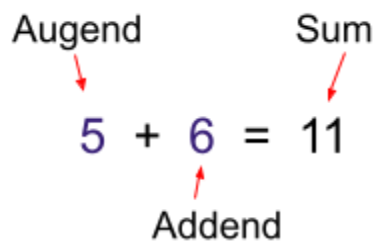


Diagram illustrating the components of an addition equation:

$$\begin{array}{c} \text{Augend} \quad \quad \quad \text{Sum} \\ \downarrow \quad \quad \quad \downarrow \\ 5 + 6 = 11 \\ \uparrow \\ \text{Addend} \end{array}$$

What I learned: (what was new? AFTER doing Right side)
- what is my take-away? What is the important 'thing' about all this vocabulary?

PS. Update your BOK vocab!

1.2 (RIGHT-SIDE)

Vocabulary:

Product – the answer to a multiplication Question

Two **Factors** multiply to result in a **Product**

$$\text{Ex. } 2 \times 5 = 10$$

Sum – the Answer to an addition Question

An **Augend** and an **Addend** add to → a **Sum**

$$\text{Ex. } 3 + 4 = 7$$

Difference – the answer to a subtraction Question

A **Minuend** & a **Subtrahend** subtract to → a **Difference**

$$\text{Ex. } 9 - 6 = 3$$

Quotient – the answer to a division Question

A **Dividend** is divided by a **Divisor** to → a **Quotient**

$$\text{Ex. } 12 \div 4 = 3$$

1.3 (LEFT SIDE)

Prime vs Composite:

1. Prove whether the following numbers Prime or Composite:
 - a. 37
 - b. 156
 - c. 81
 - d. 13
 - e. 61
2. Find the factors of the following
 - a. 15:
 - b. 76:
 - c. 121
 - d. 200
3. Find the first 10 multiple of each
 - a. 7
 - b. 9
 - c. 11
 - d. 13
4. Find the Greatest common factor of each pair of numbers
 - a. 18 & 26
 - b. 12 & 50
 - c. 15 & 60
 - d. 15 & 25
5. Find the Lowest Common Multiple of each pair
 - a. 4 & 18
 - b. 12 & 18
 - c. 15 & 20
 - d. 7 & 9

1.3 (RIGHT SIDE)

Prime Numbers: only have 1 and themselves as factors

Composite Numbers: have more than 2 factors

Factoring a Number = making a list of all factors (using the divisibility rules)



Eg. 36: 1, 2, 3, 4, 6, 9, 12, 18, 36

NB: Ascending Order, Commas b/w

Greatest Common Factor: (GCF) the largest factor shared by two numbers, when comparing their factor lists

12: 1, 2, 3, **4**, 6, 12

20: 1, 2, **4**, 5, 10, 20 GCF: 4

Multiples: the result of 'counting by' a number

Eg. 4: 8, 12, 16, 20, 24, 28, 32, 36, 40, etc.

Lowest Common Multiple: (LCM) the smallest multiple shared by two numbers when comparing their multiple lists

5: 10, 15, 20, 25, **30**, 35, 40

6: 12, 18, 24, **30**, 36, 42, 48,

1.4 SIEVE

Print the Sieve [here](#)

1. Highlight multiples in each row
2. Circle primes in RED
3. Fold the sieve and paste in here for reference

1.4 SIEVE OF ERATOSTHENES

Eratosthenes of Cyrene (276 BCE - 194 BCE)

- Greek mathematician, astronomer, poet
- Compared directionality of sun at noon from various places
- Derived a measurement of the earth based on those differences
- Left behind detailed notes of his process & calculations

The Sieve of Eratosthenes: ancient algorithm for finding Prime Numbers

- Dates from 2nd Century C.E. Greece, named for Eratosthenes
- Marks off multiples of small numbers, leaving Primes unmarked
- Helps show relationships between common factors and multiples
- Helps identify patterns

Why use it?

- Practice multiples (yay times tables!)
- Help put Primes in active working memory
- Helps show sets of related numbers

1.5 APPLYING LCM & GCF (LEFT)

Word Problems:

Two neon signs are turned on at the same time. Both signs blink as they are turned on. One sign blinks every 9 seconds. The other sign blinks every 15 seconds. In how many seconds will they blink together again? (ask: is this a growing pattern? Or looking inside for small numbers)

9: 18, 27, 36, 45, 54, 63, 72, 81 ∴ every 45 seconds

15: 30, 45, 60, 75

Lisa is making activity baskets to donate to charity. She has 12 colouring books, 28 markers, and 36 crayons. What is the greatest number of baskets she can make if each type of toy is equally distributed among the baskets? How many of each supply will go into the baskets? (growing pattern? Or breaking down into smaller numbers?)

The school cafeteria serves tacos every sixth day and cheeseburgers every eight day. If tacos and cheeseburgers are both on today's menu, how many days will it be before they are both on the menu again? (growing pattern or smaller sets?)

1.5 APPLYING LCM & GCF (RIGHT)

Goal: know when to use LCM & GCF in word problems

First: identify the archetype (the 'type' of question)

1. GCF looks for smaller numbers than those stated in the question, & 'how many in a set'
2. LCM uses growing patterns to count numbers of sets

Archetype 1: Hot Dogs & Buns (LCM)

Hot dogs come in packs of 12. Buns come in packs of 8.

What is the smallest number of hot-dog-in-a-bun combos can you make and have none of either left over?

12: 24, 36, 48, 60, 72, 84, 96 etc.

8: 16, 24, 32, 40, 48, 56, 64, etc. → 24 hot-dog-in-buns

Archetype 2: Making Bouquets (GCF)

Sara has 16 red flowers and 24 yellow flowers. She wants to make bouquets with the same number of each colour flower in each bouquet. What is the greatest number of bouquets she can make?

16: 1, 2, 4, 8, 16

$16/8 = 2$ red

24: 1, 2, 3, 4, 6, 8, 12, 24

$24/8 = 3$ yellow

GCF = 8 bouquets (each with 2 red and 3 yellow flowers)

1.6 (RIGHT SIDE) MATH PROPERTIES

[Paste Foldable here](#)

Add your new definitions to your BOK!!!

1.6 (RIGHT SIDE) ADDING & SUBTRACTING USING MATHEMATICAL PROPERTIES

Goal: to be efficient when adding & subtracting

Associative Property: parentheses and grouping don't change the result when adding

$$(a + b) + c = d \rightarrow \text{the same result as } a + (b + c) = d$$

When to use is? Difficult: $(13 + 19) + 7 = ___$

Easier: $(13 + 7) + 19 =$

$$\text{"20"} + 19 = 39$$

Commutative Property: when addends trade places, the sum is the same: $a + b = b + a$

When to use it? Difficult: $33 + 121 = ___$

Easier: $121 + 33 = ___$

Equality Property: when you add or subtract the same to both sides, the equation is still true - both sides are still equal:

$$17 + 9 = 26 \quad \text{and} \quad 17 + 9 + \mathbf{3} = 26 + \mathbf{3}$$

When to use it: in algebra: $x + 3 = 8$

$$x + 3 \mathbf{- 3} = 8 \mathbf{- 3}$$

$$x = 5$$

Identity Property: any number plus or minus zero is the original number:

$$5 + 0 = 5 \quad 9 - 0 = 9$$

1.7 (LEFT)

Task:

In Collaborative Groups:

Remembering Order of Operations:

Brackets, Exponents, Division & Multiplication $L \rightarrow R$

Addition & Subtraction $L \rightarrow R$

Using only the number '4' and all mathematical processes:

Find a way to make every digit from 1 to 25:

- | | |
|--------------|-----|
| 1. | 14. |
| 2. | 15. |
| 3. | 16. |
| 4. | 17. |
| 5. $4 + 4/4$ | 18. |
| 6. | 19. |
| 7. | 20. |
| 8. | 21. |
| 9. | 22. |
| 10. | 23. |
| 11. | 24. |
| 12. | 25. |
| 13. | |

1.7 MULTIPLYING AND DIVIDING USING MATHEMATICAL PROPERTIES (RIGHT)

Associative: $(4 \times 3) \times 5 = 4 \times (3 \times 5)$

Commutative: the order of factors does not affect the product:
 $4 \times 7 = 7 \times 4$ (so choose the fact order that is easier)

Identity: Any number multiplied by 1 is the original number &
Any number divided by 1 is the original number

Inverse: since any whole number can be written over 1, all whole numbers have a reciprocal that is a fraction. Any whole number multiplied by its reciprocal equals 1.

$$\frac{4}{1} \times \frac{1}{4} = 1$$

Zero: Any number multiplied by 0 is zero.
Any number divided by 0 is **undefined**

Distributive: A factor can be applied to each place value individually

$$4 \times 128 = (4 \times 100) + (4 \times 20) + (4 \times 8)$$

(we will use this in expanded notation soon)

1.8 LEFT PLACE VALUE & ROUNDING (LEFT)

1. Round to the nearest 10

- a. 36 b. 92 c. 114 d. 129 e. 452 f. 3 923

2. Round to the nearest 100

- a. 429 b. 291 c. 4 295 d. 71 e. 2 356

3. Multiply

- | | | |
|--------------------|--------------------|---------------------|
| a. 32×10 | b. 71×100 | c. 64×10 |
| d. 479×10 | e. 15×10 | f. 478×100 |

4. Solve:

- a. $4 \times 2 \times 10$ b. $18 \times 3 \times 10$ c. 12×50

1.8 LEFT PLACE VALUE & ROUNDING (RIGHT)

Given The number 4 263 182

Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones
4	2	6	3	1	8	2

Rounding:

1. Look at digit to the right of your 'round to' place
2. Is it 5 or higher? Add 1 to the place digit
3. Is it 4 or less? Leave the digit as it is

Eg. 2 563 Round to the nearest 10 (ask: 2 560? Or 2 570?)

3 is '4 or less' → 2 560

2 563 Round to the nearest 100 (ask: 2 500? Or 2 600?)

6 is '5 and up' → 2 600

Multiplying by 10, 100, 1 000:

By 10, add a 0 after the number

By 100, add 00 after the number

By 1 000, add 000 after the number

1.9 DIVISIBILITY (LEFT)

1. Which of these numbers is 1792 divisible by?

- a. 2 b. 3 c. 4 d. 5 e. 6

2. Which of the following numbers is divisible by 6?

- a. 870 b. 232 c. 681

3. Which numbers are divisible by nine?

- a. 6 723 b. 864 c. 29 934

1.9 DIVISIBILITY (RIGHT)

Divisibility Rules: A number is divisible by:

- 2 if the number is even
- 3 if the sum of its digits is divisible by 3
- 4 if the number represented of the last 2 digits is divisible by 4
- 5 if the last digit is 0 or 5
- 6 if the number is divisible by 2 and 3
- 8 if the number represented by the last 3 digits is divisible by 8
- 9 if the sum of the digits is divisible by 9
- 10 if the last digit is 0

1.10 LONG DIVISION

(NB: working on graph paper keeps things lined up - paste your work here when done)

1. Solve:

a. $279 \overline{) 7}$

b. $4529 \overline{) 13}$

c. $7\ 449 \overline{) 7}$

d. $84 \overline{) 3}$

e. $128 \overline{) 24}$

f. $193 \overline{) 16}$

2. Solve:

a. Jashan has a backyard pond that needs filling. The pond holds 1125 L of water when full. Jashan has a 7L bucket. How many times will he need to fill (and then pour out) that bucket while filling the pond?

b. Vienna mows her lawn every 8th day. Her neighbour, Mx. G, mows their lawn every 9 days. How long until they both mow their lawns on the same day?

1.10 LONG DIVISION TRADITIONAL ALGORITHM (RIGHT) [SONG](#)

Divide: does 4 go into 2? No Does 4 go into 26? Yes (6 times)	$\begin{array}{r} 6 \\ 4 \overline{) 268.47} \end{array}$
Multiply: $6 \times 4 = 24$	$\begin{array}{r} 6 \\ 4 \overline{) 268.47} \\ 24 \end{array}$
Subtract: $26 - 24 = 2$	$\begin{array}{r} 6 \\ 4 \overline{) 268.47} \\ - 24 \\ \hline 2 \end{array}$
Bring it down: transfer then next digit in the dividend down to your working layer	$\begin{array}{r} 6 \\ 4 \overline{) 268.47} \\ - 24 \downarrow \\ \hline 28 \end{array}$
Bring it on Back: Focus on the Divisor (4), the partial quotient (6...) and the evolving dividend (28)	$\begin{array}{r} 6 \\ 4 \overline{) 268.47} \\ - 24 \\ \hline 28 \end{array}$
Divide: does 4 go into 28? Yes (7 times)	$\begin{array}{r} 6 \text{ } 7 \\ 4 \overline{) 268.47} \\ - 24 \\ \hline 28 \end{array}$
Multiply: $(7 \times 4 = 28)$	$\begin{array}{r} 6 \text{ } 7 \\ 4 \overline{) 268.47} \\ - 24 \\ \hline 28 \\ 28 \end{array}$
Subtract: when you hit a difference of 0, you're done. Or, once you have 3 decimal places in the quotient (and can round to 2) you're done. *Always round to 2 decimal places	$\begin{array}{r} 6 \text{ } 7 \text{ } 1 \\ 4 \overline{) 268.47} \text{ not done!} \\ - 24 \\ \hline 28 \\ - 28 \downarrow \\ \hline 0 \text{ bring down 4} \end{array}$

1.11 POWERS & EXPONENTS (LEFT)

Consider:

$1^{-1} = 1$	$1^0 = 1$	$1^1 = 1$	$1^2 = 1$	$1^3 = 1$
$10^{-1} = 0.1$	$10^0 = 1$	$10^1 = 10$	$10^2 = 100$	$10^3 = 1000$
$4^{-1} = 0.25$	$4^0 = 1$	$4^1 = 4$	$4^2 = 16$	$4^3 = 64$
$5^{-1} = 0.2$	$5^0 = 1$	$5^1 = 5$	$5^2 = 25$	$5^3 = 125$

Solve the following: eg. $3^2 + 10^4 = 9 + 10\ 000 = 10\ 009$

1. $10^2 + 10^4 =$
2. $10^4 + 7^2 =$
3. $4^3 + 7^0 =$
4. $3^3 + 2^3 + 5^2 =$
5. $10^{-1} + 6^2 =$

Find the following Square roots:

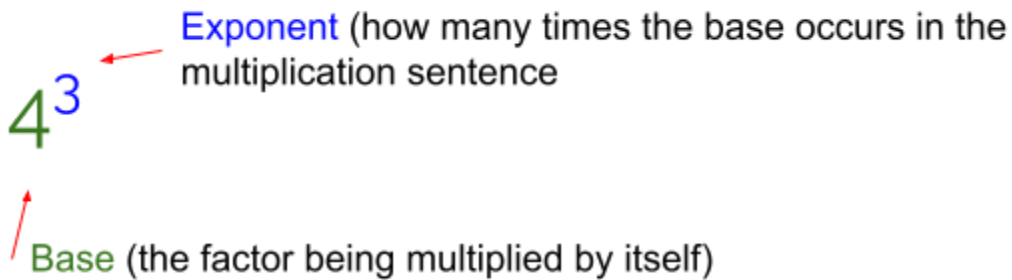
1. $\sqrt{81} =$
2. $\sqrt{169} =$
3. $\sqrt{225} =$
4. $\sqrt{49} =$
5. $\sqrt{36} =$

Using what you know of perfect squares:

1. Estimate the square root of 42
2. Estimate the square root of 90
3. Estimate the square root of 12

1.11 POWERS & EXPONENTS (RIGHT)

Powers: an expression of a product of equal factors ($4 \times 4 \times 4 = 4^3$)



Perfect Squares: the product of any factor multiplied by itself ONCE

$$1 \times 1 = 1$$

$$2 \times 2 = 4$$

$$3 \times 3 = 9$$

$$4 \times 4 = 16$$

$$5 \times 5 = 25$$

$$6 \times 6 = 36$$

$$7 \times 7 = 49$$

$$8 \times 8 = 64$$

$$9 \times 9 = 81$$

$$10 \times 10 = 100$$

$$11 \times 11 = 121$$

$$12 \times 12 = 144$$

The square root $\sqrt{\quad}$ of a perfect square is the original factor

Eg. $4 \times 4 = 16 \quad \therefore \sqrt{16} = 4$

$\therefore n^2$ and $\sqrt{\quad}$ are Inverse Operations

1.12 FRACTIONS (LEFT):

Put the following fractions in order from least to greatest

O $\frac{5}{6}$ $\frac{1}{2}$ $\frac{1}{1}$ $\frac{1}{6}$ $\frac{2}{3}$

Put the following fractions in order from least to greatest

$\frac{3}{4}$ $\frac{1}{2}$ $\frac{3}{12}$ $\frac{1}{4}$ $\frac{1}{12}$ $\frac{2}{2}$ $\frac{5}{12}$

Use < > and = to compare the following

$\frac{5}{6}$ $\frac{1}{2}$ $\frac{1}{6}$ $\frac{2}{3}$ $\frac{3}{12}$ $\frac{1}{4}$

Draw the following fractions using a bar model:

$\frac{5}{6}$ $\frac{2}{3}$ $\frac{1}{6}$ $\frac{3}{3}$

1.12 FRACTIONS (RIGHT):

Fractions: numbers less than a whole

Denominator: how many parts one whole was divided into

Numerator: how many of those smaller pieces you have

Eg. $\frac{3}{4}$ a whole was divided into 4 parts, you have 3 of them

Big Ideas to remember:

The smaller the numerator the fewer pieces you have (are shaded)

$\frac{1}{5}$ has fewer pieces shaded than $\frac{4}{5}$

The larger the numerator the more pieces you have (are shaded)

$\frac{3}{4}$ has more pieces shaded than $\frac{1}{4}$

The smaller the denominator the larger the pieces are

$\frac{1}{2}$ is a larger chunk than $\frac{1}{12}$

The larger the denominator the smaller the pieces are

$\frac{1}{1}$ means one whole and you have all of it -- it means 'one whole'

$\frac{3}{3}$ means you have one whole, cut into 3 pieces, & you have all 3 pieces

$\frac{3}{1}$ means 3 wholes..... 3 separate wholes, and you have all of them

$\frac{1}{3}$ means you have one whole, cut into 3 pieces, & you have one of those pieces

1.13 EQUIVALENT FRACTIONS (LEFT):

Find the equivalent fraction with a larger denominator

$$\frac{4}{9} = \frac{\quad}{54}$$

$$\frac{1}{10} = \frac{\quad}{60}$$

$$\frac{3}{8} = \frac{\quad}{32}$$

$$\frac{3}{4} = \frac{\quad}{44}$$

$$\frac{8}{9} = \frac{\quad}{81}$$

$$\frac{3}{4} = \frac{\quad}{48}$$

$$\frac{3}{5} = \frac{\quad}{45}$$

$$\frac{5}{9} = \frac{\quad}{108}$$

$$\frac{2}{5} = \frac{\quad}{15}$$

Find the equivalent fraction with a smaller denominator. Simplest Form.

$$\frac{28}{50} =$$

$$\frac{32}{48} =$$

$$\frac{36}{216} =$$

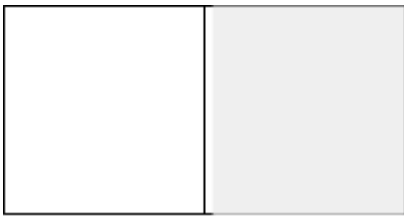
$$\frac{8}{24} =$$

$$\frac{36}{54} =$$

$$\frac{35}{42} =$$

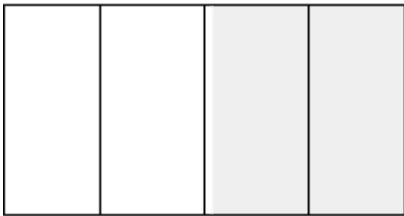
1.13 EQUIVALENT FRACTIONS (RIGHT):

Equivalent fractions:

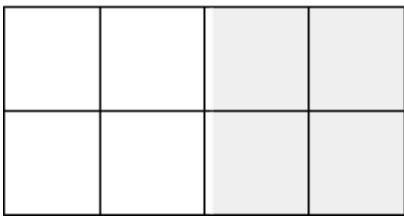


$$\frac{1}{2}$$

The size of the Whole does not change, the overall size of the shaded section does not change, but the number of pieces that shaded chunk is broken into, changes



$$\frac{2}{4}$$



$$\frac{4}{8}$$

To find equivalent fractions, multiply N and D by a common scale factor:

$$\begin{array}{ccccccc} \frac{1}{2} & \times 2 = & \frac{2}{4} & \times 3 = & \frac{6}{12} & \times 2 = & \frac{12}{24} \\ 1 & \times 2 = & 2 & \times 3 = & 6 & \times 2 = & 12 \\ 2 & \times 2 = & 4 & \times 3 = & 12 & \times 2 = & 24 \end{array}$$

Or

Divide by a common factor:

$$\begin{array}{ccccccc} \frac{100}{120} & / 2 = & \frac{50}{60} & / 10 = & \frac{5}{6} \\ 100 & / 2 = & 50 & / 10 = & 5 \\ 120 & / 2 = & 60 & / 10 = & 6 \end{array}$$

1.14 ADDING FRACTIONS (LEFT):

Task 1: add the following fractions

$$\frac{5}{6} + \frac{3}{5} =$$

$$1\frac{5}{6} + 2\frac{3}{4} =$$

$$\frac{7}{12} + \frac{2}{3} =$$

$$\frac{5}{6} + \frac{7}{8} =$$

$$\frac{15}{21} + \frac{1}{7} =$$

$$1\frac{1}{20} + 2\frac{2}{3} =$$

$$\frac{11}{20} + \frac{2}{3} =$$

$$2\frac{15}{16} + \frac{1}{4} =$$

Extensions:

$$\frac{x}{2} + \frac{5}{7} = \quad \frac{7}{x} + \frac{3}{5} = \quad \frac{2x}{5} + \frac{3}{4} =$$

1.14 ADDING FRACTIONS (RIGHT):

Adding Fractions:

1. In order to add fractions, denominators must be the same

$$\frac{1}{3} + \frac{2}{4} = \frac{4}{12} + \frac{6}{12} = \frac{10}{12}$$

2. ALL fraction answers must be reduced to Lowest Form
 - a. Ask: is there a number that goes into both N and D?
 - b. Divide both N and D by that number
 - c. Repeat

$$\frac{10}{12} \div 2 = \frac{5}{6} \text{ (lowest form)}$$

3. Adding fractions can result in Improper Fractions (N is larger than D) $\rightarrow$

$$\frac{4}{5} + \frac{2}{3} = \frac{12}{15} + \frac{10}{15} = \frac{22}{15}$$

4. Improper fractions must be written as mixed numbers (fraction must be in lowest terms):

$$\frac{22}{15} \text{ take } \frac{15}{15} \text{ (or one whole) away from the N}$$


$$1 \frac{7}{15}$$

1.15 SUBTRACTING FRACTIONS (LEFT):

Solve:

$$\frac{11}{12} - \frac{3}{4} =$$

$$2\frac{1}{12} - \frac{1}{4} =$$

$$3\frac{3}{4} - 1\frac{3}{8} =$$

$$\frac{7}{8} - \frac{1}{3} =$$

$$3\frac{5}{6} - 1\frac{2}{3} =$$

$$3\frac{9}{10} - \frac{3}{15} =$$

$$\frac{2}{3} - \frac{7}{12} =$$

$$1\frac{7}{15} - 1\frac{2}{5} =$$

1.15 SUBTRACTING FRACTIONS (RIGHT):

Subtracting Fractions:

1. In order to subtract fractions, denominators must be the same

$$\frac{2}{3} - \frac{2}{12} = \frac{8}{12} - \frac{2}{12} = \frac{6}{12}$$

2. ALL fraction answers must be reduced to Lowest Form
 - a. Ask: is there a number that goes into both N and D?
 - b. Divide both N and D by that number
 - c. Repeat

$$\frac{2}{12} \div 2 = \frac{1}{6} \text{ (lowest form)}$$

e.converting mixed numbers into improper fractions:

$3\frac{2}{5}$ Three wholes, chopped into fifths, is 15 fifths, plus 2 fifths is $\frac{17}{5}$
"Bottom times Big plus Top"

1.16 EQUIVALENT FRACTIONS (LEFT):

Solve

$$5 \times \frac{2}{3} =$$

$$\frac{1}{2} \times 7 =$$

$$\frac{5}{6} \times \frac{6}{7} =$$

$$\frac{4}{5} \times \frac{3}{6} =$$

$$\frac{8}{9} \times \frac{3}{7} =$$

$$\frac{4}{11} \times \frac{3}{4} =$$

$$\frac{10}{13} \times \frac{2}{3} =$$

$$\frac{1}{2} \times \frac{14}{20} =$$

1.16 EQUIVALENT FRACTIONS (RIGHT):

Multiplying Fractions:

Remember: **'of'** means multiply

And multiplying by a fraction gives you 'less'

eg. one-half of 8 is..... 4 (you got smaller)

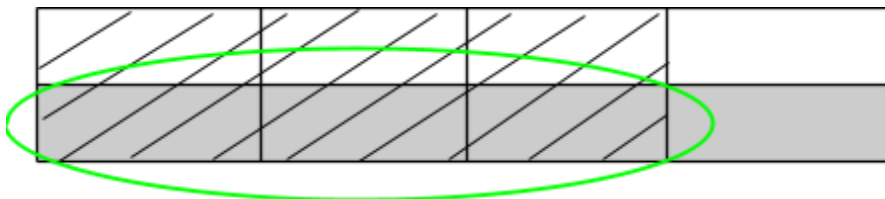
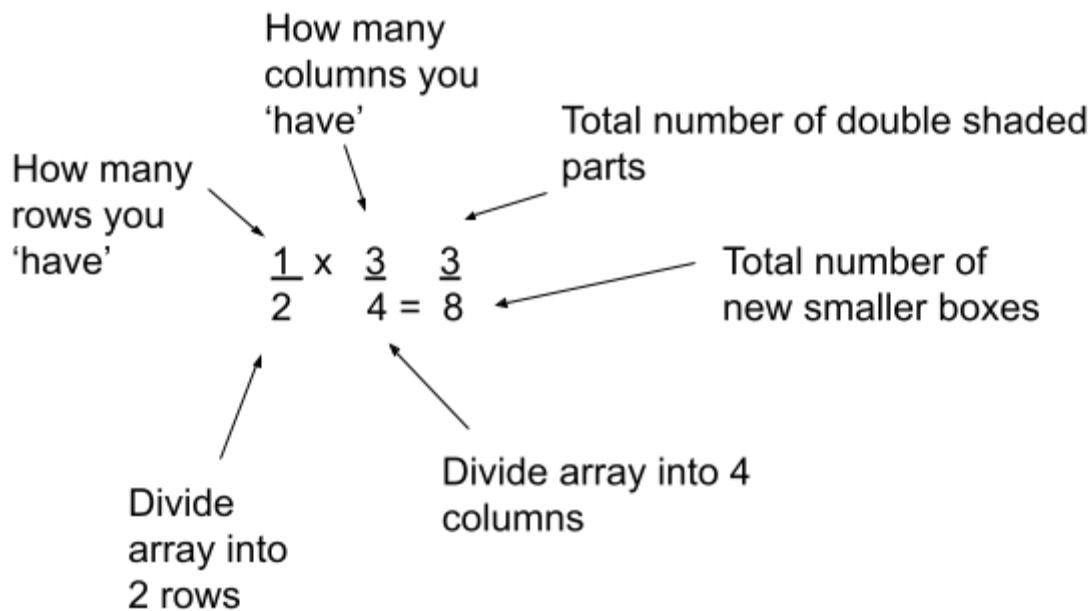
$$\frac{1}{2} \times \frac{8}{1} = \frac{8}{2} = \frac{8}{2} = 4$$

Step 1: Top times top = new top

Step 2: Bottom times bottom = new bottom

Step 3: Reduce

Multiplying Fractions with Fractions:



1.18 Integers (Left)

practice: model and solve:

1. $(+4) + (+7)$
2. $(-2) + (-5)$
3. $(+8) + (+2)$
4. $(+13) + (+12)$
5. $(+1) + (+25)$
6. $(-14) + (-3)$
7. $(-2) + (+9)$
8. $(-5) + (-9)$
9. $(-12) + (-4)$
10. $(+5) + (-2)$

What does the model show? solve

1. ++++++ +++ -----
2. ---- +++++ --
3. --
4. +++ --- ++++++
5. ++++++ ++++++ ++++++
----- ----- --
6. ++++++ ++++++ ++++++ +++
----- --

more practice and some more

1.18 (Right) Integers

integers: an integer is a number quantity that has -- in addition -- a positive or negative quality

Positive: a whole number greater than zero

- represents a gain, what you have ●

Negative: a whole number less than zero

- represents a loss or a debt, what you take away, "in the red" ●

Zero principle: inverse additives cancel to zero: $-x$ and $+x = 0$



Addition of positives: $(+4) + (+3) = \text{●●●●} + \text{●●●} = (+7)$

Addition of negatives: $(-2) + (-5) = \text{●●} + \text{●●●●●} = (-7)$

Addition of mixed integers: $(-4) + (+2) = \text{●●●●} + \text{●●} = (-2)$

$(-3) + (+7) = \text{●●●} + \text{●●●●●●} = (+4)$

1.19 (Left) Practice:

Addition (combining) of Positives and Negatives:

1. $(+3) + (-1) =$
2. $(+6) + (-2) =$
3. $(-2) + (-6) =$
4. $(-12) + (+3) =$
5. $(-10) + (+3) + (-1) =$
6. $(+5) + (-4) + (+6) =$
7. $(+20) + (-12) =$
8. $(-11) + (-6) + (+5) =$
9. $(+5) + (-2) + (+7) + (-6) =$
10. $(+25) + (-15) + (+32) =$

[more practice](#)

Represent the following in math-language:

1. John owes Tim four dollars
2. Kyle found 20\$ on the road
3. Marcy lost her cat
4. Bob got 50\$ for his birthday. He gave 20\$ to the humane society.
5. Kim wants to buy a new coat, but she doesn't have the money.
She uses her brand new credit card to buy it.

And.... watch the signs....

1. $(+5) - (-3) = (+8)$
2. $(-3) + (-2) = (-5)$
3. $(+6) - (-4) = (+10)$
4. $(-3) + (-5) = (-8)$
5. $(+7) - (-3) = (+10)$
6. $(+7) - (-6) = (+13)$
7. $(+5) - (-7) = (+12)$
8. $(+8) - (-4) = (+12)$
9. $(-3) + (-6) = (-9)$
10. $(-5) - (-8) = (+3)$

$$(+5) - (-7) = \begin{array}{ccccccccc} + & + & + & + & + & & + & + & + & + & + & + & + \\ & & & & & & & & & & & & & \\ - & - & - & (& - & - & - & - & - &) \end{array}$$

1.21 [Left side]

Reflection: How do you think of negative and positive numbers? What are some examples now?

How does having a real-world model for integers help you solve world problems?

1. Katherine is very interested in cryogenics (the science of very low temperatures). With the help of her science teacher she is doing an experiment on the effect of low temperatures on bacteria. She cools one sample of bacteria to a temperature of -51°C and another to -76°C . What was the temperature difference in the two experiments? $(-51) - (-76) =$

$$(-50) - (-70) + (-1) - (-6) =$$

2. On Tuesday the mailman delivers 3 checks for \$5 each and 2 bills for \$2 each. If you had a starting balance of \$25, what is the ending balance?
3. You owe \$225. on your credit card. You make a \$55 payment and then purchase \$87 worth of clothes at Dillards. What is the integer that represents the balance owed (-) on the credit card?
4. If it is -25°F in Rantoul and it is $+75^{\circ}\text{F}$ in Honolulu, what is the temperature difference between the two cities?
5. on Monday morning when you wake up at 8 am, it is $+2^{\circ}\text{C}$. The temperature rises steadily 2 degrees per hour. At 1 pm the temperature holds steady. At 5 pm the temperature begins to drop a steady 2 degrees per hour. at 10 o'clock it reaches it's lowest point. What temperature is it at 11 pm?

1.20 Integers take 2 (combining Integers)

A. model and solve:

1. $(-4) + (+3) = -1$ - - - -
 + + + =
2. $(+2) + (-4) =$ ●●
 ●● ●● -2
3. $(-6) + (+5) =$ ●●●●●● ● -1
 ●●●●●●
4. $(+2) + (+5) =$ ●● ●●●●●● +7
5. $(-4) + (-5) =$ ●●●● ●●●●●●●● -9

Show all work (in math language and symbols) to communicate HOW you solved these questions:

1. Snuffy leaves his penthouse and hops on the elevator in his building on the 13th floor. The elevator goes down two floors, then goes back to the 13th floor. The Bird family gets on, and they all journey 7 floors down. The manager, Luis, gets on and takes everyone back up one floor. Then, finally, Elmo gets on, and they all travel down three floors together, at which point they all get off to go to a birthday party for Oscar. Which floor is the party on? $(+13) + (-2) + (+2) + (-7) + (+1) + (-3) = (+4)$

2. A submarine dives 836 ft. It rises at a rate of 22 ft per minute. What was the depth of the submarine after 12 minutes?

$$(-836) + (+264) = (-572) \quad (-836) + (22)(12) =$$

1.20 (Left side:)

$$(+)\times(+)=(+)$$

adding groups of positives

$$(+)\times(-)=(-)$$

adding groups of negatives

.....all from the field of 'zeroes'

$$(-)\times(-)=(+)$$

removing groups of negatives

$$(-)\times(+)=(-)$$

removing groups of positives

try modelling them:

1. $(+2)\times(-5)=$

2. $(-3)\times(-4)=$

3. $(-4)\times(+2)=$

4. $(-4)\times(-5)=$

5. $(-10)\times(-3)=$

6. $(+9)\times(+3)=$

7. $(+3)\times(-11)=$

8. $(+2)\times(-20)=$

1.21 (Right Side) Multiplying Integers

Multiplying is...

- repeated addition $(5 \times 4) = (5 + 5 + 5 + 5)$

- adding or subtracting 'groups' of stuff

$$(+++++) (+++++) (+++++) (+++++) = +20$$

$$(+5) \times (-4) = ??? (----) (----) (----) (----) = -20$$

You 'have' added 5 groups of '4 negatives'

$(-3) \times (-2) = +6$ start with a field of zeroes, then

Take away 3 groups of (-2)

$$\begin{array}{cccccccccccccccc} + & + & + & + & + & + & + & + & + & + & + & + & + & + & + \\ - & - & - & - & - & - & - & - & - & - & - & - & - & - & - \end{array} (\quad \quad \quad)$$

$$(-4) \times (+2) = -8$$

$$\begin{array}{cccccccc} + & + & + & + & + & + & + & + \\ - & - & - & - & - & - & - & - \end{array}$$

$$(+2) \times (-4) = -8$$

$$---- \quad ---- = -8$$

$$+ \times + = +$$

$$+ \times - = -$$

$$- \times + = -$$

$$- \times - = +$$

1.22 (Left)

Practice :)

- a. $(+4) - (+3) =$
- b. $3 \times (2 + 6)$
- c. $10 - (2 \times 4)$
- d. $10 + (-2 \cdot 4) =$
- e. $(-2)(-4 + -7) =$
- f. $5 \cdot 3 - 12 =$
- g. $50 - 2 \cdot (+3) =$
- h. $+2 \cdot (+3 \cdot +5) =$
- i. $[(+6) - (-4)] + 4(12-5) =$
- j. $8 \times (-2) - (-4)2$
- k. $34 \div 9 + 2 \times 5$
- l. $3^3 + (-1)(+7) - (+6) + 32 =$
- m. $(\sqrt{16 + 5^2}) - 3^2 - (-6)(-2) =$

turn into mathematical sentences and solve:

1. Joe babysits for \$5 an hour, plus a flat tip of 4\$ per child.

2. A certain small factory employs 98 workers. Of these, 10 receive a wage of \$150 per day and the rest receive \$85.50 per day. To the management, a week is equal to 6 working days. How much does the factory pay out for each week?

3. A certain Math Club makes 35 bars of laundry soap a week and sells these at \$20 each. Before the soap can all be sold, the pupils found out that 6 bars were destroyed by mice. How much will be the total sale at the end of a four-week month?

4. Lilia scores 15 points fewer than Bob, who scores 35 points. Carol scores half as many points as Lilia. How many points does Carol score?

1.22(Right) 'Givens' in Numeracy

1. Order of Operations ALWAYS needs to be followed:
Brackets (Parentheses)
Exponents (squares, square roots, etc)
Division and Multiplication **in the order they appear from Left to Right**
Addition and Subtraction **in the order they appear from Left to Right**
2. a number with no sign is 'understood' to be 'positive': $5 = (+5)$
3. a number immediately in front of a bracket implies multiplication $3(6) = 18$
4. brackets can be placed around a digit and it's sign to clarify 'quality' from 'process'
 $(+6) + (-3) =$ vs $+6 + -3 =$
5. because 'x' will soon represent a variable, we can use $\cdot$ for multiplication
 Xxx (yuck~!) $x \cdot x$ (much more clear) or $5(x-1)$
6. +

ex. 1 $5 \cdot 9 + 4 - 2 + (5-2) =$
 $5 \cdot 9 + 4 - 2 + (3) =$
 $45 + 4 - 2 + 3 =$
 $45 + 4 - 2 + 3 = 50$

Ex. 2 $3 + 7 - 3 \cdot 2 + 12/4 =$
 $3 + 7 - 6 + 3 = 7$



1.23 - order of operations Left practice: BEDMAS

Goal - in own words

Reflect: Solve the following in order Left to Right, and again using bedmas.

$$9 + (-5) \cdot 2 + 7 \cdot (-3) =$$

Now direct input on a calculator: does your calculator use BEDMAS logic?

Practice:

1. 5 times the product of negative 3 and five is... $5[(-3)(5)]$
2. The difference of 7 and 3 squared is... $7 - 3^2 =$
3. The sum of 15 and the product of 3 and negative 8 is... $15 + [(3)(-8)] =$
4. The product of eight and the sum of nine and negative two, subtract 7, is...
5. The square root of the product of eight and two is...

1.23 Bedmas Investigation (Right)

Goal: communicate mathematical thinking orally, visually, and in writing, using mathematical vocabulary and a variety of appropriate representations, and observing mathematical conventions.

Investigate how BEDMAS effects answers:

Solve from Left to Right: $7 + 3 \cdot (-6) + 2 \cdot 10 = (-580)$

Solve using Bedmas:

$$\begin{aligned} 7 + 3 \cdot (-6) + 2 \cdot 10 &= \\ 7 + (-18) + 2 \cdot 10 &= \\ 7 + (-18) + 20 &= \\ (-11) + 20 &= (+9) \end{aligned}$$
$$\begin{aligned} (2^2 + 2) - (+2)(2-5)^2 + (4 - 6 + 6)^2 &= \\ 6 - (2)(-3)^2 + (4)^2 &= \\ 6 - (2)(9) + 16 &= \\ 6 - 18 + 16 &= (+4) \end{aligned}$$

1.24 Combination skills:

1. $(+3)(-4) + (-9)(-2) =$
2. $+2(-5) - (-6)(-4) =$
3. $(+6) + (-9)(-2) =$
4. $(-3)(-5)(-3) =$ watch the signs!!!!
5. $(-4)(+7) - (+5) =$
6. $+14 - (-2)(-7) =$
7. $(-15) \div (+3) =$
8. $(-20) \div (+4) =$
9. $(+30) \div (-5) =$
10. $(-7)(-3)(-4) + (-2)(-3) =$

1. Marilyn's class played a game that is scored a little like golf. As you may know, golf is scored based on a number called par. Each player gets a score that is at par or is some number of points above or below par. The lower the score the better, so it's best to get below par.

For Marilyn's game, par was 54. The players on Marilyn's team had the following scores in terms of par: -3, +9, -10, -1, +15, -7, +4, and -6. What was the winning score? What was the worst score?

2. Which product is farther from 0? $(45) \cdot (-105)$ or $(-58) \cdot (-95)$

3. Which sum is closer to 0? $(186) + (-36)$ or $(-98) + (-124)$

4. Larry had possession of the ball four times in a recent football game. In one play, he gained 21 yards for his team. In another play, he was pushed back by the defense and gained -6 yards. In the last two plays, he gained 18 and -9 yards. What was his total yardage gained?

What was his average yardage gain for the four plays?

$$(+21) + (-6) + (18) + (-9) = \underline{+24} = 6$$

1.25

1) $(+7) + (+2) =$

2) $(-35) \div (-7) =$

3) $(+3) - (-5) =$

4) $(-4)(+2) =$

5) $(+2)(+4) =$

6) $(+7) + (+6) =$

7) $(-5) - (-8) =$

8) $(-7)(+9) =$

9) $(+9) - (-5) =$

10) $(+18) \div (-6) =$

11) $(-5)(+4) =$

12) $(+18) \div (+2) =$

13) $(+9) - (-7) =$

14) $(-5) - (-9) =$

15) $(-72) \div (-9) =$

16) $(-8) + (+8) =$

17) $(-5) - (+3) =$

18) $(+4)(-4) =$

19) $(+7)(+7)(-2) =$

20) $(+5) + (+6) =$

21) $(-2) + (+5) =$

22) $(-3)(-8) =$

23) $(+6)(-5) =$

24) $(+8) \div (-2) =$

25) $(-2) - (+4) =$

26) $(-2) + (+8)(-2) =$

27) $(-8) + (-5)(-3) =$

28) $(-21) \div (+7) =$

29) $(+4) - (+2)(-3) =$

30) $(+8) + (-4) =$