# Analyzing Exponential and Logarithmic Models Writing Task

Natural logarithms and exponential functions appear throughout science, engineering, finance, and many other fields. The mathematical constant *e* serves as the base for natural exponential growth and decay models, while hyperbolic functions describe important physical phenomena like hanging cables.

In this assignment, you will analyze and compare different exponential and logarithmic models in the context of real-world applications. You'll use calculus tools to understand the behavior of these functions, make predictions, and evaluate their effectiveness in various scenarios.

## The Assignment

### Part 1: Exponential Growth Analysis

An ecological research team is studying two different bacterial populations in a laboratory. Population A grows according to the function $A(t) = 500e^{0.05t}$, while Population B grows according to the function $B(t) = 350e^{0.08t}$, where *t* is measured in hours.

1. Calculate the initial size of each bacterial population.
2. Find the rate of change of each population at *t* = 10 hours. What does this tell you about how the populations are growing at this point?
3. Determine when the two populations will be equal in size. Show your work and verify your answer graphically.
4. Calculate the doubling time for each population and explain the significance of this value in the context of bacterial growth.
5. If the lab can sustain a maximum of 15,000 bacteria, when will this capacity be reached for each population? Which population reaches capacity first?

### Part 2: Exponential Decay and Half-Life

A medical researcher is studying the breakdown of two different medications in the bloodstream. Medication C has a concentration that follows the model $C(t) = 25e^{-0.12t}$, and Medication D follows the model $D(t) = 25e^{-0.08t}$, where *t* is measured in hours.

1. Calculate the half-life of each medication, showing all your work.
2. What percentage of each medication remains in the bloodstream after 6 hours?

3. The therapeutic range for these medications requires a minimum concentration of 5 mg/L. Determine how long each medication remains effective.
4. Compare the rates of decay using derivatives, and explain how these rates change over time.
5. A doctor wants to maintain a minimum concentration of 8 mg/L for Medication D. If additional doses can be given every 8 hours, how much medication (in mg/L) should be administered with each dose? Assume the effects are additive.

**Part 3: Natural Logarithms and Inverse Relationships**

1. The pH of a solution is defined as pH = $-\log_{10}[H^+]$, where $[H^+]$ is the hydrogen ion concentration in moles per liter. Convert this to a natural logarithm expression.
2. If the pH of a solution changes from 6 to 4:
    a. What is the factor by which the hydrogen ion concentration increases?
    b. Find the derivative of pH with respect to $[H^+]$ and interpret its meaning.
3. The relationship between the intensity of sound (I) and the decibel level (β) is given by $\beta = 10\log_{10}(I/I_0)$, where $I_0$ is the reference intensity.
    a. Express this relationship using natural logarithms.
    b. If the decibel level increases by 20 decibels, by what factor does the intensity increase?
    c. Find the derivative dβ/dI and explain its significance.

**Part 4: Hyperbolic Functions Application**

An electrical power cable hangs between two towers that are 200 meters apart. The cable forms a catenary curve described by the function $h(x) = 40\cosh(\frac{x}{40}) - 30$, where *x* is the horizontal distance from the center point between the towers (in meters) and *h(x)* is the height of the cable (in meters).

1. Find the height of the cable at the center point (*x* = 0) and at the towers (*x* = ±100).
2. Calculate the slope of the cable at the towers using derivatives.
3. Find the total length of the cable using the arc length formula. (Hint: Use the property that the derivative of $\cosh(\frac{x}{40})$ is $(\frac{1}{40})\sinh(\frac{x}{40})$)
4. If the cable weighs 5 kg per meter, calculate the total weight of the cable.
5. Use the derivative of the function to find where the cable is horizontal and explain why this result makes physical sense.

**Part 5: Comparison and Reflection**

In 1-2 paragraphs, compare and contrast the behavior of exponential growth, exponential decay, and hyperbolic functions. Discuss:

1. How the derivatives of these functions relate to the original functions
2. Why *e* is particularly useful as a base for these models (as opposed to other bases)
3. The significance of horizontal asymptotes in these functions
4. How these mathematical models both illuminate and simplify the real-world phenomena they describe

In another 1-2 paragraphs, reflect on how calculus techniques (differentiation, integration, inverses) help us analyze and understand these models. Include at least three specific examples from the assignment where calculus provided insights that might not be immediately obvious from just looking at the functions.

## Your submission should include:

- All mathematical work shown clearly
- Correct use of appropriate calculus techniques
- Well-labeled graphs where appropriate
- Clear explanations of mathematical results in context
- Thoughtful comparisons and reflections that demonstrate understanding of both the mathematics and applications

This assignment is worth 20 points. Your work will be assessed on mathematical accuracy, proper application of calculus concepts, clarity of explanations, and the depth of your analysis and reflections.

## Rubric:

| Criteria | Proficient | Developing | Not Evident | Points |
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| **Exponential Growth Analysis** | Correctly calculates all growth parameters and accurately determines population intersections. Provides clear interpretation of doubling times and capacity limits with insightful connections to calculus concepts. | Calculations mostly correct with minor errors. Basic interpretation of results with some connections to calculus. May be missing deeper insights into exponential behavior. | Multiple significant errors in calculations. Weak or missing interpretations of results. Little connection to calculus concepts. | ___/4 |
| **Exponential Decay Analysis** | Accurately determines half-lives and decay characteristics with proper calculus techniques. Clear, insightful analysis of medication effectiveness with thorough explanation of rates of change. | Most calculations correct with some minor errors. Basic interpretation of pharmaceutical context. Some connection to calculus but analysis could be deeper. | Significant errors in half-life or concentration calculations. Minimal or incorrect interpretation of results. Poor connection to calculus concepts. | ___/4 |
| **Natural Logarithm Applications** | Correctly converts logarithm bases and accurately analyzes pH and decibel relationships. Derivatives calculated correctly with meaningful interpretations that show deep understanding. | Most conversions and calculations correct with minor errors. Basic interpretation of logarithmic relationships. Some connection to practical applications. | Significant errors in logarithm conversions or derivative calculations. Poor understanding of logarithmic relationships. Little connection to applications. | ___/4 |
| **Hyperbolic Functions Application** | Accurately models the catenary problem with correct calculations of heights, slopes, and arc length. Shows thorough understanding of hyperbolic functions and their physical significance. | Most catenary calculations correct with minor errors. Basic understanding of hyperbolic functions demonstrated. Some connection to physical reality of the cable. | Significant errors in catenary calculations. Poor understanding of hyperbolic functions. Weak connection to physical interpretation. | ___/4 |

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| **Comparison and Reflection** | Provides insightful comparison of function types with clear connections to derivatives, significance of e, asymptotic behavior, and modeling limitations. Reflection demonstrates deep understanding of how calculus illuminates these models with specific, well-explained examples. | Basic comparison of function types with some insights. Reflection shows understanding of calculus applications but may lack depth or specific examples. Some connections made but analysis could be more thorough. | Superficial or incorrect comparison of function types. Reflection shows minimal understanding of how calculus relates to these models. Few or no specific examples provided. | ___/4 |
| **Total** | | | | ___/20 |