A IAL Pure Maths 4
Vector
MS

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1.Jan 2025-8

8(a)	States or implies that $a = -3$ **or** $b = 10$	B1
	States or implies that $a = -3$ **and** $b = 10$	B1
		(2)
(b)	Attempts $= \begin{pmatrix} "10" \\ 3 \\ 1 \end{pmatrix} - \begin{pmatrix} -2 \\ "-3" \\ 4 \end{pmatrix} = \ldots$ either way around	M1
	$\overrightarrow{AB} = \begin{pmatrix} 12 \\ 6 \\ -3 \end{pmatrix}$ or $12\mathbf{i} + 6\mathbf{j} - 3\mathbf{k}$	A1
		(2)
(c)	Attempts $\overrightarrow{AC} = \begin{pmatrix} 4 \\ 7 \\ -2 \end{pmatrix} - \begin{pmatrix} -2 \\ "-3" \\ 4 \end{pmatrix} = \begin{pmatrix} 6 \\ 10 \\ -6 \end{pmatrix}$	M1
	Attempts $\overrightarrow{AC}.\overrightarrow{AB} = \begin{pmatrix} 6 \\ 10 \\ -6 \end{pmatrix} \bullet \begin{pmatrix} 12 \\ 6 \\ -3 \end{pmatrix} = 72 + 60 + 18$	dM1
	Attempts $\overrightarrow{AC}.\overrightarrow{AB} = \lvert AC \rvert \lvert AB \rvert \cos\theta \Rightarrow 150 = \sqrt{172} \times \sqrt{189} \cos\theta \Rightarrow \theta = \ldots$ $\left(NB \ \ \sqrt{172} = 2\sqrt{43},\ \sqrt{189} = 3\sqrt{21} \right)$	ddM1
	$\theta =$ awrt $33.7°$	A1

	(c) Alternative using the cosine rule:	
	$\overrightarrow{AC} = \begin{pmatrix} 4 \\ 7 \\ -2 \end{pmatrix} - \begin{pmatrix} -2 \\ "-3" \\ 4 \end{pmatrix} = \begin{pmatrix} 6 \\ 10 \\ -6 \end{pmatrix}$	M1
	$\overrightarrow{AC} = \begin{pmatrix} 4 \\ 7 \\ -2 \end{pmatrix} - \begin{pmatrix} -2 \\ "-3" \\ 4 \end{pmatrix} = \begin{pmatrix} 6 \\ 10 \\ -6 \end{pmatrix}$ **and** $\overrightarrow{BC} = \begin{pmatrix} 4 \\ 7 \\ -2 \end{pmatrix} - \begin{pmatrix} "10" \\ 3 \\ 1 \end{pmatrix} = \begin{pmatrix} -6 \\ 4 \\ -3 \end{pmatrix}$ $\Rightarrow AB^2 = 12^2 + 6^2 + 3^2,\ BC^2 = 6^2 + 4^2 + 3^2,\ AC^2 = 6^2 + 10^2 + 6^2$	dM1
	$BC^2 = AB^2 + AC^2 - 2AB \times AC\cos\theta$ $61 = 189 + 172 - 2\sqrt{189}\sqrt{172}\cos\theta \Rightarrow \cos\theta = \ldots$	ddM1
	$\theta =$ awrt $33.7°$	A1
		(4)

(a)

B1: States or implies that $a = -3$ **or** $b = 10$ (Note that a comes from $\lambda = -1$ and b from $\lambda = 2$)

B1: States or implies that $a = -3$ **and** $b = 10$ (Note that a comes from $\lambda = -1$ and b from $\lambda = 2$)

In the rest of the question, you can condone slips in writing down vectors as long as the intention is clear.

(b)

M1: Attempts to subtract their vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$ either way round.
If no method is shown, it can be implied by at least 2 correct components for their vectors.

A1: Correct vector using correct notation.

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Do **not** allow as coordinates and do **not** allow $\begin{pmatrix} 12\mathbf{i} \\ 6\mathbf{j} \\ -3\mathbf{k} \end{pmatrix}$ but apply isw once a correct vector is seen.

(c)

M1: Attempts to subtract $\overrightarrow{OC}$ and their vector $\overrightarrow{OA}$ either way around

dM1: Attempts to find the scalar product of $\pm\overrightarrow{AC}$ and their vector $\pm\overrightarrow{AB}$
This may be implied by their value so you may need to check.
If the value is incorrect and no method is shown score M0

ddM1: Full attempt to find angle CAB using the scalar product of $\pm\overrightarrow{AC}$ and their vector $\pm\overrightarrow{AB}$

A1: $\theta =$ awrt 33.7° and no other angles (Degrees symbol **not** required)

Note that in (a) they can use any multiples of $\pm\overrightarrow{AC}$ and $\pm\overrightarrow{AB}$ to find the required angle.

Alternative:

M1: Attempts to subtract $\overrightarrow{OC}$ and their vector $\overrightarrow{OA}$ either way around

dM1: Attempts to subtract $\overrightarrow{OC}$ and their vector $\overrightarrow{OA}$ either way around **and** attempts to subtract $\overrightarrow{OC}$ and their vector $\overrightarrow{OB}$ either way around and attempts the lengths or lengths^2 of AB, BC and AC

ddM1: Full attempt to find angle CAB using the cosine rule

A1: $\theta =$ awrt 33.7° and no other angles (Degrees symbol **not** required)

(d)	Attempts a correct method of finding one position for D. See notes for possible approaches.	M1
	$(22, 9, -2)$ **or** $(-26, -15, 10)$	A1
	Attempts a correct method of finding both positions for D. See notes for possible approaches.	dM1
	$(22, 9, -2)$ **and** $(-26, -15, 10)$	A1
		(4)
		(12 marks)

(d)

M1: Attempts a complete and correct method for finding one position for D.
(May be implied by at least 2 correct or correct ft components)

Examples:

Starting from $\begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$: $\begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} + 5\begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix}$ **or** $\begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} - 7\begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix}$

Starting from A $\begin{pmatrix} -2 \\ \text{"}-3\text{"} \\ 4 \end{pmatrix}$: $\begin{pmatrix} -2 \\ \text{"}-3\text{"} \\ 4 \end{pmatrix} + 6\begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix}$ **or** $\begin{pmatrix} -2 \\ \text{"}-3\text{"} \\ 4 \end{pmatrix} - 6\begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix}$

Starting from B $\begin{pmatrix} \text{"}10\text{"} \\ 3 \\ 1 \end{pmatrix}$: $\begin{pmatrix} \text{"}10\text{"} \\ 3 \\ 1 \end{pmatrix} + 3\begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix}$ **or** $\begin{pmatrix} \text{"}10\text{"} \\ 3 \\ 1 \end{pmatrix} - 9\begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix}$

Note that some candidates may use their $\overrightarrow{AB}$ rather than the direction vector e.g.

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Starting from $\begin{pmatrix}2\\-1\\3\end{pmatrix}$: $\begin{pmatrix}2\\-1\\3\end{pmatrix}+\frac{5}{3}\overrightarrow{AB}$ **or** $\begin{pmatrix}2\\-1\\3\end{pmatrix}-\frac{7}{3}\overrightarrow{AB}$

Starting from A $\begin{pmatrix}-2\\"-3"\\4\end{pmatrix}$: $\begin{pmatrix}-2\\"-3"\\4\end{pmatrix}+2\overrightarrow{AB}$ **or** $\begin{pmatrix}-2\\"-3"\\4\end{pmatrix}-2\overrightarrow{AB}$

Starting from B $\begin{pmatrix}"10"\\3\\1\end{pmatrix}$: $\begin{pmatrix}"10"\\3\\1\end{pmatrix}+\overrightarrow{AB}$ **or** $\begin{pmatrix}"10"\\3\\1\end{pmatrix}-3\overrightarrow{AB}$

A1: One correct point $(22, 9, -2)$ or $(-26, -15, 10)$

Condone if given as a vector e.g. $22\mathbf{i}+9\mathbf{j}-2\mathbf{k}$ or $-26\mathbf{i}-15\mathbf{j}+10\mathbf{k}$

dM1: Attempts a complete and correct method for finding both possible positions for D.
(May be implied by at least 2 correct or correct ft components)

Examples:

Starting from $\begin{pmatrix}2\\-1\\3\end{pmatrix}$: $\begin{pmatrix}2\\-1\\3\end{pmatrix}+5\begin{pmatrix}4\\2\\-1\end{pmatrix}$ **and** $\begin{pmatrix}2\\-1\\3\end{pmatrix}-7\begin{pmatrix}4\\2\\-1\end{pmatrix}$

Starting from A $\begin{pmatrix}-2\\"-3"\\4\end{pmatrix}$: $\begin{pmatrix}-2\\"-3"\\4\end{pmatrix}+6\begin{pmatrix}4\\2\\-1\end{pmatrix}$ **and** $\begin{pmatrix}-2\\"-3"\\4\end{pmatrix}-6\begin{pmatrix}4\\2\\-1\end{pmatrix}$

Starting from B $\begin{pmatrix}"10"\\3\\1\end{pmatrix}$: $\begin{pmatrix}"10"\\3\\1\end{pmatrix}+3\begin{pmatrix}4\\2\\-1\end{pmatrix}$ **and** $\begin{pmatrix}"10"\\3\\1\end{pmatrix}-9\begin{pmatrix}4\\2\\-1\end{pmatrix}$

Note that some candidates may use their $\overrightarrow{AB}$ rather than the direction vector e.g.

Starting from $\begin{pmatrix}2\\-1\\3\end{pmatrix}$: $\begin{pmatrix}2\\-1\\3\end{pmatrix}+\frac{5}{3}\overrightarrow{AB}$ **and** $\begin{pmatrix}2\\-1\\3\end{pmatrix}-\frac{7}{3}\overrightarrow{AB}$

Starting from A $\begin{pmatrix}-2\\"-3"\\4\end{pmatrix}$: $\begin{pmatrix}-2\\"-3"\\4\end{pmatrix}+2\overrightarrow{AB}$ **and** $\begin{pmatrix}-2\\"-3"\\4\end{pmatrix}-2\overrightarrow{AB}$

Starting from B $\begin{pmatrix}"10"\\3\\1\end{pmatrix}$: $\begin{pmatrix}"10"\\3\\1\end{pmatrix}+\overrightarrow{AB}$ **and** $\begin{pmatrix}"10"\\3\\1\end{pmatrix}-3\overrightarrow{AB}$

A1: Gives both possible coordinates $(-26, -15, 10)$ and $(22, 9, -2)$

Condone if given as vectors e.g. $22\mathbf{i}+9\mathbf{j}-2\mathbf{k}$ and $-26\mathbf{i}-15\mathbf{j}+10\mathbf{k}$

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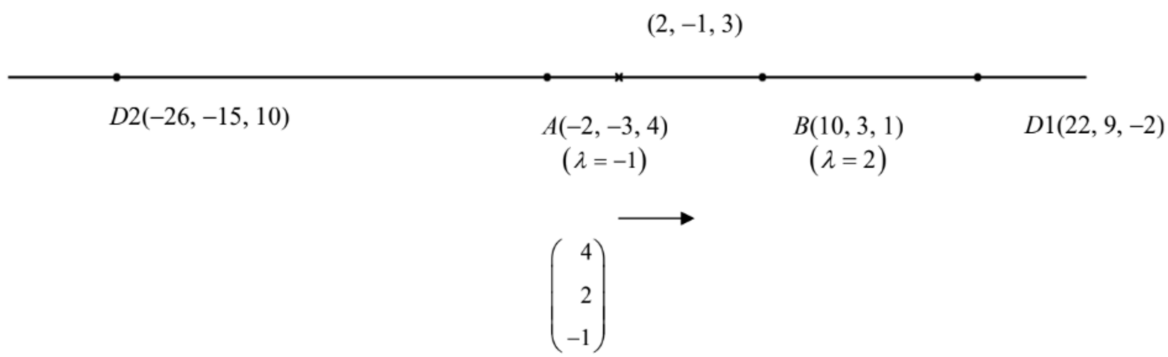
Configuration in (d):



Note that there may be more convoluted methods in (d) e.g.

$$\text{Area } CAD = 2 \times \text{Area } CAB \Rightarrow AD = 2\sqrt{12^2+6^2+3^2} \Rightarrow AD^2 = 4\times 189$$

$$\overrightarrow{AD} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix} - \begin{pmatrix} -2 \\ -3 \\ 4 \end{pmatrix} = \begin{pmatrix} 4\lambda+4 \\ 2\lambda+2 \\ -\lambda-1 \end{pmatrix}$$

$$(4\lambda+4)^2 + (2\lambda+2)^2 + (-\lambda-1)^2 = 756$$

$$\Rightarrow 21\lambda^2 + 42\lambda - 735 = 0 \Rightarrow \lambda = 5,\ -7$$

$$\overrightarrow{OD} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} + 5 \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix} \quad \textbf{or} \quad \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} - 7 \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix}$$

$$\rightarrow (-26, -15, 10) \text{ and } (22, 9, -2)$$

In such cases, marks can be awarded as above for:

- **M1: A complete and correct method to find one position for *D***
  (May be implied by at least 2 correct or correct ft components)
- **A1: One correct position for *D***
- **dM1: A complete and correct method to find both positions for *D***
  (May be implied by at least 2 correct or correct ft components)
- **A1: As main scheme**

If you are in any doubt whether a method is sound or not use review.

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Can also be done via the area e.g.

$$\text{Area } CAB = \frac{1}{2} AB \times AC \sin\theta = \frac{1}{2}\sqrt{189}\sqrt{172}\sin\theta$$

$$\cos\theta = \frac{25}{\sqrt{43}\sqrt{21}} \Rightarrow \sin\theta = \sqrt{\frac{278}{903}}$$

$$\text{Area } CAD = \frac{1}{2} AD \times AC \sin\theta = \sqrt{189}\sqrt{172}\sqrt{\frac{278}{903}}$$

$$\overrightarrow{AD} = \begin{pmatrix} 4\lambda+4 \\ 2\lambda+2 \\ -\lambda-1 \end{pmatrix} \therefore \frac{1}{2}\sqrt{(4\lambda+4)^2+(2\lambda+2)^2+(-\lambda-1)^2}\sqrt{172}\sqrt{\frac{278}{903}} = \sqrt{189}\sqrt{172}\sqrt{\frac{278}{903}}$$

$$\Rightarrow (4\lambda+4)^2+(2\lambda+2)^2+(-\lambda-1)^2 = 756$$

$$\Rightarrow 21\lambda^2 + 42\lambda - 735 = 0 \Rightarrow \lambda = 5, -7$$

$$\overrightarrow{OD} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} + 5\begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix} \text{ or } \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} - 7\begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix}$$

$$\rightarrow (-26, -15, 10) \text{ and } (22, 9, -2)$$

The same marking principles apply

- **M1: A complete and correct method to find one position for *D***
  (May be implied by at least 2 correct or correct ft components)
- **A1: One correct position for *D***
- **dM1: A complete and correct method to find both positions for *D***
  (May be implied by at least 2 correct or correct ft components)
- **A1: As main scheme**

For this method it is unlikely that candidates will work in exact terms and will revert to decimals.
This is acceptable but is unlikely to result in correct exact coordinates but the method marks are available as long as the method is complete and correct.

If candidates do work in decimals and then round their inexact values for the coordinates to the correct ones the A marks should be withheld.

2.Jan 2024

6(a)	$(3\mathbf{i}+p\mathbf{j}+7\mathbf{k})+\lambda(2\mathbf{i}-5\mathbf{j}+4\mathbf{k})=(8\mathbf{i}-2\mathbf{j}+5\mathbf{k})+\mu(4\mathbf{i}+\mathbf{j}+2\mathbf{k})$ $3+2\lambda=8+4\mu \quad (1)$ $\Rightarrow p-5\lambda=-2+\mu \quad (2)$ $7+4\lambda=5+2\mu \quad (3)$	**M1**
	e.g. $(3)-2\times(1): 1=-11-6\mu \Rightarrow \mu=-2 \quad \left(\text{or } \lambda=-\frac{3}{2}\right)$	**M1**
	$\mu=-2, \lambda=-\frac{3}{2} \Rightarrow p=-2-2-\frac{15}{2}=...$	**M1**
	$p=-\frac{23}{2}$	**A1**
		(4)
(b)	Intersect at $(8\mathbf{i}-2\mathbf{j}+5\mathbf{k})+-2(4\mathbf{i}+\mathbf{j}+2\mathbf{k})=..$	**M1**
	$=-4\mathbf{j}+\mathbf{k}$	**A1**
		(2)
(c)	$(2\mathbf{i}-5\mathbf{j}+4\mathbf{k}).(4\mathbf{i}+\mathbf{j}+2\mathbf{k})=2\times4-5\times1+4\times2=...$	**M1**
	$\cos\theta=\frac{"11"}{\sqrt{2^2+5^2+4^2}\sqrt{4^2+1^2+2^2}}=...\left(=\frac{11}{3\sqrt{105}}=0.3578...\right)$	**M1**
	$\theta=\cos^{-1}\frac{11}{3\sqrt{105}}=69.0°$	**A1**
		(3)
(d)	$\lambda=2\Rightarrow \overrightarrow{OA}=\left(7\mathbf{i}-\frac{43}{2}\mathbf{j}+15\mathbf{k}\right)$	**B1ft**
	$\overrightarrow{AB}=\pm\left((8\mathbf{i}-2\mathbf{j}+5\mathbf{k})+\mu(4\mathbf{i}+\mathbf{j}+2\mathbf{k})-\left(7\mathbf{i}-\frac{43}{2}\mathbf{j}+15\mathbf{k}\right)\right)$ $=\pm\left((1+4\mu)\mathbf{i}+\left(\frac{39}{2}+\mu\right)\mathbf{j}+(-10+2\mu)\mathbf{k}\right)$	**M1**
	$\left((1+4\mu)\mathbf{i}+\left(\frac{39}{2}+\mu\right)\mathbf{j}+(-10+2\mu)\mathbf{k}\right).(4\mathbf{i}+\mathbf{j}+2\mathbf{k})=0\Rightarrow\mu=..\left(=-\frac{1}{6}\right)$	**M1**
	$\overrightarrow{OB}=(8\mathbf{i}-2\mathbf{j}+5\mathbf{k})+"-\frac{1}{6}"(4\mathbf{i}+\mathbf{j}+2\mathbf{k})=...$	**dM1**
	$=\frac{22}{3}\mathbf{i}-\frac{13}{6}\mathbf{j}+\frac{14}{3}\mathbf{k}$ or $\left(\frac{22}{3},-\frac{13}{6},\frac{14}{3}\right)$	**A1**
		(5)
		(14 marks)

Notes: Accept alternative notations, such as column or row vector notation, throughout. Condone if **i**, **j** and **k** are left in columns etc. except for in the final answers to (b) and (d).
(a) Note: work for (a) must be shown in part (a) not recovered from later parts. **M1:** Equates equations of the lines and extracts at least two simultaneous equations (usually (1) and (3)) by equating coefficients. **M1:** Solves the equations to find at least one of λ or μ **M1:** Uses their values or other suitable method to find p allowing for slips. **A1:** Correct value.

(b) **M1:** Uses either parameter to find the point of intersection. May be implied by two correct entries if no method shown. **A1:** Correct point. Accept as coordinates or in vector form. **(c)** **M1**: Attempts the scalar product with the two direction vectors (or any multiples of them). Allow if there are miscopies etc if the intent of the two correct vectors is clear. Note that $-3\mathbf{i}+\frac{15}{2}\mathbf{j}-6\mathbf{k}$ and $-8\mathbf{i}-2\mathbf{j}-4\mathbf{k}$ are commonly seen correct direction vectors. **M1:** Applies the scalar product formula to find the value for cos θ. Allow if wrong "direction" vectors are used, or if a slip is made but the intent is clear. This mark is for demonstrating a correct use of the formula with any vectors. **A1:** Correct angle, awrt 69.0°, and isw if they give go on to give the obtuse angle. But A0 if another incorrect angle is found. **(d)** **B1ft:** Correct coordinates for A, follow through their p **M1:** Attempts to find $\overrightarrow{AB}$ in terms of μ with their $\overrightarrow{OA}$ and a general point on l_2. Allow subtraction either way round. May be implied by two correct entries if no method shown. Note, if $\overrightarrow{AB}=(x-7)\mathbf{i}+\left(y+"\frac{43}{2}"\right)\mathbf{j}+(z-15)\mathbf{k}$ is used this mark is not scored until the point where they identify x, y and z in terms of μ. **M1:** Takes scalar product of their $\overrightarrow{AB}$ with direction of l_2, sets equal to zero and solves to find μ **dM1:** Depends on previous M mark. Uses their parameter in the correct line to find the coordinates. Implied by 2 correct coordinates if no method is shown. **A1:** Correct answer. Accept as coordinates or in vector form.

Alt (d)	$\lambda = 2 \Rightarrow \overrightarrow{OA} = \left(7\mathbf{i} - \frac{43}{2}\mathbf{j} + 15\mathbf{k}\right)$	**B1ft**
E.g.	$\overrightarrow{XA} = \pm\left(\left(7\mathbf{i} - \frac{43}{2}\mathbf{j} + 15\mathbf{k}\right) - (-4\mathbf{j} + \mathbf{k})\right)$ $\overrightarrow{XB} = \pm\left((8+4\mu)\mathbf{i} + (-2+\mu)\mathbf{j} + (5+2\mu)\mathbf{k} - (-4\mathbf{j}+\mathbf{k})\right)$	**M1**
	$AX\cos\theta = BX \Rightarrow \left(7^2 + \left(-\frac{35}{2}\right)^2 + 14^2\right)\left(\frac{121}{945}\right) = (8+4\mu)^2 + (2+\mu)^2 + (4+2\mu)^2$ $\Rightarrow 36\mu^2 + 144\mu + 23 = 0 \Rightarrow \mu = ..\left(= -\frac{1}{6}, -\frac{23}{6}\right)$	**M1**
	$\overrightarrow{OB} = (8\mathbf{i} - 2\mathbf{j} + 5\mathbf{k}) + \text{"}-\frac{1}{6}\text{"}(4\mathbf{i} + \mathbf{j} + 2\mathbf{k}) = \ldots$	**dM1**
	$= \frac{22}{3}\mathbf{i} - \frac{13}{6}\mathbf{j} + \frac{14}{3}\mathbf{k}$ or $\left(\frac{22}{3}, -\frac{13}{6}, \frac{14}{3}\right)$	**A1**
		(5)
Notes		

Alt (d) – by right triangle

B1ft: Correct coordinates for A, follow through their p

M1: Attempts to find at least two sides of the triangle AXB where X is the intersection point. (Note if no further progress in this method is made the main scheme can apply to allow M1 for just $\overrightarrow{AB}$) in terms of μ where relevant, using their $\overrightarrow{OA}$ and a general point on l_2. Allow subtraction either way round. May be implied by two correct entries if no method shown.

M1: For a full, correct method leading to a value for μ. There are variation to that shown in the scheme, such as use of Pythagoras on all three sides, or via $AX\sin\theta = AB$. The correct side must be used for the hypotenuse, but allow use of decimals in the trig work but must be working in the correct mode to 3s.f..

dM1: Depends on previous M mark. Uses one of their parameters in the correct line to find the coordinates. Implied by 2 correct coordinates if no method is shown. They may use either solution to their quadratic for this mark.

A1: Correct answer only. Must have selected the root which corresponds to A and reject the other. Accept as coordinates or in vector form. Allow awrt 3 s.f. for the coordinates.

3.June 2024-2

2	$\overrightarrow{OA}=\begin{pmatrix}7\\2\\-5\end{pmatrix}$ $\overrightarrow{AB}=\begin{pmatrix}-2\\4\\3\end{pmatrix}$ $\overrightarrow{OC}=\begin{pmatrix}a\\5\\-1\end{pmatrix}$	
(a)	$\overrightarrow{OB}=\overrightarrow{OA}+\overrightarrow{AB}=\begin{pmatrix}7\\2\\-5\end{pmatrix}+\begin{pmatrix}-2\\4\\3\end{pmatrix}=...$	**M1**
	$=\begin{pmatrix}5\\6\\-2\end{pmatrix}$, $5\mathbf{i}+6\mathbf{j}-2\mathbf{k}$ or $(5,6,-2)$	**A1**
		(2)
(b)	$\overrightarrow{BC}=\overrightarrow{OC}-"\overrightarrow{OB}"=\begin{pmatrix}a\\5\\-1\end{pmatrix}-"\begin{pmatrix}5\\6\\-2\end{pmatrix}"=...$	**M1**
	$\overrightarrow{OC}."\overrightarrow{BC}"=0\Rightarrow\begin{pmatrix}a\\5\\-1\end{pmatrix}."\begin{pmatrix}a-5\\-1\\1\end{pmatrix}"=0\Rightarrow a(a-5)-5-1=0$	**dM1**
	$a^2-5a-6=0\Rightarrow(a-6)(a+1)=0\Rightarrow a=...$	**ddM1**
	$a=-1,\ 6$	**A1**
		(4)
		(6 marks)

(a)

M1: Attempts to add $\overrightarrow{OA}$ and $\overrightarrow{AB}$, which may be implied by two correct coordinates out of three if there is no direct statement as in the main scheme.

A1: Correct coordinates, in either vector or Cartesian form. (See main scheme).

Withhold the mark for hybrid versions such as $(5\mathbf{i}, 6\mathbf{j}, -2\mathbf{k})$ but isw after you have seen a correct answer

(b)

M1: Attempts $\overrightarrow{BC}$ by subtracting their $\overrightarrow{OB}$ (look at their answer to (a)) and $\overrightarrow{OC}$, subtracted either way round, which may be implied by two correct coordinates out of three following through on their (a).

dM1: Applies the scalar product with $\overrightarrow{OC}$ and their $\overrightarrow{BC}$, sets equal to 0 and forms a quadratic equation in *a*. Dependent upon the previous M.

An alternative method is via Pythagoras' theorem $OC^2 + BC^2 = OB^2 \Rightarrow$

$\left(a^2+5^2+1^2\right)+\left((a-5)^2+1^2+1^2\right)=5^2+6^2+(-2)^2 \Rightarrow 2a^2-10a-12=0$

ddM1: Depends on both previous Ms. Solves their 3- term quadratic equation (which must have real roots) by any valid means including a calculator (check roots if done this way).

A1: Correct values for *a*. If the candidate goes on to reject a value it is A0.

4.June 2024-6

6(a)	$\left\lvert\overrightarrow{OA}\right\rvert^2=(1+8\lambda)^2+(2-\lambda)^2+(5+4\lambda)^2$	M1
	$\left\lvert\overrightarrow{OA}\right\rvert=5\sqrt{10}\Rightarrow(1+8\lambda)^2+(2-\lambda)^2+(5+4\lambda)^2=250$	M1
	$\Rightarrow 64\lambda^2+16\lambda+1+\lambda^2-4\lambda+4+16\lambda^2+40\lambda+25=250$ $\Rightarrow 81\lambda^2+52\lambda-220=0$ *	A1*
		(3)
(b)	$81\lambda^2+52\lambda-220=0\Rightarrow(81\lambda-110)(\lambda+2)=0\Rightarrow\lambda=...\left(-2,\frac{110}{81}\right)$ $\Rightarrow\overrightarrow{OA}=\begin{pmatrix}1+8\times\text{"their}\lambda\text{"}\\2-\text{"their}\lambda\text{"}\\5+4\times\text{"their}\lambda\text{"}\end{pmatrix}$	M1
	E.g $\Rightarrow\overrightarrow{OA}=\begin{pmatrix}1\\2\\5\end{pmatrix}-2\begin{pmatrix}8\\-1\\4\end{pmatrix}=\begin{pmatrix}-15\\4\\-3\end{pmatrix}$ * ; or $\overrightarrow{OA}=\begin{pmatrix}1+8\times\frac{110}{81}\\2-\frac{110}{81}\\5+4\times\frac{110}{81}\end{pmatrix}=\begin{pmatrix}\frac{961}{81}\\\frac{52}{81}\\\frac{845}{81}\end{pmatrix}$	A1*; A1
		(3)
(c)	$\cos\theta=\pm\frac{\begin{pmatrix}-15\\4\\-3\end{pmatrix}.\begin{pmatrix}8\\-1\\4\end{pmatrix}}{\sqrt{15^2+4^2+3^2}\sqrt{8^2+1^2+4^2}}=...$	M1
	$=\pm\frac{-136}{45\sqrt{10}}\ (=\pm0.9557...)$ or e.g. $\theta=$ awrt $0.299(rad)/17.1°$	A1
	Area $OAB=\frac{1}{2}5\sqrt{10}\times4\sqrt{10}\sin\theta=...$	M1
	$=$ awrt 29.4	A1
		(4)
		(10 marks)

(a)

M1: Attempts to find the distance, or distance squared, from O to a general point on the line, or to A. Condone slips. It is acceptable to use a different variable rather than λ.

M1: Attempts to use $\left|\overrightarrow{OA}\right| = 5\sqrt{10}$ to set up a quadratic equation in λ, need not be expanded for this mark.

Condone slips on the 250 for OA^2 on this mark. Allow it to appear as $5\times10=50$

A1*: Expands and reaches the correct equation from correct work.

An intermediate line equivalent to $64\lambda^2+16\lambda+1+\lambda^2-4\lambda+4+16\lambda^2+40\lambda+25=250$ must be written out before the final given answer is seen.

(b)

M1: Correct attempt at solving the quadratic equation, (allow a calculator), and substitutes at least one of their values for λ into the equation for the line to find a possible position vector. The $(-15, 4, -3)$ is given so they cannot just write $\lambda=-2$, $A=(-15, 4, -3)$ without some evidence. However, sight of a correct second coordinate $\left(\frac{961}{81}, \frac{52}{81}, \frac{845}{81}\right)$ would imply M1

A1*: Correct value $\lambda=-2$ found, and substituted to reach the given position vector.

Scored for sight of $\overrightarrow{OA} = \begin{pmatrix} 1+8\times-2 \\ 2-(-2) \\ 5+4\times-2 \end{pmatrix} = \begin{pmatrix} -15 \\ 4 \\ -3 \end{pmatrix}$ o.e.

Also allow the coordinate $(-15, 4, -3)$ but not an incorrect hybrid version. E.g. $(-15\mathbf{i}, 4\mathbf{j}, -3\mathbf{k})$

A1: Correct second possible position vector but condone in coordinate form. $\left(\frac{961}{81}, \frac{52}{81}, \frac{845}{81}\right)$ Must be exact. Do not allow hybrid versions but if this form has been penalised already, allow here.

(c)

M1: Attempts the scalar product of $\pm\overrightarrow{OA} = \pm(-15\mathbf{i}+4\mathbf{j}-3\mathbf{k})$ and the direction vector of the line $\pm(8\mathbf{i}-1\mathbf{j}+4\mathbf{k})$ to find $\cos\theta$ for the angle between $\overrightarrow{OA}$ and l_2. Condone slips but the intention must be clear. If the method of finding modulus is not shown award if one of the two is correct. For example, $4\sqrt{10}$ is a common error/slip on the modulus of $\pm(8\mathbf{i}-1\mathbf{j}+4\mathbf{k})$ as this length is given in the question.

Note that it is acceptable to find the scalar product of $\pm\overrightarrow{OA}$ with $\pm(8\mu\mathbf{i}-1\mu\mathbf{j}+4\mu\mathbf{k})$ as long as μ is used consistently within the calculation on the numerator. Note any variable may be used here.

A1: Correct value for $\cos\theta$ (either sign acceptable). Implied by use of $\theta=$ awrt 0.299 rads or awrt 17.1°

M1: Uses formula for area of triangle in a complete method to find the area required. It is dependent upon having found $\cos\theta$ via a scalar product of the two correct vectors.

A1: Correct area, awrt 29.4

Alt: There are alternative methods most of which involve finding the perpendicular distance between the two lines. An example of such a method is shown below.

Alt (c)	Midpoint of possible A's is $M = \frac{1}{81}\begin{pmatrix}-127\\188\\301\end{pmatrix}$ and $OM = \frac{1}{81}\sqrt{127^2+188^2+301^2} = \ldots$	**M1**
	$= 4.6534\ldots$	**A1**
	Area $= \frac{1}{2}\times 4\sqrt{10}\times 4.6534...;= 29.43\ldots$	**M1; A1**
		(4)

M1: Finds the midpoint for the two possible positions for A and proceeds to find the distance from O to this point.
A1: Correct distance between lines.
M1: Uses formula for area of triangle in a complete method to find the area required.
A1: Correct area, awrt 29.4
There may be longer alternatives, such as finding point B first and using a different angle, but the scheme will apply in like manner.

5.Oct 2024-8

8(a)	$\pm(-6\mathbf{i}+4\mathbf{j}-\mathbf{k}-(-10\mathbf{i}+5\mathbf{j}-4\mathbf{k}))$	M1
	e.g. $\mathbf{r}=-10\mathbf{i}+5\mathbf{j}-4\mathbf{k}\pm\lambda(4\mathbf{i}-\mathbf{j}+3\mathbf{k})$ e.g. $\mathbf{r}=-6\mathbf{i}+4\mathbf{j}-\mathbf{k}\pm\lambda(4\mathbf{i}-\mathbf{j}+3\mathbf{k})$	A1
		(2)
(b)	$3+3\mu=-6\Rightarrow\mu=-3$	M1
	$p+12=4\Rightarrow p=\ldots$ or $q-3=-1\Rightarrow q=\ldots$	dM1
	$p=-8,\quad q=2$	A1
		(3)
(c)	$(4\mathbf{i}-\mathbf{j}+3\mathbf{k})\cdot(3\mathbf{i}-4\mathbf{j}+\mathbf{k})=12+4+3$	M1
	$(4\mathbf{i}-\mathbf{j}+3\mathbf{k})\cdot(3\mathbf{i}-4\mathbf{j}+\mathbf{k})=\sqrt{26}\sqrt{26}\cos\theta=19\Rightarrow\cos\theta=\ldots$	M1
	$(\cos\theta=)\ \frac{19}{26}$	A1
		(3)
(d)	$AC=AB\sin\theta=\sqrt{26}\sqrt{1-\left(\frac{19}{26}\right)^2}$	M1
	$\frac{3\sqrt{910}}{26}$ o.e.	A1
		(2)
		Total 10

(a)

M1: Attempts the direction of l_1. The expression is sufficient condoning one sign slip or may be implied by 2 correct components of $\pm(4\mathbf{i}-\mathbf{j}+3\mathbf{k})$

A1: Any correct equation including '$\mathbf{r}=$'. Allow any parameter to be used including μ.

Allow $\mathbf{r}=-10\mathbf{i}+5\mathbf{j}-4\mathbf{k}\pm\lambda(4\mathbf{i}-\mathbf{j}+3\mathbf{k})$, $\mathbf{r}=\begin{pmatrix}-10\\5\\-4\end{pmatrix}\pm\lambda\begin{pmatrix}4\\-1\\3\end{pmatrix}$, $\mathbf{r}=\begin{pmatrix}-6\\4\\-1\end{pmatrix}\pm\lambda\begin{pmatrix}-4\\1\\-3\end{pmatrix}$

Do not accept i, j, k notation in either bracket using column vector notation

e.g. $\mathbf{r}=\begin{pmatrix}-10\\5\\-4\end{pmatrix}\pm\lambda\begin{pmatrix}4\mathbf{i}\\-1\mathbf{j}\\3\mathbf{k}\end{pmatrix}$

(b)

M1: Equates the x component of l_2 to -6 and proceeds to find a value for μ.

dM1: Uses the value of μ to find p or q. It is dependent on the previous method mark. Alternatively,

- forms a correct equation for the x component using their l_1 and l_2
- proceeds to find λ
- uses this value of λ with the value of μ correctly to find p or q by forming an equation for the y or z component.

Do not be concerned by the mechanics of the rearrangement in solving.
A1: Correct values

Alt (b)

M1: Forms three correct equations $3+3\mu=-6,\ p-4\mu=4,\ q+\mu=-1$

dM1: Solves simultaneously to find a value for p or q. Do not be concerned by the mechanics of the rearrangement.

A1: Correct values

(c)

M1: Attempts the scalar product using their direction vectors or may find another direction vector which should be a multiple of the one for l_1 or l_2 (condoning slips e.g. if they restart using another point on l_2 to find a direction vector). Look for at least two correct products using their l_1. May be implied by "±19". May find the obtuse angle which can still score this mark.

M1: Completes the scalar product method using their direction vectors to find cos θ (acute or obtuse). Attempts the magnitude of both of their direction vectors and uses these values in the correct positions in the equation for cos θ

A1: $\frac{19}{26}$ or exact equivalent. Do not accept fractions where the numerator or denominator is not an integer (square roots must be evaluated). Isw if they proceed to find θ

Alt (c) There will be many alternatives such as the one below. Send to review if unsure.

M1: The first mark should be for finding the required components to use in the cosine rule e.g. attempts to find another point P on l_2 and attempts to find the lengths of AB, BP and PA

M1: The second mark should be using all the components required in the cosine rule. E.g.uses their lengths for AB, BP and PA in the correct positions in the cosine rule.

A1: As above in the main scheme notes.

(d)

M1: Fully correct method to find the exact value for the length of AC.
May alternatively attempt e.g.

$$\begin{pmatrix}3\\-4\\1\end{pmatrix}\bullet\begin{pmatrix}13+3\mu\\-13-4\mu\\6+\mu\end{pmatrix}=0\Rightarrow\mu=-\frac{97}{26}\Rightarrow\overrightarrow{AC}=\begin{pmatrix}\frac{47}{26}\\\frac{25}{13}\\\frac{59}{26}\end{pmatrix}\Rightarrow AC=\sqrt{\left(\frac{47}{26}\right)^2+\left(\frac{25}{13}\right)^2+\left(\frac{59}{26}\right)^2}$$

Note the coordinates for C are $\left(-\frac{213}{26},\frac{90}{13},-\frac{45}{26}\right)$ and they may use their coordinates for C with point A to find the distance AC. Condone one sign slip in the distance formula. i.e. there must be sufficient evidence that they were attempting the differences between appropriate coordinates.
Look out for other fully correct methods to find the exact value for the length of AC.
e.g. attempts to find $|BC|$ and uses either trigonometry or Pythagoras' Theorem.

A1: Correct value. Allow $\sqrt{\frac{315}{26}}$ or $3\sqrt{\frac{35}{26}}$

6.Jan 2023-6

6 (a)(i)	$\overrightarrow{AB} = (\pm)\left[\left(8\mathbf{i}+3\mathbf{j}-7\mathbf{k}\right)-\left(2\mathbf{i}-3\mathbf{j}+5\mathbf{k}\right)\right] = \ldots$	**M1**
	$\overrightarrow{AB} = 6\mathbf{i}+6\mathbf{j}-12\mathbf{k}$	**A1**
(ii)	$\mathbf{r} = \begin{pmatrix}2\\-3\\5\end{pmatrix} + \lambda\begin{pmatrix}1\\1\\-2\end{pmatrix}$ o.e. such as $\mathbf{r} = \begin{pmatrix}8\\3\\-7\end{pmatrix} + \lambda\begin{pmatrix}6\\6\\-12\end{pmatrix}$	**B1ft**
		(3)
(b)	Attempts $\pm\overrightarrow{CP} = \pm\begin{pmatrix}2+\lambda-3\\-3+\lambda-5\\5-2\lambda-2\end{pmatrix}$	**M1**
	$\overrightarrow{CP}\bullet k\begin{pmatrix}1\\1\\-2\end{pmatrix} = 0 \Rightarrow \begin{pmatrix}\lambda-1\\\lambda-8\\-2\lambda+3\end{pmatrix}\bullet k\begin{pmatrix}1\\1\\-2\end{pmatrix} = 0 \Rightarrow 1(\lambda-1)+1(\lambda-8)-2(-2\lambda+3)=0$ Alt: $(\lambda-1)^2+(\lambda-8)^2+(-2\lambda+3)^2 = 6\lambda^2-30\lambda+74 = 6\left(\lambda-\frac{5}{2}\right)^2+\frac{73}{2}$	**dM1**
	$\Rightarrow \lambda = \frac{5}{2}$ $\left[\text{use of } \overrightarrow{AB} \text{ in } \overrightarrow{CP} \text{ gives } \lambda = \frac{5}{12}, \text{use of } \overrightarrow{OB}\ \lambda = \frac{-7}{2} \text{ or } \frac{-7}{12}\right]$	**A1**
	$\overrightarrow{OP} = \begin{pmatrix}2\\-3\\5\end{pmatrix} + \frac{5}{2}\begin{pmatrix}1\\1\\-2\end{pmatrix} = \frac{9}{2}\mathbf{i} - \frac{1}{2}\mathbf{j}$	**ddM1, A1** **(5)**
		(8 marks)

Notes:

Accept either form of vector notation throughout. Accept with **i**,**j** and **k** in their column vectors.

(a)(i)

M1: Attempts to subtract vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$ either way around. May be implied by two correct components.

A1: $\overrightarrow{AB} = 6\mathbf{i}+6\mathbf{j}-12\mathbf{k}$ o.e.

(a)(ii)

B1ft: Any correct equation for the line, may use a correct or follow through multiple of $\overrightarrow{AB}$ for direction and with any point on the line. Must start $\mathbf{r} = \ldots$ or accept $x\mathbf{i}+y\mathbf{j}+z\mathbf{k} = \ldots$ ($l = \ldots$ is B0).

(b)

M1: Attempts $\pm\overrightarrow{CP}$ using point C and a general point on their l

dM1: Sets the scalar product of their $\overrightarrow{CP}$ (either direction) and their direction of l (or $\overrightarrow{AB}$) to 0 and proceeds to an equation in λ. Condone sign slips in components if the intention is clear.

Alternatively attempts to minimise the distance CP (by completing square as shown, or by differentiation) to obtain a linear equation in λ.

A1: Finds a correct value of λ for their l. Note if they use $\overrightarrow{AB}$ the correct value is $\frac{5}{12}$

ddM1: Substitutes their λ (from a correct method) into their l

A1: $\overrightarrow{OP} = \frac{9}{2}\mathbf{i} - \frac{1}{2}\mathbf{j}$ Accept as coordinates, and accept $P = \ldots$ instead of $\overrightarrow{OP}$.

7.June 2023-4

4(a)	Attempts direction vector by subtracting $(5\mathbf{i}+6\mathbf{j}+3\mathbf{k})$ and $(4\mathbf{i}+8\mathbf{j}+\mathbf{k})$ either way around.	M1
	E.g. $\mathbf{r}=4\mathbf{i}+8\mathbf{j}+\mathbf{k}+\lambda(\mathbf{i}-2\mathbf{j}+2\mathbf{k})$, $\mathbf{r}=5\mathbf{i}+6\mathbf{j}+3\mathbf{k}+\mu(\mathbf{i}-2\mathbf{j}+2\mathbf{k})$	A1
		(2)
(b)	E.g $\overrightarrow{PC}=\begin{pmatrix}4+\lambda\\8-2\lambda\\1+2\lambda\end{pmatrix}-\begin{pmatrix}2\\-2\\1\end{pmatrix}=\begin{pmatrix}2+\lambda\\10-2\lambda\\2\lambda\end{pmatrix}$	M1
	Uses $\overrightarrow{PC}.(\mathbf{i}-2\mathbf{j}+2\mathbf{k})=0$ E.g. $\Rightarrow 1(2+\lambda)-2(10-2\lambda)+2\times 2\lambda=0\Rightarrow\lambda=...$	dM1
	E.g I $\lambda=2\Rightarrow\mathbf{c}=(4+2)\mathbf{i}+(8-4)\mathbf{j}+(1+4)\mathbf{k}$ E.g II $\mu=1\Rightarrow\mathbf{c}=(5+1)\mathbf{i}+(6-2)\mathbf{j}+(3+2)\mathbf{k}$	ddM1
	$(6,\ 4,\ 5)$	A1
		(4)
(c)	$\overrightarrow{OP'}=\mathbf{p}+2\overrightarrow{PC}=2\mathbf{i}-2\mathbf{j}+\mathbf{k}+2(4\mathbf{i}+6\mathbf{j}+4\mathbf{k})$	M1
	$(10,\ 10,\ 9)$	A1
		(2)
(d)	$\lvert\overrightarrow{PP'}\rvert=2\lvert\overrightarrow{PC}\rvert=2\sqrt{4^2+6^2+4^2}=...$	M1
	$4\sqrt{17}$	A1
		(2)
		Total 10

General rule in this question: If no method is shown look for two "correct" components for their vector

(a) **There are many correct versions so please check the candidates answer carefully**

M1: Attempts the direction of l by subtracting $(5\mathbf{i}+6\mathbf{j}+3\mathbf{k})$ and $(4\mathbf{i}+8\mathbf{j}+\mathbf{k})$ either way around.

If no method is shown look for two correct components of $(\pm 1\mathbf{i}\pm 2\mathbf{j}\pm 2\mathbf{k})$ or a multiple of this

A1: Any correct equation including the lhs of '$\mathbf{r}=$'

Allow in the form $\mathbf{r}=\begin{pmatrix}4\\8\\1\end{pmatrix}+\lambda\begin{pmatrix}1\\-2\\2\end{pmatrix}$ but $l=\begin{pmatrix}4\\8\\1\end{pmatrix}+\lambda\begin{pmatrix}1\\-2\\2\end{pmatrix}$ and is $\mathbf{r}=\begin{pmatrix}4\\8\\1\end{pmatrix}+\lambda\begin{pmatrix}i\\-2j\\2k\end{pmatrix}$ A0

(b)

M1: Attempts the general vector from P to l forming $\overrightarrow{PC}$ where C is a general point on l.

Look for an attempt to subtract their "$\begin{pmatrix}4+\lambda\\8-2\lambda\\1+2\lambda\end{pmatrix}$" and $\begin{pmatrix}2\\-2\\1\end{pmatrix}$ either way around

dM1: Attempts $\overrightarrow{PC}$.their$(\mathbf{i}-2\mathbf{j}+2\mathbf{k})=0$ and solves for λ.

It is dependent upon the previous M and the scalar product of this vector must be attempted with their gradient for the line l to produce and solve a linear equation for λ.

ddM1: Uses their λ to find C. Scored for substituting their correctly found λ into their equation for l

A1: Correct coordinates or vector.

Only penalise an error such as $\begin{pmatrix}6i\\4j\\5k\end{pmatrix}$ once, the first time that it is made. Hence award if a mark has already been withheld for such a mistake.

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(c)
M1: Correct strategy for $\overrightarrow{OP'}$ Can be implied by two correct coordinates using their P and C

For example look for $\overrightarrow{OP'} = \overrightarrow{OP} + 2\times\overrightarrow{PC}$ using the coordinates of C found in part (b).

A1: Correct coordinates or vector

(d)
M1: Correct method for $\left|\overrightarrow{PP'}\right|$ using their values for P and P' or P and C or P' and C. E.g $\left|\overrightarrow{PP'}\right| = 2\times\left|\overrightarrow{PC}\right|$

There are many ways to do this but it must be a complete method, not a distance squared.

A1: CAO

Alternative part (b) using shortest distance

M1: Attempts the general vector from P to l.

Look for an attempt to subtract their "$\begin{pmatrix} 4+\lambda \\ 8-2\lambda \\ 1+2\lambda \end{pmatrix}$" and $\begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}$ either way around

dM1: Finds an expression for d or d^2, differentiates and sets = 0 to find λ.

As it is a quadratic expression an equivalent method would be via completing the square .

$$\text{FYI}: D^2 = (2+\lambda)^2 + (10-2\lambda)^2 + (2\lambda)^2 = 9\lambda^2 - 36\lambda + 104 \Rightarrow \frac{dD^2}{d\lambda} = 18\lambda - 36 = 0 \Rightarrow \lambda = 2$$

ddM1: Uses their λ to find C.

A1: Correct coordinates or vector

Alternative part (b) using Pythagoras' Theorem

M1: Attempts the general vector from P to l.

Look for an attempt to subtract their "$\begin{pmatrix} 4+\lambda \\ 8-2\lambda \\ 1+2\lambda \end{pmatrix}$" and $\begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}$ either way around

dM1: Uses $PC^2 + CB^2 = PB^2$ or $PC^2 + CA^2 = PA^2$ to set up and solve an equation in λ.

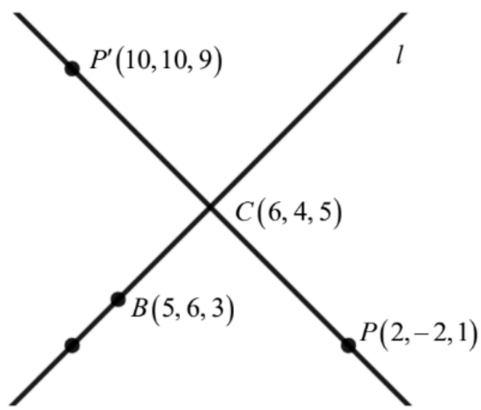
E.g Using $PC^2 + CB^2 = PB^2$ the equations are

$$(2+\lambda)^2 + (10-2\lambda)^2 + (2\lambda)^2 + (\lambda-1)^2 + (2-2\lambda)^2 + (2\lambda-2)^2 = 77$$

$$\Rightarrow 18\lambda^2 - 54\lambda + 36 = 0 \Rightarrow \lambda = 2(1)$$

ddM1: Uses their λ to find C.

A1: Correct coordinates or vector



8.Oct 2023

6(a)	$(2, 3, -7)$	B1
		(1)
(b)	Attempts $\begin{pmatrix}1\\2\\2\end{pmatrix}\bullet\begin{pmatrix}4\\-1\\8\end{pmatrix}=1\times4+2\times-1+2\times8=(18)$	M1
	Attempts $\mathbf{a.b}=\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\cos\theta$: $18=\sqrt{1^2+2^2+2^2}\times\sqrt{4^2+(-1)^2+8^2}\cos\theta$	**d**M1
	$\cos\theta=\frac{2}{3}$	A1
		(3)
(b) Alt	e.g. $\pm\left(\begin{pmatrix}4\\-1\\8\end{pmatrix}-\begin{pmatrix}1\\2\\2\end{pmatrix}\right)=\pm\begin{pmatrix}3\\-3\\6\end{pmatrix}$	M1
	$3^2+3^2+6^2=1^2+2^2+2^2+4^2+(-1)^2+8^2-2\sqrt{1^2+2^2+2^2}\times\sqrt{4^2+(-1)^2+8^2}\cos\theta$	dM1
	$\cos\theta=\frac{2}{3}$	A1
(c)	Uses $\lambda=6$ to find length PQ E.g. $\overrightarrow{PQ}=6\mathbf{i}+12\mathbf{j}+12\mathbf{k}\Rightarrow PQ=\sqrt{6^2+12^2+12^2}=(18)$ Or $PQ=6\times\sqrt{1^2+2^2+2^2}=(18)$	M1
	Area $QPR=\frac{1}{2}ab\sin C=\frac{1}{2}\times18^2\times\sqrt{1-\left(\frac{2}{3}\right)^2}=54\sqrt{5}$	M1 A1
		(3)
(d)	Attempts a correct method of finding at least one value for μ e.g. $(4\mu)^2+\mu^2+(8\mu)^2=6^2+12^2+12^2$ $\Rightarrow\mu=\frac{\text{"18"}}{\sqrt{4^2+(-1)^2+8^2}}=(\pm)2$	M1
	Attempts one correct position $\mathbf{r}=\begin{pmatrix}2\\3\\-7\end{pmatrix}\pm2\begin{pmatrix}4\\-1\\8\end{pmatrix}$	dM1
	Possible coordinates $(10, 1, 9)$ and $(-6, 5, -23)$	A1
		(3)
		(10 marks)

(a)

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B1: Correct coordinates or position vector e.g. as shown or $x = 2$, $y = 3$, $z = -7$ or $2\mathbf{i} + 3\mathbf{j} - 7\mathbf{k}$ or $\begin{pmatrix} 2 \\ 3 \\ -7 \end{pmatrix}$

Condone $(2\mathbf{i}, 3\mathbf{j}, -7\mathbf{k})$

(b)

M1: Attempts $\begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} \bullet \begin{pmatrix} 4 \\ -1 \\ 8 \end{pmatrix}$ condoning slips. Note that any non-zero multiples of these vectors can be used.

If the method is not explicit then this mark may be implied by 2 correct components.

dM1: Full attempt at $\mathbf{a.b} = |\mathbf{a}||\mathbf{b}|\cos\theta$ where **a** and **b** are the direction vectors or multiples of them.

Depends on the first method mark.

Note that $\cos\theta = \dfrac{4 - 2 + 16}{\sqrt{1^2 + 2^2 + 2^2}\sqrt{4^2 + 1^2 + 8^2}}$ would imply both method marks.

A1: $\cos\theta = \dfrac{2}{3}$

(b) Alternative using the cosine rule:

M1: Attempts $\pm\left(\alpha\begin{pmatrix} 4 \\ -1 \\ 8 \end{pmatrix} - \beta\begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}\right)$

dM1: Full attempt at the cosine rule using the appropriate lengths.

Depends on the first method mark.

A1: $\cos\theta = \dfrac{2}{3}$

(c)

M1: Uses $\lambda = 6$ to find the length of PQ. E.g. uses Pythagoras to calculate $|6\mathbf{i} + 12\mathbf{j} + 12\mathbf{k}|$ or $6|\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}|$

If the method is not explicit then the attempt at $\overrightarrow{PQ}$ may be implied by 2 correct components.

M1: Full attempt at $\frac{1}{2}ab\sin C$ where $a = b = |\overrightarrow{PQ}|$ and C is their θ which may be attempted in decimals.

Their θ or $\sin\theta$ must follow an attempt to use their $\cos\theta$.

A1: $54\sqrt{5}$ or exact equivalent e.g. $\dfrac{162\sqrt{5}}{3}$

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(d)

M1: Attempts a **correct** method of finding at least one value for μ.

Pythagoras must be used correctly and both sides of their equation must be consistent e.g.

$(4\mu)^2 - \mu^2 + (8\mu)^2 = 18^2$ and $(4\mu)^2 + \mu^2 + (8\mu)^2 = 18$ both score M0

Must be correct work here so must be using the length of PQ not OQ.

dM1: Attempts one correct position $\mathbf{r} = \begin{pmatrix} 2 \\ 3 \\ -7 \end{pmatrix} \pm 2\begin{pmatrix} 4 \\ -1 \\ 8 \end{pmatrix}$ **Depends on the first method mark.**

A1: Gives both possible coordinates $(10, 1, 9)$ and $(-6, 5, -23)$

Allow as coordinates or vectors or as $x = \ldots, y = \ldots, z = \ldots$

9.Jan 2022-8

8(a)	Attempts $\pm\left(\begin{pmatrix}6\\0\\7\end{pmatrix}-\begin{pmatrix}6\\6\\2\end{pmatrix}\right)=\pm\begin{pmatrix}0\\-6\\5\end{pmatrix}$	**M1**
	$\mathbf{r}=\begin{pmatrix}6\\6\\2\end{pmatrix}+\lambda\begin{pmatrix}0\\-6\\5\end{pmatrix}$ or $\mathbf{r}=\begin{pmatrix}6\\0\\7\end{pmatrix}+\lambda\begin{pmatrix}0\\6\\-5\end{pmatrix}$ oe	**A1**
		(2)
(b)	Meet if $\begin{pmatrix}6\\6\\2\end{pmatrix}+\lambda\begin{pmatrix}0\\-6\\5\end{pmatrix}=\begin{pmatrix}3\\1\\4\end{pmatrix}+\mu\begin{pmatrix}1\\5\\9\end{pmatrix}$ so $\begin{cases}6=3+\mu\\6-6\lambda=1+5\mu\\2+5\lambda=4+9\mu\end{cases}$	**M1**
	From first equation $\mu=3$ Alt: solves equations 2 and 3 to give either $\mu=\frac{13}{79}$ or $\lambda=\frac{55}{79}$	**A1**
	Then need $\lambda=\frac{5-5\times3}{6}=-\frac{5}{3}$ from 2nd equation, $\lambda=\frac{2+9\times3}{5}=\frac{29}{5}$ from 3rd equation Alt: checks their μ in the first equation	**M1**
	Values of λ do not agree and hence lines do not meet.	**A1**
		(4)
(c)	$\overrightarrow{OC}=\begin{pmatrix}3\\1\\4\end{pmatrix}-\begin{pmatrix}1\\5\\9\end{pmatrix}\left(=\begin{pmatrix}2\\-4\\-5\end{pmatrix}\right)$	**M1**
	$\overrightarrow{AC}=\overrightarrow{OC}-\overrightarrow{OA}=\begin{pmatrix}2\\-4\\-5\end{pmatrix}-\begin{pmatrix}6\\6\\2\end{pmatrix}=\begin{pmatrix}-4\\-10\\-7\end{pmatrix}$ Accept $\pm$. **Depends on first mark.**	**dM1**
	$\frac{\pm\overrightarrow{AC}\bullet(\mathbf{i}+5\mathbf{j}+9\mathbf{k})}{\lvert\overrightarrow{AC}\rvert\lvert\mathbf{i}+5\mathbf{j}+9\mathbf{k}\rvert}=\pm\frac{-4\times1+-10\times5+-7\times9}{\sqrt{16+100+49}\sqrt{1+25+81}}$ **Depends on both previous marks.**	**ddM1A1**
	$\Rightarrow\theta=\arccos\left\lvert\frac{-117}{\sqrt{165}\sqrt{107}}\right\rvert=28.3°$ or e.g. $\Rightarrow\theta=90°-\arcsin\left\lvert\frac{-117}{\sqrt{165}\sqrt{107}}\right\rvert=28.3°$	**A1**
		(5)
		(11 marks)

Notes:

(a)

M1: Attempts the difference between the given two vectors. Implied by 2 out of three correct coordinates if no method is shown. Can be either way round.

A1: Any correct equation for the line in the form given and **must have** "**r** = ….". Two examples are shown in the scheme, but they may use any point on the line as the starting point, and any multiple of the direction vector.

Allow in **i**, **j**, **k** form but not e.g. $\mathbf{r}=\begin{pmatrix}6\mathbf{i}\\6\mathbf{j}\\2\mathbf{k}\end{pmatrix}+\lambda\begin{pmatrix}0\\-6\mathbf{j}\\5\mathbf{k}\end{pmatrix}$ but condone missing brackets on the position vector so

condone e.g. $\mathbf{r}=\begin{matrix}6\\6\\2\end{matrix}+\lambda\begin{pmatrix}0\\-6\\5\end{pmatrix}$ but not on the direction so $\mathbf{r}=\begin{pmatrix}6\\6\\2\end{pmatrix}+\lambda\begin{matrix}0\\-6\\5\end{matrix}$ is A0

(b)

M1: Equates their line and the given line and extracts at least one equation in λ **and** μ

A1: **"**Correct" value for one parameter. This will depend on which equation(s) they solve – see main scheme **but must follow M1**.

M1: Substitutes into both other equations to compare results.

A1: Correct values for λ found with conclusion that the lines do not meet.

Alt: If equation 1 is not first used to find μ then the 1st A mark can be awarded for any correct value for λ or μ from the equations they solve, and the M mark for checking their solutions for consistency in the remaining equation.

For the final A, all work must have been correct with a suitable conclusion made.

When checking, allow e.g. $2-\frac{25}{3}\neq 4+27$ **so they do not meet. I.e. the calculations may not be fully carried out as long as the result is obvious. However if either side is subsequently evaluated incorrectly, withhold the final mark.**

FYI: equations (1) and (2) give $\lambda=-\frac{5}{3}$ or $\mu=3$; equations (1) and (3) give $\lambda=\frac{29}{5}$ or $\mu=3$ and equations (2) and (3) give $\lambda=\frac{55}{79}$ or $\mu=\frac{13}{79}$ (use of the negative direction vectors reverse the signs of λs)

If $\mathbf{r}=\begin{pmatrix}6\\0\\7\end{pmatrix}+\lambda\begin{pmatrix}0\\6\\-5\end{pmatrix}$ is used equations (1) and (2) give $\lambda=\frac{8}{3}$ or $\mu=3$; equations (1) and (3) give $\lambda=-\frac{24}{5}$ or $\mu=3$ and equations (2) and (3) give $\lambda=\frac{24}{79}$ or $\mu=\frac{13}{79}$ (use of the negative direction vectors reverse the signs of λs)

Note that the maximum marks in (b) if their line in (a) is incorrect is 1010

There will be many different approaches to part (b).
The general structure for marking is as follows:
M1: Equates lines and obtains at least one equation in λ and μ
A1: Correct value for one of the parameters
M1: Attempts the checking process to show the lines do not meet
A1: All correct with a conclusion

(c)

M1: Substitutes $\mu = -1$ into l_2 to find the coordinates for C or the vector $\overrightarrow{OC}$

dM1: Uses their $\overrightarrow{OC}$ to find $\pm\overrightarrow{AC}$. Look for an attempt at the difference of vectors with at least two correct coordinates.

ddM1: Attempts $\dfrac{\pm\overrightarrow{AC}\bullet(\mathbf{i}+5\mathbf{j}+9\mathbf{k})}{|\overrightarrow{AC}||\mathbf{i}+5\mathbf{j}+9\mathbf{k}|}$ with their $\overrightarrow{AC}$

This requires an attempt at the scalar product of their $\overrightarrow{AC}$ with $\mathbf{i}+5\mathbf{j}+9\mathbf{k}$ in the numerator with at least 2 "components" correct if no method is shown, and correct attempts at the product of the magnitudes of $\overrightarrow{AC}$ and the direction of $\mathbf{i}+5\mathbf{j}+9\mathbf{k}$ in the denominator.

A1: A correct expression simplified or unsimplified e.g. $\pm\left(\dfrac{-4\times1+-10\times5+-7\times9}{\sqrt{16+100+49}\sqrt{1+25+81}}\right)$, $\pm\dfrac{117}{\sqrt{17655}}$

A1: Correct answer. Accept awrt 28.3

10.June 2022-6

6(a)	$\overrightarrow{AB}=\begin{pmatrix}5-1\\3--4\\-2-3\end{pmatrix}=\begin{pmatrix}4\\7\\-5\end{pmatrix}=4\mathbf{i}+7\mathbf{j}-5\mathbf{k}$	M1
	e.g. $\mathbf{r}=\mathbf{i}-4\mathbf{j}+3\mathbf{k}+\lambda(4\mathbf{i}+7\mathbf{j}-5\mathbf{k})$ or $\mathbf{r}=5\mathbf{i}+3\mathbf{j}-2\mathbf{k}+\lambda(4\mathbf{i}+7\mathbf{j}-5\mathbf{k})$	M1A1
		(3)
(b)	$\overrightarrow{AC}=\begin{pmatrix}3-1\\p--4\\-1-3\end{pmatrix}=\begin{pmatrix}2\\p+4\\-4\end{pmatrix}=2\mathbf{i}+(p+4)\mathbf{j}-4\mathbf{k}$	M1
	$\begin{pmatrix}2\\p+4\\-4\end{pmatrix}\bullet\begin{pmatrix}4\\7\\-5\end{pmatrix}=8+7p+28+20=0\Rightarrow p=-8$	M1A1
		(3)
(c)	$\lvert AB\rvert=\sqrt{4^2+7^2+(-5)^2}=\sqrt{90}$ or $\lvert AC\rvert=\sqrt{2^2+(-4)^2+(-4)^2}=6$	M1
	Area $\frac{1}{2}\times"\sqrt{90}"\times"6"=9\sqrt{10}$	dM1A1
		(3)
		(9 marks)

Notes
Accept either vector form throughout but extra i, j k in column vectors will lose A mark in (a). **(a) This is now being marked MMA** **M1:** Attempts to find $\overrightarrow{AB}$. Score for subtracting either way round. Implied by 2 out of 3 correct coordinates. **M1:** Attempts equation for the line, score for $\overrightarrow{OA}+\lambda\times$ their $\overrightarrow{AB}$ or $\overrightarrow{OB}+\lambda\times$ their $\overrightarrow{AB}$ No need for **r** = for this mark. **A1:** Any correct equation. Must be **r** = … (l = .. is A0) **(b)** **M1:** Attempts to find $\overrightarrow{AC}$. Score for subtracting either way round. Implied by 2 out of 3 correct coordinates. **M1:** Takes scalar product of their $\overrightarrow{AB}$ and their $\overrightarrow{AC}$ to form and solve a linear equation in p **A1:** $p=-8$ **(c)** **M1:** Attempts to find the magnitude of either their $\overrightarrow{AB}$ or their $\overrightarrow{AC}$ using their p **dM1:** Attempts to find the exact area of the triangle ABC. It is dependent on the previous method mark. There most common method will be $\frac{1}{2}\lvert\overrightarrow{AB}\rvert\lvert\overrightarrow{AC}\rvert$ as in scheme but other methods are possible. E.g. $\cos\angle ABC=\frac{\overrightarrow{BA}.\overrightarrow{BC}}{\lvert\overrightarrow{BA}\rvert\lvert\overrightarrow{BC}\rvert}\Rightarrow A=\frac{1}{2}\lvert\overrightarrow{BA}\rvert\lvert\overrightarrow{BC}\rvert\sin\angle ABC$. Such a method must be complete, including use of Pythagorean identity to find $\sin\angle ABC$. Other more advanced methods (such as cross products) are also possible. If you see something you feel is worthy of some credit but does not fit the scheme, send to Review. **A1:** $9\sqrt{10}$

11.Oct 2022-3

3 (a)	Attempts $(2\mathbf{i}-3\mathbf{j}+4\mathbf{k})-(8\mathbf{i}-5\mathbf{j}+3\mathbf{k})$	M1
	$\overrightarrow{RQ}=-6\mathbf{i}+2\mathbf{j}+\mathbf{k}$	A1
		(2)
(b)	Attempts $\overrightarrow{PQ}\cdot\overrightarrow{RQ}=2\times-6+-3\times2+4\times1$	M1
	Full attempt to find $\cos PQR$ E.g. $2\times-6+-3\times2+4\times1=\sqrt{29}\sqrt{41}\cos PQR$	dM1
	Angle $PQR = 114°$	A1
		(3)
		(5 marks)

(a)

M1: Attempts to subtract vectors $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ either way around. Look for $\overrightarrow{PQ}-\overrightarrow{PR}$ or $\overrightarrow{PR}-\overrightarrow{PQ}$. If a method is not shown it can be implied by two correct components of $\pm 6\mathbf{i}\pm 2\mathbf{j}\pm \mathbf{k}$ Note that an attempt such as $\overrightarrow{PR}-\overrightarrow{QP}$ is M0

A1: $\overrightarrow{RQ}=-6\mathbf{i}+2\mathbf{j}+\mathbf{k}$ o.e. such as $\begin{pmatrix}-6\\2\\1\end{pmatrix}$ Do not accept coordinates or indeed $\begin{pmatrix}-6\mathbf{i}\\2\mathbf{j}\\\mathbf{k}\end{pmatrix}$

(b)

M1: Attempts scalar product of $\overrightarrow{PQ}$ and their $\overrightarrow{RQ}$. Look for an attempt at multiplying together the components and adding. There will be some confusion over direction so allow for sight of

$$\left(\pm 2\times "\pm 6"\right)+\left(\pm 3\times "\pm 2"\right)+\left(\pm 4\times "\pm 1"\right)$$

This cannot be scored if they attempt a scalar product of $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ for instance.

dM1: Full attempt to find $\cos PQR$ using $\mathbf{a}.\mathbf{b}=|\mathbf{a}|\,|\mathbf{b}|\cos\theta$ using vectors $\pm\overrightarrow{PQ}$ and their $\pm\overrightarrow{RQ}$.

There must be an attempt at both moduli with at least one correct (which may be unsimplified)

but you should ft on their $\overrightarrow{RQ}$. Don't be concerned whether the angle is acute or obtuse.

A1: Angle PQR = awrt 114° ISW after sight of this, e.g followed by 66°

Alt (b)

M1: Attempts all three lengths or all three lengths^2 using Pythagoras' Theorem. Look for an attempt to square and add with at least one modulus or modulus^2 correct.

dM1: Attempts to use the cosine rule with the lengths in the correct positions in order to find angle PQR

Look for $\cos PQR=\dfrac{PQ\ +QR\ -PR}{2\times PQ\times QR}$

There are more round about methods including finding other angles first and then using the sine rule

but this method mark can only be awarded when an attempt is made at angle PQR

A1: Angle PQR = awrt 114°

It must be found using "correct" vectors, e.g . $\pm\left(-6\mathbf{i}+2\mathbf{j}+\mathbf{k}\right)$ and $\pm\left(2\mathbf{i}-3\mathbf{j}+4\mathbf{k}\right)$

If, for example $\overrightarrow{RQ}$ is incorrect, e.g. $6\mathbf{i}-2\mathbf{j}+\mathbf{k}$, A0 will be awarded even if 114° is stated

12.Oct 2022-9

9	$\begin{pmatrix}2-\lambda\\8+2\lambda\\10+3\lambda\end{pmatrix}=\begin{pmatrix}-4+5\mu\\-1+4\mu\\2+8\mu\end{pmatrix}$ Attempts to solve any two of the three equations Either (1) and (2) $\left.\begin{matrix}2-\lambda=-4+5\mu\\8+2\lambda=-1+4\mu\end{matrix}\right\}\Rightarrow\lambda=-\frac{3}{2},\mu=\frac{3}{2}$ (1) and (3) $\left.\begin{matrix}2-\lambda=-4+5\mu\\10+3\lambda=2+8\mu\end{matrix}\right\}\Rightarrow\lambda=\frac{8}{23},\mu=\frac{26}{23}$ (2) and (3) $\left.\begin{matrix}8+2\lambda=-1+4\mu\\10+3\lambda=2+8\mu\end{matrix}\right\}\Rightarrow\lambda=-10,\mu=-\frac{11}{4}$	M1, A1
	Substitutes their values of λ and μ into both sides of the "third" equation E.g. $\lambda=-\frac{3}{2}$ into $10+3\lambda=\frac{11}{2}$ and $\mu=\frac{3}{2}$ into $2+8\mu=14$	dM1
	Concludes that lines don't intersect with correct calculations and minimal reason	A1
	Additionally states that $\begin{pmatrix}-1\\2\\3\end{pmatrix}$ is not parallel to $\begin{pmatrix}5\\4\\8\end{pmatrix}$ with a minimal reason So lines are skew CSO *	A1*
		(5) **(5 marks)**

Notes:
Main method seen
M1: Attempts to solve two of the three equations.
Accept as an attempt, writing down two of the three equations (condoning slips) followed by values for
both λ and μ
A1: Solves two of the three equations to find correct values for both λ and μ, Allow equivalent fractions
dM1: Either: Substitutes their values of λ and μ into both sides of the third equation....or into the equations of both lines to find both coordinates
A1: Having achieved correct values for λ and μ, the values for the third equation are found to enable a comparison to be made. E.g. solving equations (1) and (2) and using equation (3) stating $10+3\times-\frac{3}{2}\neq 2+8\times\frac{3}{2}$ is sufficient. If the values are found they must be correct.
Important: Additionally, to score this mark, a minimal statement must be made that states that the lines do not intersect /cross. Condone statements such as $l_1 \neq l_2$
Stating that the lines are skew at this point is not sufficient to score this mark
In the alternative stating that "as the values are not the same, the lines cannot intersect" is sufficient.

A1*: CSO . Hence all previous marks must have been scored.
In addition to not intersecting there must be a statement, with a minimal reason, that the lines are not parallel and hence skew. Accept statements like, not intersecting, not parallel (with reason), hence proven.

Reasons could be $\begin{pmatrix}5\\4\\8\end{pmatrix}\neq k\begin{pmatrix}-1\\2\\3\end{pmatrix}$ o.e such as $5 = -5\times-1$ but $4 = 2\times2$ so they are not parallel.

Accept an argument based around the scalar product of the direction vectors. If parallel $\cos\theta = 1$

A reason for the lines not being parallel cannot be $\begin{pmatrix}5\\4\\8\end{pmatrix}\neq\begin{pmatrix}-1\\2\\3\end{pmatrix}$

Note: Other methods are possible and it is important that you look at their complete attempt at proving that they don't intersect.
Alternative 1
For example it is possible to solve equations (1) and (2) to find just λ
then solve equations (1) and (3) to find just λ
and then conclude that "as the two values are not the same, the lines don't intersect"

M1 dM1 marks are scored together. Both aspects have to be attempted
Attempts to solve two of the three equations to find λ (or μ)
Attempts to solve a different pair of equations to find λ (or μ)
A1: Correct values for λ (or μ).
A1: conclude that "as the two values are not the same, the lines don't intersect"

If you see something that you feel deserves credit AND that you cannot mark, then please send to review

13.Jan 2021-8

8	$\begin{pmatrix}-1\\5\\4\end{pmatrix}+\lambda\begin{pmatrix}2\\-1\\5\end{pmatrix}=\begin{pmatrix}2\\-2\\-5\end{pmatrix}+\mu\begin{pmatrix}4\\-3\\b\end{pmatrix}\Rightarrow \begin{matrix}-1+2\lambda=2+4\mu\ (1)\\5-\lambda=-2-3\mu\ (2)\\4+5\lambda=-5+\mu b\ (3)\end{matrix}$	
	Uses equations (1) and (2) to find either λ **or** μ e.g. $(1)+2(2)\Rightarrow\mu=...$ or $3(1)+4(2)\Rightarrow\lambda=...$	M1
	Uses equations (1) and (2) to find both λ and μ	**d**M1
	$\mu=-\frac{11}{2}$ and $\lambda=-\frac{19}{2}$	A1
	$4+5\lambda=-5+\mu b\Rightarrow 4+5\times-\frac{19}{2}=-5-\frac{11}{2}b$ or $4+5\lambda=-5+7\mu\Rightarrow 4+5\times-\frac{19}{2}=-5-\frac{11}{2}\times 7$	**dd**M1
	$\Rightarrow 11b=77\Rightarrow b=7$ or obtains $-\frac{87}{2}=-\frac{87}{2}$	A1
	States that when $b=7$, lines intersect or when $b\neq 7$, lines do not intersect Lines are not parallel so when $b\neq 7$ lines are skew. *	A1 **Cso**
		(6)

	Alternative assuming $b=7$:	
	$\begin{pmatrix}-1\\5\\4\end{pmatrix}+\lambda\begin{pmatrix}2\\-1\\5\end{pmatrix}=\begin{pmatrix}2\\-2\\-5\end{pmatrix}+\mu\begin{pmatrix}4\\-3\\7\end{pmatrix}\Rightarrow \begin{matrix}-1+2\lambda=2+4\mu\ (1)\\5-\lambda=-2-3\mu\ (2)\\4+5\lambda=-5+7b\ (3)\end{matrix}$	
	Uses any 2 equations to find either λ **or** μ	M1
	Uses any 2 equations to find both λ and μ	**d**M1
	$\mu=-\frac{11}{2}$ and $\lambda=-\frac{19}{2}$	A1
	Checks in the 3rd equation e.g. equation 3: $4+5\left(-\frac{19}{2}\right)=-5+7\left(-\frac{11}{2}\right)=...$ equation 1: $-1+2\left(-\frac{19}{2}\right)=2+4\left(-\frac{11}{2}\right)=...$ equation 2: $5-\left(-\frac{19}{2}\right)=-2-3\left(-\frac{11}{2}\right)=...$	**dd**M1
	Equation 3: $-\frac{87}{2}$ Equation 1: -20 Equation 2: $\frac{29}{2}$	A1
	States that when $b=7$, lines intersect or when $b\neq 7$, lines do not intersect Lines are not parallel so when $b\neq 7$ lines are skew. *	A1 **Cso**
		(6 marks)

EDUCATION (KHDA APPROVED)

M1: For attempting to solve equations (1) and (2) to find **either** λ **or** μ

dM1: For attempting to solve equations (1) and (2) to find **both** λ **and** μ **Depends on the first M.**

A1: $\mu = -\frac{11}{2}$ and $\lambda = -\frac{19}{2}$

ddM1: Attempts to solve $4+5\lambda = -5+\mu b$ for their values of λ and μ. Or uses $b = 7$ with their λ and μ in an attempt to show equality. **Depends on both previous M's.**

A1: Achieves (without errors) that they will intersect when $b = 7$

Note that the previous 3 marks may be scored without explicitly seeing the values of both parameters e.g.

$\mu = -\frac{11}{2}$, (2) → $\lambda = 3\mu + 7$ → $4 + 5(3\mu + 7) = -5 + \mu b$ → $b = 7$

A1*:**Cso** States that when $b = 7$, lines intersect and since lines are not parallel it shows that when $b \neq 7$ lines are skew.

Alternative:

M1: Uses $b = 7$ and attempts to solve 2 equations to find **either** λ **or** μ

dM1: For attempting to solve 2 equations to find **both** λ **and** μ **Depends on the first M.**

A1: $\mu = -\frac{11}{2}$ and $\lambda = -\frac{19}{2}$

ddM1: Attempts to show that the 3rd equation is true for their values of λ and μ

Depends on both previous M's.

A1: Achieves (without errors) that the 3rd equation gives the same values for (or equivalent)

A1*: **Cso** States that when $b = 7$, lines intersect and since lines are not parallel it shows that when $b \neq 7$ lines are skew.

To score the final mark there must be some statement that the lines intersect (or equivalent e.g. meet at a point, cross, etc.) when $b = 7$ or that they do not intersect if $b \neq 7$ **and** that the lines are not parallel which may appear anywhere (reason not needed but may be present) so lines are skew when $b \neq 7$.

Ignore any work attempting to show that the lines are perpendicular or not.

14.Jan 2021-2

2(a)	$\overrightarrow{BA}.\overrightarrow{BC}=-6\times2+2\times5-3\times8=(-26)$	M1
	Uses $\overrightarrow{BA}.\overrightarrow{BC}=\lvert\overrightarrow{BA}\rvert\lvert\overrightarrow{BC}\rvert\cos\theta\Rightarrow-26=\sqrt{49}\times\sqrt{93}\cos\theta\Rightarrow\theta=...$	**d**M1
	$\theta=112.65°$	A1
		(3)
(b)	Attempts to use $\lvert\overrightarrow{BA}\rvert\lvert\overrightarrow{BC}\rvert\sin\theta$ with their θ	M1
	Area = awrt 62.3	A1
		(2)
		(5 marks)

(a)

M1: Attempts the scalar product of $\pm\overrightarrow{AB}.\pm\overrightarrow{BC}$ condone slips as long as the intention is clear

Or attempts the vector product $\pm\overrightarrow{AB}\times\pm\overrightarrow{BC}$ **(see alternative 1)**

Or attempts vector AC **(see alternative 2)**

dM1: Attempts to use $\pm\overrightarrow{AB}.\overrightarrow{BC}=\lvert\overrightarrow{AB}\rvert\lvert\overrightarrow{BC}\rvert\cos\theta$ AND proceeds to a value for θ

Expect to see at least one correct attempted calculation for a modulus.

For example $\sqrt{2^2+5^2+8^2}\left(=\sqrt{93}\right)$ or $\sqrt{6^2+2^2+3^2}\ (=7)$

Note that we condone poor notation such as: $\cos\theta=\frac{26}{7\sqrt{93}}=67.35°$ **Depends on the first mark.**

<u>**Must be an attempt to find the correct angle.**</u>

A1: $\theta=$ awrt $112.65°$ Versions finishing with $\theta=$ awrt $67.35°$ will normally score M1 dM1 A0

Angles given in radians also score A0 (NB $\theta=1.9661...$ or acute $1.1754...$)

Allow e.g. $\theta=67.35°\Rightarrow\theta=180-67.35°=112.65$ and allow $\cos\theta=\frac{26}{7\sqrt{93}}\Rightarrow\theta=112.65$

<u>**1. Alternative using the vector product:**</u>

M1: Attempts the vector product $\pm\overrightarrow{AB}\times\pm\overrightarrow{BC}=\pm\begin{pmatrix}6\\-2\\3\end{pmatrix}\times\pm\begin{pmatrix}2\\5\\8\end{pmatrix}=\pm\begin{pmatrix}-31\\-42\\34\end{pmatrix}$ condone slips as long as the intention is clear

dM1: Attempts to use $\pm\overrightarrow{AB}\times\overrightarrow{BC}=\lvert\overrightarrow{AB}\rvert\lvert\overrightarrow{BC}\rvert\sin\theta$ AND proceeds to a value for θ

Expect to see at least one correct attempted calculation for a modulus on rhs and attempt at the modulus of the vector product

For example $\sqrt{2^2+5^2+8^2}$ or $\sqrt{6^2+2^2+3^2}$ and $\sqrt{31^2+42^2+34^2}\left(=\sqrt{3881}\right)$

Note that we condone poor notation such as: $\sin\theta = \frac{\sqrt{3881}}{7\sqrt{93}} = 67.35°$ **Depends on the first mark.**

Must be an attempt to find the correct angle.

A1: $\theta =$ awrt $112.65°$ Versions finishing with $\theta =$ awrt $67.35°$ will normally score M1 dM1 A0

2. Alternative using cosine rule:

M1: Attempts $\pm\overrightarrow{AC} = \pm\left(\overrightarrow{AB} + \overrightarrow{BC}\right) = \pm(8\mathbf{i} + 3\mathbf{j} + 11\mathbf{k})$ condone slips and poor notation as long as the intention is clear e.g. allow $\begin{pmatrix} 8\mathbf{i} \\ 3\mathbf{j} \\ 11\mathbf{k} \end{pmatrix}$

dM1: Attempts to use $AC^2 = AB^2 + BC^2 - 2AB.BC\cos\theta$ AND proceeds to a value for θ

Must be an attempt to find the correct angle.

A1: $\theta =$ awrt $112.65°$

(b)

M1: Attempts to use $\left|\overrightarrow{AB}\right|\left|\overrightarrow{BC}\right|\sin\theta$ with their θ. You may see $\frac{1}{2}\left|\overrightarrow{AB}\right|\left|\overrightarrow{BC}\right|\sin\theta$ found first before it is doubled.

or attempts the magnitude of their vector product e.g. $\sqrt{3881}$

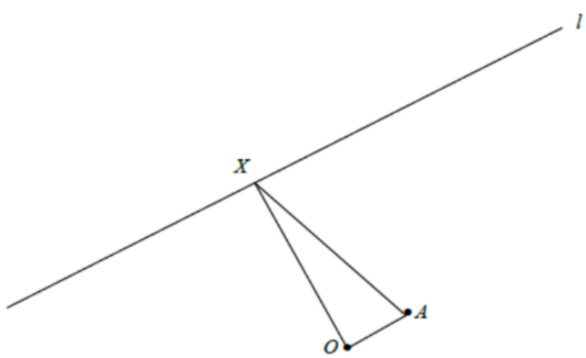
A1: Area = awrt 62.3. If this is achieved from an angle of $\theta =$ awrt $67.35°$ full marks can be scored

Note that there are other more convoluted methods for finding the area – score M1 for a complete and correct method using their values and send to review if necessary.

15.June 2021-7

7 (a)	Finds $\left\lvert\begin{pmatrix}4\\4\\2\end{pmatrix}\right\rvert = \sqrt{4^2+4^2+2^2} = 6$ and attempts $\frac{1}{"6"}\begin{pmatrix}4\\4\\2\end{pmatrix}$	M1
	$\overrightarrow{OA} = \begin{pmatrix}2/3\\2/3\\1/3\end{pmatrix}$ oe	A1
		(2)
(b)	Co-ordinates or position vector of point $X = \begin{pmatrix}1+4\lambda\\-10+4\lambda\\-9+2\lambda\end{pmatrix}$	M1
	$\overrightarrow{OX}.\begin{pmatrix}4\\4\\2\end{pmatrix} = 0 \Rightarrow 4(1+4\lambda)+4(-10+4\lambda)+2(-9+2\lambda)=0$	dM1
	$36\lambda = 54 \Rightarrow \lambda = 1.5$	ddM1 A1
	$X = (7,-4,-6)$	A1
		(5)
(c)	Finds $OX = \sqrt{7^2+(-4)^2+(-6)^2} = \sqrt{101}$ and $OA = 1$	M1
	Area $OXA = \frac{1}{2}\times 1\times\sqrt{101} = \frac{\sqrt{101}}{2}$	dM1 A1
		(3)
		(10 marks)

Handy diagram



(a)

M1: Correct attempt at the unit vector. Look for an attempt at $\sqrt{4^2+4^2+2^2}$ and use of $\frac{\mathbf{r}}{|\mathbf{r}|}$ where $\mathbf{r} = \begin{pmatrix}4\\4\\2\end{pmatrix}$

or equivalent vector form such as $\mathbf{r} = (4\mathbf{i}+4\mathbf{j}+2\mathbf{k})$. Condone an attempt to find the coordinates of A

UCATION
DA APPROVED)

A1: $\overrightarrow{OA}=\begin{pmatrix}2/3\\2/3\\1/3\end{pmatrix}$ or equivalent such as $\overrightarrow{OA}=\frac{1}{6}\begin{pmatrix}4\\4\\2\end{pmatrix}$ or $\overrightarrow{OA}=\frac{1}{6}(4\mathbf{i}+4\mathbf{j}+2\mathbf{k})$ but **must be in vector form and not in coordinate form**.

(b)

M1: For an attempt at the co-ordinates or position vector of point $X=(1+4\lambda,-10+4\lambda,-9+2\lambda)$ **or** $\begin{pmatrix}1+4\lambda\\-10+4\lambda\\-9+2\lambda\end{pmatrix}$

dM1: For using $\overrightarrow{OX}.\begin{pmatrix}4\\4\\2\end{pmatrix}=0$ to set up an equation in λ

Alternatively finds $OX^2=(1+4\lambda)^2+(-10+4\lambda)^2+(-9+2\lambda)^2$ and attempts to differentiate and set = 0

May use $OX^2+OA^2=AX^2$ to set up an equation in λ

ddM1: Solves for λ This is dependent upon having scored both previous M's

A1: Correct value for $\lambda=1.5$

A1: Correct coordinates for $X=(7,-4,-6)$. Condone position vector form $7\mathbf{i}-4\mathbf{j}-6\mathbf{k}$ o.e.

(c)

M1: Finds all elements required to calculate the area.

In the main scheme this would be the distance OX or OX^2 AND distance OA or OA^2 (which is 1)

dM1: Correct method of finding the area of $OXA=\frac{1}{2}\times OX\times 1$

A1: Area = $\frac{\sqrt{101}}{2}$ o.e

There are various alternatives for part (c). Amongst others are;

Alt I **via vector product**.

$$\frac{1}{2}\times\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\7&-4&-6\\2/3&2/3&1/3\end{vmatrix}=\frac{1}{2}\times\left|\frac{8}{3}\mathbf{i}-\frac{19}{3}\mathbf{j}+\frac{22}{3}\mathbf{i}\right|=\frac{1}{2}\times\sqrt{\left(\frac{8}{3}\right)^2+\left(-\frac{19}{3}\right)^2+\left(\frac{22}{3}\right)^2}=\frac{\sqrt{101}}{2}$$

M1: For an attempt at $\frac{1}{2}\times\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\7&-4&-6\\2/3&2/3&1/3\end{vmatrix}=\frac{1}{2}\times\left|\frac{8}{3}\mathbf{i}-\frac{19}{3}\mathbf{j}+\frac{22}{3}\mathbf{i}\right|$

dM1: Followed by an attempt at finding the modulus of the resulting vector multiplied by 1/2

Alt II **via scalar products**

M1: Attempts to find all three components required to find the area triangle OXA

E.g. Angle OXA with length of side OX and XA

Alternatively angle OAX with length of side OA and XA

For this to be scored

- appropriate gradient vectors need to be attempted by subtracting
- a correct attempt at using $\mathbf{a}.\mathbf{b}=|\mathbf{a}||\mathbf{b}|\cos\theta$ to find $\cos\theta$ or θ

dM1: Full attempt at area of triangle using $\frac{1}{2}|OX||XA|\sin(OXA)$ or equivalent

16.Oct 2021-7

7 (a)	Co-ordinates or position vector of a point on l = $\begin{pmatrix} 4-4\lambda \\ 2-3\lambda \\ -3+5\lambda \end{pmatrix}$	
	$\overrightarrow{AX} = \begin{pmatrix} 4-4\lambda \\ 2-3\lambda \\ -3+5\lambda \end{pmatrix} - \begin{pmatrix} 9 \\ -3 \\ 2 \end{pmatrix} = \begin{pmatrix} -5-4\lambda \\ 5-3\lambda \\ -5+5\lambda \end{pmatrix}$	M1
	Uses $\overrightarrow{AX}.\begin{pmatrix} -4 \\ -3 \\ 5 \end{pmatrix} = 0 \Rightarrow 20+16\lambda-15+9\lambda-25+25\lambda=0 \Rightarrow \lambda = \frac{2}{5}$	dM1 A1
	(i) Substitutes their λ into $\begin{pmatrix} 4-4\lambda \\ 2-3\lambda \\ -3+5\lambda \end{pmatrix} \Rightarrow X = \left(\frac{12}{5}, \frac{4}{5}, -1\right)$	dM1 A1
	(ii) $\overrightarrow{AX} = -\frac{33}{5}\mathbf{i} + \frac{19}{5}\mathbf{j} - 3\mathbf{k}$	
	Shortest distance $= \sqrt{\left(-\frac{33}{5}\right)^2 + \left(\frac{19}{5}\right)^2 + (-3)^2} = \sqrt{67}$	M1 A1
		(7)
(b)	Uses $\overrightarrow{AX} = \overrightarrow{XB}$ or similar correct method	M1
	point B has position vector $-\frac{21}{5}\mathbf{i} + \frac{23}{5}\mathbf{j} - 4\mathbf{k}$	A1
		(2) **(9 marks)**

7 (a) ALT way	Co-ordinates or position vector of a point on l = $\begin{pmatrix} 4-4\lambda \\ 2-3\lambda \\ -3+5\lambda \end{pmatrix}$	
	$\overrightarrow{AX} = \begin{pmatrix} 4-4\lambda \\ 2-3\lambda \\ -3+5\lambda \end{pmatrix} - \begin{pmatrix} 9 \\ -3 \\ 2 \end{pmatrix} = \begin{pmatrix} -5-4\lambda \\ 5-3\lambda \\ -5+5\lambda \end{pmatrix}$	M1
	Uses $d^2 = (-5-4\lambda)^2 + (5-3\lambda)^2 + (-5+5\lambda)^2$ which is smallest when its gradient =0 $\Rightarrow -8(-5-4\lambda) - 6(5-3\lambda) + 10(-5+5\lambda) = 0 \Rightarrow \lambda = \frac{2}{5}$	dM1 A1
	(Note that this could be done by completing the square)	
	(i) Substitutes their λ into $\begin{pmatrix} 4-4\lambda \\ 2-3\lambda \\ -3+5\lambda \end{pmatrix} \Rightarrow X = \left(\frac{12}{5}, \frac{4}{5}, -1\right)$	dM1 A1
	(ii) Shortest distance = $\left(-5-4\times\frac{2}{5}\right)^2 + \left(5-3\times\frac{2}{5}\right)^2 + \left(-5+5\times\frac{2}{5}\right)^2 \Rightarrow d = \sqrt{67}$	M1 A1
		(7)

(a)

M1: States or uses a general point on $l = \begin{pmatrix} 4-4\lambda \\ 2-3\lambda \\ -3+5\lambda \end{pmatrix}$ and attempt $\overrightarrow{AX} = \begin{pmatrix} 4-4\lambda \\ 2-3\lambda \\ -3+5\lambda \end{pmatrix} - \begin{pmatrix} 9 \\ -3 \\ 2 \end{pmatrix} = \begin{pmatrix} -5-4\lambda \\ 5-3\lambda \\ -5+5\lambda \end{pmatrix}$

either way around with their general point. Condone slips

dM1: Uses a correct method to find a value for λ

This could be

- via scalar products $\overrightarrow{AX}.\begin{pmatrix} -4 \\ -3 \\ 5 \end{pmatrix} = 0$
- via differentiation using minimum distance of $(-5-4\lambda)^2 + (5-3\lambda)^2 + (-5+5\lambda)^2$ in this method condone slips on the differentiation

A1: $\lambda = \frac{2}{5}$

(i)

dM1: Uses their λ to find the coordinates (or position vector) of X This is dependent upon the previous M

A1: Finds coordinates for $X = \left(\frac{12}{5}, \frac{4}{5}, -1\right)$ Condone use of position vector

(ii)

M1: Uses a correct method to find distance AX or distance AX^2 using A and their X.

Award if a correct method is seen for two of the three coordinates

Usually look for an attempt at $(-5-4\lambda)^2 + (5-3\lambda)^2 + (-5+5\lambda)^2$ with their λ.

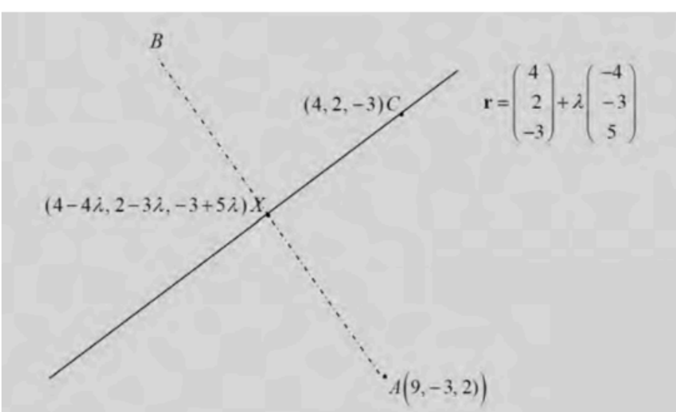
A1: $\sqrt{67}$ Note that $d = \sqrt{67}$ so $d = 67$ is quite common. This is fine as d is the constant given in the question

(b)

M1: Uses a correct method to find B.

The method can be implied by two correct coordinates for their coordinates for X and A.

A1: Correct position vector for B. $-\frac{21}{5}\mathbf{i} + \frac{23}{5}\mathbf{j} - 4\mathbf{k}$. Condone use of coordinates



Note that it is possible (though unlikely) to attempt this questions via scalar product using CA and CX.

FYI $\left|CX\right| = \left|CA\right|\cos\theta = \frac{\overrightarrow{CA}.\overrightarrow{CX}}{\left|CX\right|} = 2\sqrt{2}$

Hence $\lambda = \frac{2\sqrt{2}}{\sqrt{50}} = \frac{2}{5}$

17.Oct 2020-8

8 (a)	$\begin{pmatrix}4\\-3\\2\end{pmatrix}+\lambda\begin{pmatrix}3\\-2\\-1\end{pmatrix}=\begin{pmatrix}2\\0\\-9\end{pmatrix}+\mu\begin{pmatrix}2\\-1\\-3\end{pmatrix}\Rightarrow \begin{matrix}4+3\lambda=2+2\mu\\-3-2\lambda=0-1\mu\\2-1\lambda=-9-3\mu\end{matrix}$ **any two** of these	M1
	Full method to find either λ **or** μ eg (1) +3(3) $\Rightarrow \mu=\ldots$	M1
	Either $\lambda=-4$ or $\mu=-5$	A1
	Position vector of intersection is $\begin{pmatrix}4\\-3\\2\end{pmatrix}-4\begin{pmatrix}3\\-2\\-1\end{pmatrix}=$ OR $\begin{pmatrix}2\\0\\-9\end{pmatrix}-5\begin{pmatrix}2\\-1\\-3\end{pmatrix}=$	dM1
	$=\begin{pmatrix}-8\\5\\6\end{pmatrix}$	A1
		(5)
(b)	Co-ordinates or position vector of point $Q=\begin{pmatrix}2+2\mu\\0-1\mu\\-9-3\mu\end{pmatrix}$	
	$\overrightarrow{PQ}=\begin{pmatrix}2+2\mu\\0-1\mu\\-9-3\mu\end{pmatrix}-\begin{pmatrix}10\\-7\\0\end{pmatrix}=\begin{pmatrix}2\mu-8\\7-1\mu\\-9-3\mu\end{pmatrix}$	M1
	Uses $\overrightarrow{PQ}.\begin{pmatrix}2\\-1\\-3\end{pmatrix}=0\Rightarrow 4\mu-16-7+\mu+27+9\mu=0\Rightarrow \mu=-\frac{2}{7}$	dM1 A1
	Substitutes their μ into $\begin{pmatrix}2+2\mu\\0-1\mu\\-9-3\mu\end{pmatrix}\Rightarrow Q=\left(\frac{10}{7},\frac{2}{7},-\frac{57}{7}\right)$	ddM1 A1
		(5)
		(10 marks)

EDUCATION
(KHDA APPROVED)

(a)

M1: For writing down **any two equations** that give the coordinates of the point of intersection.

Accept two of $4+3\lambda=2+2\mu$, $-3-2\lambda=0-1\mu$, $2-1\lambda=-9-3\mu$

There must be an attempt to set the coordinates equal but condone one slip.

Setting the vectors equal to each other is insufficient for this but correct calculations will imply this.

M1: A full method to find **either** λ **or** μ. There are a few answers with limited or no working.

In such cases this M mark can be implied only if **either** λ **or** μ are correct for their equations.

A1: Either value correct $\mu=-5$ or $\lambda=-4$. Correct value(s) following correct equations implies M1 A1

dM1: Substitutes their value of λ into l_1 to find the coordinates or position vector of the point of intersection. It is dependent upon having scored second method mark. Alternatively substitutes their value of μ into l_2 to find the coordinates or position vector of the point of intersection. It can be implied in cases where there is limited working by two correct coordinates for their λ **or** μ.

A1: Either $\begin{pmatrix}-8\\5\\6\end{pmatrix}$ or $-8\mathbf{i}+5\mathbf{j}+6\mathbf{k}$ but not the coordinate $(-8, 5, 6)$

However ISW after sight of the correct **vector****

(b)

M1: States or uses a general point on $l_2=\begin{pmatrix}2+2\mu\\0-1\mu\\-9-3\mu\end{pmatrix}$ and attempt $\overrightarrow{PQ}=\begin{pmatrix}2+2\mu\\0-1\mu\\-9-3\mu\end{pmatrix}-\begin{pmatrix}10\\-7\\0\end{pmatrix}=\begin{pmatrix}2\mu-8\\7-1\mu\\-9-3\mu\end{pmatrix}$ either

way around with their general point. Condone slips

dM1: Uses the scalar product with fact that $\overrightarrow{PQ}$ and $\begin{pmatrix}2\\-1\\-3\end{pmatrix}$ are perpendicular vectors to find μ

way around with their general point. Condone slips

dM1: Uses the scalar product with fact that $\overrightarrow{PQ}$ and $\begin{pmatrix}2\\-1\\-3\end{pmatrix}$ are perpendicular vectors to find μ

See scheme but again condone slips.

Alternatively uses the scalar product with fact that $\overrightarrow{PQ}$ and $\overrightarrow{XQ}$ are perpendicular vectors to find μ

$$\overrightarrow{PQ}.\overrightarrow{XQ}=0\Rightarrow(2\mu-8)(2\mu+10)+(7-\mu)(-\mu-5)+(-9-3\mu)(-15-3\mu)=0\Rightarrow\mu=\left(-\frac{2}{7}\right)$$

A1: $\mu=-\frac{2}{7}$

ddM1: Uses their μ to find the coordinates of Q.

If no method is seen imply by two correct coordinates for their μ

A1: $Q=\left(\frac{10}{7},\frac{2}{7},-\frac{57}{7}\right)$ but not (**) $\frac{10}{7}\mathbf{i}+\frac{2}{7}\mathbf{j}-\frac{57}{7}\mathbf{k}$ o.e

** Only penalise incorrect notation once, the first time that it occurs. **

Alt using distances

(b)	Co-ordinates or position vector of point $Q = \begin{pmatrix} 2+2\mu \\ 0-1\mu \\ -9-3\mu \end{pmatrix}$	
	$\overrightarrow{PQ} = \begin{pmatrix} 2+2\mu \\ 0-1\mu \\ -9-3\mu \end{pmatrix} - \begin{pmatrix} 10 \\ -7 \\ 0 \end{pmatrix} = \begin{pmatrix} 2\mu-8 \\ 7-1\mu \\ -9-3\mu \end{pmatrix}$	M1
	Uses Pythagoras with $PQ^2 + QX^2 = PX^2 \Rightarrow 28\mu^2 + 148\mu + 544 = 504 \Rightarrow \mu = \cancel{5}, -\frac{2}{7}$	dM1 A1
	Substitutes their μ into $\begin{pmatrix} 2+2\mu \\ 0-1\mu \\ -9-3\mu \end{pmatrix} \Rightarrow Q = \left(\frac{10}{7}, \frac{2}{7}, -\frac{57}{7}\right)$	ddM1 A1

Alt using minimum distance and differentiation

(b)	Co-ordinates or position vector of point $Q = \begin{pmatrix} 2+2\mu \\ 0-1\mu \\ -9-3\mu \end{pmatrix}$	
	$\overrightarrow{PQ} = \begin{pmatrix} 2+2\mu \\ 0-1\mu \\ -9-3\mu \end{pmatrix} - \begin{pmatrix} 10 \\ -7 \\ 0 \end{pmatrix} = \begin{pmatrix} 2\mu-8 \\ 7-1\mu \\ -9-3\mu \end{pmatrix}$	M1
	Uses Pythagoras with $PQ^2 = (2\mu-8)^2 + (7-\mu)^2 + (-9-3\mu)^2$ $\frac{d}{d\mu} PQ^2 = 0 \Rightarrow 4(2\mu-8) - 2(7-\mu) - 6(-9-3\mu) = 0 \Rightarrow \mu = -\frac{2}{7}$	dM1 A1
	Substitutes their μ into $\begin{pmatrix} 2+2\mu \\ 0-1\mu \\ -9-3\mu \end{pmatrix} \Rightarrow Q = \left(\frac{10}{7}, \frac{2}{7}, -\frac{57}{7}\right)$	ddM1 A1