
Concepto

Racionalizar una fracción radical con alguna raíz en el denominador es transformarla en otra fracción equivalente pero que NO tenga ninguna raíz en el denominador. Veamos varios casos que se pueden presentar:

Racionalización con denominadores monómicos

(A) *Racionalización con denominadores del tipo* $A\sqrt{B}$.

Ejemplo:

$$\frac{5a}{3\sqrt{b}} \xrightarrow{\text{Se multiplica por } \sqrt{b}} \frac{5a\ .\ \sqrt{b}}{3\sqrt{b}\ .\ \sqrt{b}} = \frac{5a\ \sqrt{b}}{3\left(\sqrt{b}\right)^2} = \frac{5a\ \sqrt{b}}{3b}$$

(B) *Racionalización de fracciones con denominadores del tipo* $A\ \sqrt[n]{B}$

Ejemplos:

1) $$\frac{2x}{3\ \sqrt[5]{y^2}} \xrightarrow{\text{Multiplico por } \sqrt[5]{y^3}} \frac{2x}{3\ \sqrt[5]{y^2}} . \frac{\sqrt[5]{y^3}}{\sqrt[5]{y^3}} = \frac{2x\ \sqrt[5]{y^3}}{3\ \sqrt[5]{y^5}} = \frac{2x\ \sqrt[5]{y^3}}{3y}$$

2) $$\frac{1}{\sqrt[4]{3^7}} \xrightarrow{\text{Multiplico por } \sqrt[4]{3^{-3}}} \frac{1}{\sqrt[4]{3^7}} . \frac{\sqrt[4]{3^{-3}}}{\sqrt[4]{3^{-3}}} = \frac{\sqrt[4]{3^{-3}}}{\sqrt[4]{3^4}} = \frac{\sqrt[4]{3^{-3}}}{3}$$

Actividad resuelta

Racionaliza:

a) $$\frac{2+\sqrt{5}}{2\sqrt{5}} \rightarrow \frac{2+\sqrt{5}}{2\sqrt{5}} . \frac{\sqrt{5}}{\sqrt{5}} = \frac{2\sqrt{5}+\sqrt{5^2}}{2\sqrt{5^2}} = \frac{2\sqrt{5}+5}{2.5} = \frac{2\sqrt{5}+5}{10}$$

b) $$\frac{12}{5\ \sqrt[4]{27}} \rightarrow \frac{12}{5\ \sqrt[4]{3^3}} = \frac{12}{5\ \sqrt[4]{3^3}} . \frac{\sqrt[4]{3}}{\sqrt[4]{3}} = \frac{12\ \sqrt[4]{3}}{5\ \sqrt[4]{3^4}} = \frac{12\ \sqrt[4]{3}}{5.3} = \frac{12\ \sqrt[4]{3}}{15} \xrightarrow{:3} \frac{4\ \sqrt[4]{3}}{5}$$

Actividades

Para racionalizar $\frac{7}{2\ \sqrt{5}}$, ¿por qué tendrías que multiplicar numerador y denominador?

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Racionaliza:

$$\frac{1}{\sqrt{3}}$$

$$\frac{2\sqrt{3}-3\sqrt{2}}{\sqrt{2}}$$

$$\frac{2+\sqrt{5}}{2\sqrt{5}}$$

$$\frac{7+\sqrt{5}}{5\sqrt{5}}$$

$$\frac{6}{\sqrt[4]{3^3}}$$

$$\frac{4}{\sqrt[5]{2^3}}$$

$$\frac{12}{5\sqrt[4]{27}}$$

$$\frac{3}{\sqrt[5]{11^2}}$$

b) $\frac{2}{\sqrt[5]{x^3}}$

c) $\frac{1}{\sqrt[3]{a}}$

b) $\frac{-1}{\sqrt[4]{b^3}}$

$$\frac{6y}{5\sqrt{3y}}$$

$$\frac{8y^3}{6\sqrt{y}}$$

$$\frac{8y}{6\sqrt{y}}$$

$$\frac{3x}{2\sqrt{x}}$$

$$\frac{18}{12\cdot\sqrt{x}}$$

$$\frac{5a}{3\sqrt{b}}$$

b) $\frac{a}{3\sqrt{a}}$

$\frac{a+2}{3\sqrt{a+2}}$

$\frac{x+1}{\sqrt{x}}$

$\frac{\sqrt{a}}{\sqrt{b}}-\frac{1}{\sqrt{a}}+\frac{1}{\sqrt{b}}-\frac{\sqrt{b}}{\sqrt{a}}$

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Racionalización de fracciones con denominadores binómicos con alguna raíz cuadrada

En el denominador hay suma/resta de dos términos en los que aparece alguna raíz cuadrada

Por ejemplo, si la fracción es del tipo $\frac{N}{A\sqrt{B}+C\sqrt{D}}$ se multiplica numerador y denominador por $A\sqrt{B}-C\sqrt{D}$ y se llega así a una fracción sin raíz en el denominador: $\frac{N}{A\sqrt{B}+C\sqrt{D}}.\frac{A\sqrt{B}-C\sqrt{D}}{A\sqrt{B}-C\sqrt{D}}=$

$$\frac{N.(A\sqrt{B}-C\sqrt{D})}{(A\sqrt{B}+C\sqrt{D}).(A\sqrt{B}-C\sqrt{D})}=\frac{N.(A\sqrt{B}-C\sqrt{D})}{(A\sqrt{B})^2-(C\sqrt{D})^2}=\frac{N.(A\sqrt{B}-C\sqrt{D})}{A^2B-C^2D}$$

Si el denominador fuese una expresión del tipo $A\sqrt{B}-C\sqrt{D}$, se multiplicaría por $A\sqrt{B}+C\sqrt{D}$

Las expresiones $A\sqrt{B}+C\sqrt{D}$ y $A\sqrt{B}-C\sqrt{D}$ se dice que son conjugadas

Ejemplos:

1) $$\frac{7}{3-\sqrt{2}}\xrightarrow{\text{Multiplico por } 3+\sqrt{2} \text{ (expresión conjugada)}}\frac{7}{3-\sqrt{2}}.\frac{3+\sqrt{2}}{3+\sqrt{2}}=\frac{7(3+\sqrt{2})}{3^2-(\sqrt{2})^2}=\frac{7(3+\sqrt{2})}{7}=3+\sqrt{2}$$

2) $$\frac{\sqrt{15}}{3\sqrt{5}+5\sqrt{3}}\xrightarrow{\text{Se multiplica por } (3\sqrt{5}-5\sqrt{3})}\frac{\sqrt{15}.(3\sqrt{5}-5\sqrt{3})}{(3\sqrt{5}+5\sqrt{3}).(3\sqrt{5}-5\sqrt{3})}=\frac{3\sqrt{75}-5\sqrt{45}}{(3\sqrt{5})^2-(5\sqrt{3})^2}=\frac{15\sqrt{3}-15\sqrt{5}}{45-75}=\frac{15(\sqrt{3}-\sqrt{5})}{-30}$$

Simplificando se obtiene: $\frac{-15(\sqrt{3}-\sqrt{5})}{30}=\frac{-(\sqrt{3}-\sqrt{5})}{2}=\frac{\sqrt{5}-\sqrt{3}}{2}$

3) $$\frac{2}{1+\sqrt{3}+\sqrt{2}}=\frac{2\left[(1+\sqrt{3})-\sqrt{2}\right]}{\left[(1+\sqrt{3})+\sqrt{2}\right]\left[(1+\sqrt{3})-\sqrt{2}\right]}=\frac{2+2\sqrt{3}-2\sqrt{2}}{1+2\sqrt{3}+3-2}=\frac{2+2\sqrt{3}-2\sqrt{2}}{2+2\sqrt{3}}=$$

$$=\frac{1+\sqrt{3}-\sqrt{2}}{1+\sqrt{3}}=\frac{(1+\sqrt{3}-\sqrt{2})(1-\sqrt{3})}{(1+\sqrt{3})(1-\sqrt{3})}=\frac{2+\sqrt{2}-\sqrt{6}}{2}$$

Actividad resuelta

Racionaliza:

c) $\dfrac{\sqrt{3}+\sqrt{2}}{3\sqrt{2}-2\sqrt{3}} \rightarrow$

$$\frac{\sqrt{3}+\sqrt{2}}{3\sqrt{2}-2\sqrt{3}}\cdot\frac{3\sqrt{2}+2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}}=\frac{3\sqrt{6}+2\sqrt{3^2}+3\sqrt{2^2}+2\sqrt{6}}{(3\sqrt{2})^2-(2\sqrt{3})^2}=\frac{3\sqrt{6}+2.3+3.2+2\sqrt{6}}{18-12}=\boxed{\frac{12+5\sqrt{6}}{6}}$$

d) $\dfrac{2\sqrt{x}+1}{\sqrt{x}-2} \rightarrow \dfrac{2\sqrt{x}+1}{\sqrt{x}-2}\cdot\dfrac{\sqrt{x}+2}{\sqrt{x}+2}=\dfrac{(2\sqrt{x}+1)(\sqrt{x}+2)}{(\sqrt{x})^2-2^2}=\dfrac{2(\sqrt{x})^2+4\sqrt{x}+\sqrt{x}+2}{x-4}=\boxed{\dfrac{2x+5\sqrt{x}+2}{x-4}}$

Actividades

Encuentra el resultado de la siguiente suma:

$$\frac{1}{\sqrt{1}+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{4}}+\cdots+\frac{1}{\sqrt{99}+\sqrt{100}}$$

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c) $\dfrac{3\sqrt{5}+2\sqrt{3}}{2\sqrt{5}+\sqrt{3}}$ (racionalizando)

c) $\dfrac{3\sqrt{5}+2\sqrt{3}}{2\sqrt{5}+\sqrt{3}}\cdot\dfrac{2\sqrt{5}-\sqrt{3}}{2\sqrt{5}-\sqrt{3}}=\dfrac{6\sqrt{5^2}-3\sqrt{15}+4\sqrt{15}-2\sqrt{3^2}}{(2\sqrt{5})^2-(\sqrt{3})^2}=\dfrac{30+\sqrt{15}-6}{20-3}=\dfrac{24+\sqrt{15}}{17}$

c) $\dfrac{2\sqrt{3}+\sqrt{2}}{2\sqrt{3}-\sqrt{2}}$ (racionalizando)

c) $\dfrac{2\sqrt{3}+\sqrt{2}}{2\sqrt{3}-\sqrt{2}}\cdot\dfrac{2\sqrt{3}+\sqrt{2}}{2\sqrt{3}+\sqrt{2}}=\dfrac{4\sqrt{3^2}+2\sqrt{6}+2\sqrt{6}+\sqrt{2^2}}{(2\sqrt{3})^2-(\sqrt{2})^2}=\dfrac{12+4\sqrt{6}+2}{12-2}=\dfrac{14+4\sqrt{6}}{10}\xrightarrow{:2}\dfrac{7+2\sqrt{6}}{5}$

c) $\dfrac{\sqrt{6}}{2\sqrt{3}-\sqrt{18}}$ (racionalizando)

c) $\dfrac{\sqrt{6}}{2\sqrt{3}-3\sqrt{2}}\cdot\dfrac{2\sqrt{3}+3\sqrt{2}}{2\sqrt{3}+3\sqrt{2}}=\dfrac{2\sqrt{18}+3\sqrt{12}}{(2\sqrt{3})^2-(3\sqrt{2})^2}=\dfrac{6\sqrt{2}+6\sqrt{3}}{12-18}=\dfrac{6(\sqrt{2}+\sqrt{3})}{-6}=-(\sqrt{2}+\sqrt{3})=-\sqrt{3}-\sqrt{2}$

d) $\dfrac{5\sqrt{2}+\sqrt{3}}{5\sqrt{2}-\sqrt{3}}$ (racionalizando)

Resolución

$$\frac{5\sqrt{2}+\sqrt{3}}{5\sqrt{2}-\sqrt{3}}\cdot\frac{5\sqrt{2}+\sqrt{3}}{5\sqrt{2}+\sqrt{3}}=\frac{25\sqrt{2^2}+5\sqrt{6}+5\sqrt{6}+\sqrt{3^2}}{(5\sqrt{2})^2-(\sqrt{3})^2}=\frac{53+10\sqrt{6}}{50-3}=\boxed{\frac{53+10\sqrt{6}}{47}}$$

d) $\dfrac{2-\sqrt{5}}{3+\sqrt{5}}$ (racionalizando)

Resolución

$$\frac{2-\sqrt{5}}{3+\sqrt{5}}\cdot\frac{3-\sqrt{5}}{3-\sqrt{5}}=\frac{6-2\sqrt{5}-3\sqrt{5}+\sqrt{5^2}}{3^2-(\sqrt{5})^2}=\frac{6-2\sqrt{5}-3\sqrt{5}+5}{9-5}=\boxed{\frac{11-5\sqrt{5}}{4}}$$

d) $\dfrac{3\sqrt{7}+\sqrt{2}}{\sqrt{7}-3\sqrt{2}}$ (racionalizando)

Resolución

$$\frac{3\sqrt{7}+\sqrt{2}}{\sqrt{7}-3\sqrt{2}}\cdot\frac{\sqrt{7}+3\sqrt{2}}{\sqrt{7}+3\sqrt{2}}=\frac{3\sqrt{7^2}+9\sqrt{14}+\sqrt{14}+3\sqrt{2^2}}{(\sqrt{7})^2-(3\sqrt{2})^2}=\frac{21+10\sqrt{14}+6}{7-18}=\boxed{\frac{27+10\sqrt{14}}{-11}=\frac{-27-10\sqrt{14}}{11}}$$

VARIOS

Racionaliza y simplifica: a) $\dfrac{2\sqrt{72}-3\sqrt{50}+4\sqrt{32}+2\sqrt{98}}{\frac{54}{25}\sqrt{\frac{25}{2}}}$ b) $\dfrac{\sqrt{a\sqrt{b}}-\sqrt{b\sqrt{a}}}{\sqrt{\sqrt{a}}}$

c) $\dfrac{bc}{\sqrt{a}\cdot\sqrt[4]{b}\cdot\sqrt[8]{c}}$ d) $\dfrac{\sqrt{6}}{2\sqrt{3}-\sqrt{18}}$ e) $\dfrac{3\sqrt{192}}{\left(\sqrt{6}-3\sqrt{2}\right)^2}$ f) $\dfrac{\left(\sqrt{6}-2\sqrt{3}\right)^2}{3\sqrt{8}+2-\sqrt{18}}$ g) $\dfrac{2\sqrt{72}-3\sqrt{24}}{\sqrt{48}-5\left(3-\sqrt{3}\right)}$

Solución: a) 5 b) $\sqrt[4]{ab}-\sqrt{b}$ c) $\dfrac{\sqrt[8]{a^4b^6c^7}}{a}$ d) $-\sqrt{2}-\sqrt{3}$ e) $4\sqrt{3}+6$ f) $\dfrac{39\sqrt{2}-54}{7}$ g) $\sqrt{6}+\sqrt{2}$

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$$\frac{\left(2\sqrt{24}-\sqrt{48}\right)\left(\sqrt{6}+\sqrt{3}\right)}{\frac{7\sqrt{3}\cdot 4\sqrt{12}}{14}}$$

$$\frac{\left(2\sqrt{54}-6\sqrt{3}\right)\left(\sqrt{6}+\sqrt{3}\right)}{\frac{2\sqrt{3}\cdot\sqrt{75}}{5}}$$

$$\frac{\sqrt{2}+\sqrt{\frac{9}{8}}-\sqrt{\frac{1}{2}}}{\frac{\sqrt{2}}{8}}$$

$$\frac{3\sqrt{15}-4\sqrt{\frac{3}{5}}+\sqrt{60}}{2\sqrt{\frac{3}{20}}}$$

$$\frac{\sqrt{10} - \sqrt{\frac{8}{5}} + \sqrt{40}}{\frac{1}{\sqrt{10}}}$$

$$\frac{\frac{3}{4}\sqrt{6} - 4\sqrt{\frac{27}{32}} + 5\sqrt{\frac{75}{2}}}{\sqrt{\frac{3}{8}}}$$

$$\frac{\left(3\sqrt{20} + 2\sqrt{27}\right)\left(\sqrt{5} - \sqrt{3}\right)}{\frac{1}{4}\sqrt{6}\ \frac{2}{5}\sqrt{150}}$$

$$\frac{9\sqrt{72} - 3\sqrt{18} + 12\sqrt{98}}{\sqrt{\frac{9}{2}}}$$

$$\frac{6\sqrt{18} - 3\sqrt{72} + 9\sqrt{98}}{\sqrt{\frac{9}{2}}}$$

$$\frac{2\sqrt{72} - 3\sqrt{50} + 4\sqrt{32} + 2\sqrt{98}}{\frac{54}{25}\sqrt{\frac{25}{2}}}$$

$$\frac{2\sqrt{128} - 3\sqrt{32} + 5\sqrt{50} - 2\sqrt{2}}{\sqrt{\frac{81}{2}}}$$

$$\frac{9\sqrt{12} + 15\sqrt{48} + 6\sqrt{75} + 2\sqrt{3}}{\sqrt{\frac{121}{3}}}$$

$$\frac{4\sqrt{80} - \sqrt{20} + 5\sqrt{125} - 5\sqrt{5}}{17\sqrt{\frac{1}{5}}}$$

$$\frac{\left(\sqrt{3} + \sqrt{2}\right)^3 - \left(\sqrt{3} - \sqrt{2}\right)^3}{11\sqrt{2}}$$

$$\frac{\left(\sqrt{2} + 2\right)^3 - \left(2 - \sqrt{2}\right)^3}{7\sqrt{2}}$$

$$\frac{\left(5 + \sqrt{3}\right)^3 - \left(5 - \sqrt{3}\right)^3}{6\sqrt{3}}$$

$$\frac{\left(2\sqrt{48} + 3\sqrt{75} - 2\sqrt{27}\right) \cdot 8\sqrt{2}}{2\sqrt{96}}$$

$$\frac{\left(3\sqrt{24} + 2\sqrt{150}\right) \cdot 2\sqrt{3}}{48\sqrt{2}}$$

$$\frac{\left(3\sqrt{80} + 5\sqrt{180}\right) \cdot 2\sqrt{3}}{\sqrt{735}}$$

$$\frac{\left(3\sqrt{6} - \sqrt{24} + 2\sqrt{54}\right) \cdot 8\sqrt{3}}{\sqrt{288}}$$

$$\frac{\left(2\sqrt{3} + 3\sqrt{2}\right)\left(\sqrt{3} - \sqrt{2}\right)}{\sqrt{6}}$$

$$\frac{\left(2\sqrt{45} + \frac{3}{2}\sqrt{72}\right) \cdot \left(2\sqrt{5} - 3\sqrt{2}\right) \cdot 10\sqrt{5}}{2\sqrt{180}}$$

$$\frac{\left(2\sqrt{2} + 5\sqrt{3}\right)^2 - \left(5\sqrt{3} - 2\sqrt{2}\right)^2}{2\sqrt{24}}$$

$$\frac{\left(2\sqrt{150} + 2\sqrt{8}\right)\left(5\sqrt{6} - 2\sqrt{2}\right) \cdot \sqrt{2}}{\sqrt{10082}}$$

$$\frac{\left(2\sqrt{3} + 5\sqrt{6}\right)\left(5\sqrt{3} - \sqrt{6}\right)}{\sqrt{18}}$$

$$\frac{5}{\sqrt[5]{x^2}\sqrt[3]{y}}$$

$$\frac{\sqrt{a\sqrt{b}} - \sqrt{b\sqrt{a}}}{\sqrt{\sqrt{a}}}$$

$$\frac{bc}{\sqrt{a}\cdot\sqrt[4]{b}\cdot\sqrt[8]{c}}$$

$$\frac{2}{\sqrt[3]{x}}$$

$$\frac{a}{5\sqrt[5]{a^2}}$$

$$\frac{10(x+y)}{5\sqrt[3]{(x+y)}}$$

¿Cuál es el conjugado de $7 + \sqrt{6}$? ¿ Y de $\sqrt{8} - \sqrt{3}$?

¿A qué es igual $(\sqrt{5} - \sqrt{2}).(\sqrt{5} + \sqrt{2})$?

¿La racionalización de la expresión $\frac{a}{b-\sqrt{c}}$ resulta $\frac{a(\sqrt{c}+b)}{b^2-c}$?

Racionaliza:

$$\frac{\sqrt{2}+\sqrt{5}}{3\sqrt{5}-\sqrt{2}}$$

$$\frac{\sqrt{32}}{3-\sqrt{8}}$$

f) $\frac{9}{\sqrt{12} - \sqrt{3}}$

d) $\frac{\sqrt{15}}{3\sqrt{5}+5\sqrt{3}}$

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h) $\dfrac{\sqrt{6}}{2\sqrt{3}-\sqrt{18}}$

$\dfrac{6-\sqrt{5}}{3-\sqrt{5}}$

$\dfrac{\sqrt{6}-\sqrt{5}}{\sqrt{6}+\sqrt{5}}$

$\dfrac{3+\sqrt{5}}{\sqrt{5}-2}$

$\dfrac{\sqrt{5}}{2-3\sqrt{3}}$

$\dfrac{7-\sqrt{2}}{2\sqrt{2}-3}$

$\dfrac{\sqrt{600}}{5\sqrt{48}-10}$

$\dfrac{4\sqrt{3}-3\sqrt{2}}{4\sqrt{3}+3\sqrt{2}}$

$\dfrac{\sqrt{240}+\sqrt{60}}{2\sqrt{45}-\sqrt{12}}$

$\dfrac{\sqrt{288}-\sqrt{243}+2\sqrt{48}}{5\sqrt{27}-\sqrt{108}}$

$\dfrac{\sqrt{2}\sqrt{10}+5\sqrt{45}}{4\sqrt{18}-\sqrt{8}}$

$\dfrac{2\sqrt{6}}{\left(\sqrt{3}+1\right)^2}$

$\dfrac{\left(1-2\sqrt{2}\right)^2}{2\sqrt{3}-\sqrt{6}}$

$\dfrac{2\sqrt{10}}{\left(\sqrt{5}-1\right)^2}$

$\dfrac{3\sqrt{192}}{\left(\sqrt{6}-3\sqrt{2}\right)^2}$

- $\dfrac{\left[\left(4\sqrt{50}-3\sqrt{72}\right)\left(5\sqrt{2}+\sqrt{18}\right)\right]\cdot\sqrt{2}}{\sqrt{32}-\sqrt{8}}=$

$$\frac{\left(3\sqrt{6} + 5\sqrt{3}\right)\left(3\sqrt{54} - 3\sqrt{27}\right)}{1 + 2\sqrt{2}}$$

$$\frac{\left(2\sqrt{5} + 3\sqrt{10}\right)\left(\sqrt{10} - \sqrt{5}\right)}{4 - \sqrt{2}}$$

- $\dfrac{\left(4\sqrt{6} + 2\sqrt{3}\right)\left(3\sqrt{6} - 2\sqrt{3}\right)}{2\left(10 - \sqrt{2}\right)} =$

$$\frac{\sqrt{3}+5\sqrt{2}}{\sqrt{\sqrt{2}+\sqrt{3}}}$$

- $\dfrac{\sqrt{2} + \sqrt{3}}{\sqrt{2} - \sqrt{3}} \cdot \dfrac{\sqrt{3} - \sqrt{2}}{\sqrt{3} + \sqrt{2}} =$

$$\frac{2}{\sqrt{3}-1}+\frac{4}{\sqrt{3}}$$

$$\frac{2}{\sqrt{2}}-\frac{1+\sqrt{2}}{1-\sqrt{2}}$$

$$\frac{3}{\sqrt{2}}-\frac{1}{1-\sqrt{2}}+\frac{2}{\sqrt{2}+2}$$

$$\frac{2}{2 - \sqrt{3}} + \frac{3}{\sqrt{3} - \sqrt{2}} - \frac{3}{\sqrt{2} - 1}$$

$$\frac{3}{\sqrt{5} - \sqrt{2}} - \frac{1}{\sqrt{2} - 1} - \frac{4}{\sqrt{5} - 1}$$

$$\frac{2}{\sqrt{5}-\sqrt{3}} - \frac{2}{\sqrt{3}-1} - \frac{4}{\sqrt{5}-1}$$

$$\frac{3}{\sqrt{6}+\sqrt{3}} + \frac{2}{\sqrt{3}+1} - \frac{5}{\sqrt{6}+1}$$

$$\frac{5}{\sqrt{7}-\sqrt{2}} - \frac{1}{\sqrt{2}+1} - \frac{6}{\sqrt{7}+1}$$

$$\frac{2}{\sqrt{7}-\sqrt{5}} - \frac{3}{\sqrt{7}-2} - \frac{1}{\sqrt{5}-2}$$

$$\frac{3}{\sqrt{5}}-\frac{2\sqrt{2}+3}{1-\sqrt{2}}+\frac{\sqrt{5}+\sqrt{2}}{\sqrt{2}}$$

$$\frac{2\sqrt{162}-3\sqrt{75}+\sqrt{108}}{\sqrt{225}-2\sqrt{54}}$$

$$\frac{1}{\sqrt{2}+\sqrt{3}+\sqrt{5}}$$

$$\frac{\sqrt{24}}{3+\sqrt{27}-\sqrt{12}}$$

$$\frac{\left(\sqrt{6}-2\sqrt{3}\right)^2}{3\sqrt{8}+2-\sqrt{18}}$$

$$\frac{2\sqrt{72}-3\sqrt{24}}{\sqrt{48}-5\left(3-\sqrt{3}\right)}$$

$$\left(\frac{\sqrt{2}+1}{\sqrt{2}-1}-\frac{\sqrt{2}-1}{\sqrt{2}+1}\right):\sqrt{8}$$

$$\left(\frac{\sqrt{3}+2}{3-\sqrt{2}}-\frac{\sqrt{2}+\sqrt{3}}{3+\sqrt{2}}\right):\sqrt{18}$$

- $\left(\dfrac{\sqrt{6}\ +\ 1}{\sqrt{6}\ -\ 1} - \dfrac{\sqrt{6}\ -\ 1}{\sqrt{6}\ +\ 1} \right) \times \dfrac{5\sqrt{24}}{8} =$

- $\left(\dfrac{2\sqrt{6}\ +\ \sqrt{3}}{2\sqrt{6}\ -\ \sqrt{3}} - \dfrac{2\sqrt{6}\ -\ \sqrt{3}}{2\sqrt{6}\ +\ \sqrt{3}} \right) \times 14\sqrt{2} =$

- $\left(\dfrac{\sqrt{3}\ +\ 1}{\sqrt{3}\ -\ 1} - \dfrac{\sqrt{3}\ -\ 1}{\sqrt{3}\ +\ 1} \right)^2 : 6 =$

$\dfrac{2\sqrt{x}+1}{\sqrt{x}-2}$ soluc: $\dfrac{2\sqrt{x}+1}{\sqrt{x}-2} \cdot \dfrac{\sqrt{x}+2}{\sqrt{x}+2} = \dfrac{2(\sqrt{x})^2+4\sqrt{x}+\sqrt{x}+2}{(\sqrt{x})^2-2^2} = \dfrac{2x+5\sqrt{x}+2}{x-4}$

$\dfrac{5\sqrt{x}+1}{\sqrt{x}\ +\ 3}$

$\dfrac{4\sqrt{x}}{1-\sqrt{x}}$

$\dfrac{3\sqrt{x}+1}{2+\sqrt{x}}$

$\dfrac{3\sqrt{x}+5}{1+\sqrt{x}}$

d) $\dfrac{4\sqrt{x}}{1-\sqrt{x}}$

e) $\dfrac{3\sqrt{x}+1}{2+\sqrt{x}}$

c) $\dfrac{2\sqrt{x}}{x-2\sqrt{x}}$

$\dfrac{\sqrt{x}}{\sqrt{x}-\sqrt{x-1}}$

$\dfrac{\sqrt{a+b}}{\sqrt{a}+\sqrt{b}}$

$\dfrac{2\sqrt{x}-2\sqrt{y}}{2\sqrt{y}-2\sqrt{x}}$

$$\frac{\sqrt{a}+\sqrt{x}}{\sqrt{x}-\sqrt{a}}$$

$$\frac{x}{\sqrt{x}-\sqrt{2}}-\frac{x}{\sqrt{x}+\sqrt{2}}$$

$$\frac{3}{3+\sqrt{x}}-\frac{3-\sqrt{x}}{9-x}$$

$$\frac{3\sqrt{a}+4\sqrt{2a}}{3\sqrt{a}-4\sqrt{2a}}$$

$$\frac{x-y}{\sqrt{x}+\sqrt{y}}$$

$$\frac{1-\sqrt{x}}{\sqrt{x}+1}$$

$$\frac{\sqrt{a}}{\sqrt{a}+\sqrt{b}}+\frac{\sqrt{b}}{\sqrt{a}-\sqrt{b}}$$

$$\frac{\sqrt{a+b}}{\sqrt{a+b}-\sqrt{a-b}}$$

$$\frac{\sqrt{a}}{2-\sqrt{a}}$$

$$\frac{\sqrt{x}+\sqrt{y}}{\sqrt{x}-\sqrt{y}}$$

$$\frac{2}{\sqrt{a+1}+\sqrt{a-3}}$$

$$\frac{3a}{a+\sqrt{b}}$$

$$\frac{\sqrt{a}}{\sqrt{a}+\sqrt{b}}+\frac{\sqrt{b}}{\sqrt{a}-\sqrt{b}}$$

$$\frac{1}{1+\sqrt{2}+\sqrt{a}}$$

$$\left(1-\frac{1}{\sqrt{a}}\right).(a+\sqrt{a})$$

$$\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{b}}{\sqrt{a}}-\sqrt{ab}+\frac{1}{\sqrt{ab}}$$

$$\left(\sqrt{\frac{\sqrt{a}}{\sqrt{b}}}+\sqrt{\frac{\sqrt{c}}{\sqrt{d}}}\right)^2$$

$$\frac{(x+2)\sqrt[4]{(x+2)^3}}{\sqrt[3]{(x+2)}}$$

$$\frac{3}{\sqrt[3]{x}-2}$$

$$\frac{1}{\sqrt[4]{a}+\sqrt[4]{b}}$$

$$\frac{1}{\sqrt{m}-\sqrt[4]{n}}$$

$$\left(\left(\frac{a-\sqrt{b}}{\sqrt{a}-\sqrt[4]{b}}-\sqrt{a}\right):\sqrt{b}-\frac{a}{\sqrt{b}}\right)\cdot b$$

Racionaliza los siguientes denominadores.

a) $\frac{5}{2\sqrt{5}}$ c) $\frac{5}{2\sqrt{5}+1}$ e) $\frac{\sqrt{6}}{2\sqrt{3}-3\sqrt{2}}$

b) $\frac{5}{2\sqrt[4]{5}}$ d) $\frac{3}{\sqrt{3}-2\sqrt{6}}$ f) $\frac{2\sqrt{3}-3\sqrt{2}}{\sqrt{8}-\sqrt{27}}$

a) $\frac{5}{2\sqrt{5}}\xrightarrow[.\sqrt{5}]{.\sqrt{5}}\frac{5\sqrt{5}}{2\sqrt{5}\sqrt{5}}=\frac{\cancel{5}\sqrt{5}}{2.\cancel{5}}=\frac{\sqrt{5}}{2}$

b) $\frac{5}{2\sqrt[4]{5}}\xrightarrow[.\sqrt[4]{5^3}]{.\sqrt[4]{5^3}}\frac{5.\sqrt[4]{5^3}}{2\sqrt[4]{5}.\sqrt[4]{5^3}}=\frac{5\sqrt[4]{5^3}}{2\sqrt[4]{5^4}}=\frac{\cancel{5}\sqrt[4]{5^3}}{2.\cancel{5}}=\frac{\sqrt[4]{125}}{2}$

c) $\frac{5}{2\sqrt{5}+1}\xrightarrow[.(2\sqrt{5}-1)]{.(2\sqrt{5}-1)}\frac{5.(2\sqrt{5}-1)}{(2\sqrt{5}+1).(2\sqrt{5}-1)}=\frac{10\sqrt{5}-5}{(2\sqrt{5})^2-1^2}=\frac{10\sqrt{5}-5}{19}$

d) $\dfrac{3}{\sqrt{3}-2\sqrt{6}}\xrightarrow[.\,(\sqrt{3}+2\sqrt{6})]{.\,(\sqrt{3}+2\sqrt{6})}\dfrac{3.\,(\sqrt{3}+2\sqrt{6})}{(\sqrt{3}-2\sqrt{6}).(\sqrt{3}+2\sqrt{6})}=\dfrac{3.\,(\sqrt{3}+2\sqrt{6})}{(\sqrt{3})^2-(2\sqrt{6})^2}=\dfrac{3.\,(\sqrt{3}+2\sqrt{6})}{3-4.6}=\dfrac{-(\sqrt{3}+2\sqrt{6})}{7}$

e) $\dfrac{\sqrt{6}}{2\sqrt{3}-3\sqrt{2}}\xrightarrow[.(2\sqrt{3}+3\sqrt{2})]{.(2\sqrt{3}+3\sqrt{2})}\dfrac{\sqrt{6}.(2\sqrt{3}+3\sqrt{2})}{(2\sqrt{3}-3\sqrt{2}).(2\sqrt{3}+3\sqrt{2})}=\dfrac{2\sqrt{18}+3\sqrt{12}}{(2\sqrt{3})^2-(3\sqrt{2})^2}=\dfrac{2.\left(3\sqrt{2}\right)+3.\left(2\sqrt{3}\right)}{4.3-9.2}=\dfrac{6(\sqrt{2}+\sqrt{3})}{-6}=-\sqrt{2}-\sqrt{3}$

f) $\dfrac{2\sqrt{3}-3\sqrt{2}}{\sqrt{8}-\sqrt{27}}=\dfrac{2\sqrt{3}-3\sqrt{2}}{2\sqrt{2}-3\sqrt{3}}\xrightarrow[.\,(2\sqrt{2}+3\sqrt{3})]{.\,(2\sqrt{2}+3\sqrt{3})}\dfrac{(2\sqrt{3}-3\sqrt{2}).\,(2\sqrt{2}+3\sqrt{3})}{(2\sqrt{2}-3\sqrt{3}).\,(2\sqrt{2}+3\sqrt{3})}=\dfrac{4\sqrt{6}+6\sqrt{3^2}-6\sqrt{2^2}-9\sqrt{6}}{(2\sqrt{2})^2-(3\sqrt{3})^2}=\dfrac{6-3\sqrt{6}}{-19}=\dfrac{3\sqrt{6}-6}{19}$

Racionaliza los denominadores.

a) $\dfrac{a}{a\sqrt[6]{a^8}}$ d) $\dfrac{\sqrt{2}}{1+\sqrt{2}}$ g) $\dfrac{1}{1-\sqrt{2}-\sqrt{3}}$

b) $\dfrac{3y}{2\sqrt[5]{y^2}}$ e) $\dfrac{2\sqrt{6}}{\sqrt{3}-\sqrt{2}}$ h) $\dfrac{1+\sqrt{2}}{\sqrt{8}+\sqrt{3}+\sqrt{2}}$

c) $\dfrac{x+2}{2\sqrt{x+2}}$ f) $\dfrac{6\sqrt{6}}{2\sqrt{3}+3\sqrt{2}}$ i) $\dfrac{\sqrt{2}+\sqrt{3}}{\sqrt{2}+\sqrt{3}+\sqrt{6}}$

a) $\dfrac{1}{\sqrt[6]{a^8}}=\dfrac{1}{\sqrt[3]{a^4}}=\dfrac{1}{a\sqrt[3]{a}}\xrightarrow[.\sqrt[3]{a^2}]{.\sqrt[3]{a^2}}\dfrac{\sqrt[3]{a^2}}{a\sqrt[3]{a}.\sqrt[3]{a^2}}=\dfrac{\sqrt[3]{a^2}}{a\sqrt[3]{a^3}}=\dfrac{\sqrt[3]{a^2}}{a^2}$

b) $\dfrac{3y}{2\sqrt[5]{y^2}}\xrightarrow[.\sqrt[5]{y^3}]{.\sqrt[5]{y^3}}\dfrac{3y.\sqrt[5]{y^3}}{2\sqrt[5]{y^2}.\sqrt[5]{y^3}}=\dfrac{3y\sqrt[5]{y^3}}{2y}=\dfrac{3\sqrt[5]{y^3}}{2}$

c) $\dfrac{x+2}{2\sqrt{x+2}}\xrightarrow[.\sqrt{x+2}]{.\sqrt{x+2}}\dfrac{(x+2).\sqrt{x+2}}{2\sqrt{x+2}.\sqrt{x+2}}=\dfrac{\cancel{(x+2)}.\sqrt{x+2}}{2\cancel{(x+2)}}=\dfrac{\sqrt{x+2}}{2}$

d) $\dfrac{\sqrt{2}}{1+\sqrt{2}}\xrightarrow[.(1-\sqrt{2})]{.(1-\sqrt{2})}\dfrac{\sqrt{2}.(1-\sqrt{2})}{(1+\sqrt{2}).(1-\sqrt{2})}=\dfrac{\sqrt{2}-(\sqrt{2})^2}{1^2-(\sqrt{2})^2}=\dfrac{\sqrt{2}-2}{-1}=2-\sqrt{2}$

e) $\dfrac{2\sqrt{6}}{\sqrt{3}-\sqrt{2}}\xrightarrow[.(\sqrt{3}+\sqrt{2})]{.(\sqrt{3}+\sqrt{2})}\dfrac{2\sqrt{6}.(\sqrt{3}+\sqrt{2})}{(\sqrt{3}-\sqrt{2}).(\sqrt{3}+\sqrt{2})}=\dfrac{2\sqrt{18}+2\sqrt{12}}{(\sqrt{3})^2-(\sqrt{2})^2}=\dfrac{2.3\sqrt{2}+2.2\sqrt{3}}{1}=6\sqrt{2}+4\sqrt{3}$

f) $\dfrac{6\sqrt{6}}{2\sqrt{3}+3\sqrt{2}}\xrightarrow[.(2\sqrt{3}-3\sqrt{2})]{.(2\sqrt{3}-3\sqrt{2})}\dfrac{6\sqrt{6}.(2\sqrt{3}-3\sqrt{2})}{(2\sqrt{3}-3\sqrt{2}).(2\sqrt{3}-3\sqrt{2})}=\dfrac{6\sqrt{6}.(2\sqrt{3}-3\sqrt{2})}{(2\sqrt{3})^2-(3\sqrt{2})^2}=$

$=\dfrac{\cancel{6}\sqrt{6}.(2\sqrt{3}-3\sqrt{2})}{-\cancel{6}}=-2\sqrt{18}+3\sqrt{12}=-2.3\sqrt{2}+3.2\sqrt{3}=6(\sqrt{3}-\sqrt{2})$

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g) $\dfrac{1}{1-\sqrt{2}-\sqrt{3}} \xrightarrow[.(1-\sqrt{2}+\sqrt{3})]{.(1-\sqrt{2}+\sqrt{3})} \dfrac{1-\sqrt{2}+\sqrt{3}}{(1-\sqrt{2})^2-(\sqrt{3})^2} = \dfrac{1-\sqrt{2}+\sqrt{3}}{1+2-2\sqrt{2}-9.2} = \dfrac{1-\sqrt{2}+\sqrt{3}}{-15-2\sqrt{2}} \xrightarrow[.(-15+2\sqrt{2})]{.(-15+2\sqrt{2})}$

$= \dfrac{(1-\sqrt{2}+\sqrt{3}).(-15+2\sqrt{2})}{(-15-2\sqrt{2}).(-15+2\sqrt{2})} = \dfrac{-15+2\sqrt{2}+15\sqrt{2}-2.2-15\sqrt{3}+2\sqrt{6}}{(-15)^2-(2\sqrt{2})^2} = \dfrac{-19+17\sqrt{2}-15\sqrt{3}+2\sqrt{6}}{217}$

h) $\dfrac{1+\sqrt{2}}{2\sqrt{2}+\sqrt{3}+\sqrt{2}} = \dfrac{1+\sqrt{2}}{3\sqrt{2}+\sqrt{3}} \xrightarrow[.(3\sqrt{2}-\sqrt{3})]{.(3\sqrt{2}-\sqrt{3})} \dfrac{(1+\sqrt{2}).(3\sqrt{2}-\sqrt{3})}{(3\sqrt{2})^2-(\sqrt{3})^2} = \dfrac{3\sqrt{2}-\sqrt{3}+3.2-\sqrt{6}}{15} = \dfrac{6+3\sqrt{2}-\sqrt{3}-\sqrt{6}}{15}$

i) $\dfrac{\sqrt{2}+\sqrt{3}}{\sqrt{2}+\sqrt{3}+\sqrt{6}} \xrightarrow[.(\sqrt{2}+\sqrt{3}-\sqrt{6})]{.(\sqrt{2}+\sqrt{3}-\sqrt{6})} \dfrac{(\sqrt{2}+\sqrt{3}).(\sqrt{2}+\sqrt{3}-\sqrt{6})}{(\sqrt{2}+\sqrt{3})^2-(\sqrt{6})^2} = \dfrac{2+\sqrt{6}-2\sqrt{2}+\sqrt{6}+3-3\sqrt{2}}{2+3+2\sqrt{6}-6} = \dfrac{5+2\sqrt{6}-5\sqrt{2}}{2\sqrt{2}-1} \xrightarrow[.(2\sqrt{2}+1)]{.(2\sqrt{2}+1)}$

$= \dfrac{(5+2\sqrt{6}-5\sqrt{2}).(2\sqrt{2}+1)}{(2\sqrt{2})^2-1^2} = \dfrac{10\sqrt{2}+5+4.2\sqrt{3}+2\sqrt{6}-10.2-5\sqrt{2}}{7} = \dfrac{5\sqrt{2}-15+8\sqrt{3}+2\sqrt{6}}{7}$

$$\frac{1}{1-\dfrac{\sqrt{2}}{1+\sqrt{2}}} + \frac{1}{1+\dfrac{\sqrt{2}}{1-\sqrt{2}}}$$

$$\left\{ \frac{\left[\left(\dfrac{-2}{3}\right)^{-2}+4\right]^{1/2}}{\sqrt[3]{\left(\dfrac{8}{5}\right)^{-1}-4}} + 2\cdot 3^{-1} \right\}^{123}$$